end of that time will be only £50. 14*s.* 5*d.*: hence, the profit accruing to the office from him if he be then alive must be £94. 9*s.* 3*d.*

Suppose D, aged 35, assured for the same sum and for the same number of years at a premium of £29. 18*s.* 3*d.*, the amount of his contributions will be £199. 5*s.* 6*d.* and the value of his policy £77. 16*s.* 5*d.*: hence, in case he survive, the office must realize by him £121. 9*s.* 1*d.*

I therefore infer that the present values of the bonuses of these two persons must, in equity, be to each other as 94·462 is to 121·454. But in order to have the present values of these bonuses to each other in the above ratio, we must divide the former number by ·457, the present value of £1 at the death of A, who must then be 26 years old, and the latter by ·5451, the present value of £1 at D's death, who will then be 41 years old; the quotients will be $\frac{94{\cdot}462}{{\cdot}457} = 206{\cdot}7$, and $\frac{121{\cdot}454}{{\cdot}5451} = 222{\cdot}8$.

Let £66. 9*s.* 5*d.* be divided—

206·7
222·8
———
429·5 : 66·471 : : 206·7 : 31·989 = £31. 19*s.* 9*d.* for A's bonus; and taking this from £66. 9*s.* 5*d.* we get for D's, £34. 9*s.* 8*d.*

We notice, by the way, a characteristic of these two actuaries: the methodical manner of Milne, and the partiality of Davies for expressing his views in figures.

Yours very truly,

T. E. YOUNG.

1 *October* 1906.

THE VALUATION OF LIMITED-PAYMENT POLICIES.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—In July 1898, Herr Altenburger, in a letter to the *Journal*, described a method of valuing special class policies by which all those on lives of the same age might be grouped together and valued as whole-life policies, a correction being afterwards made, depending on the nature of each particular contract. The method is especially applicable to Limited Payment Policies, as it avoids the separation of the sums assured and premiums into different sections, if the policies are to be valued in groups. The general formula for age attained x is $V = A_x - Pa_x + \frac{P\mathbb{N}_{x+n}}{D_x}$ where P represents the limited payment premium, and $P\mathbb{N}_{x+n}$ is inserted as a constant at the outset in the valuation registers; so that the total of the column divided by D_x gives the value of the deferred premiums to be deducted from the value of the premiums treated as payable during the whole of life. I have recently had occasion to look into the method as applied to a valuation by the O^{M} and $O^{M(5)}$ Tables, and the results are given below, in case they may be of use to others.

It is desirable that only one set of factors should be used for multiplying the premiums and loadings, in order to avoid the labour of writing off the O^{M} and writing on the $O^{M(5)}$ constants after the policies have been five years in force; and it will be seen that this may be done with results which, for all practical purposes, are exact, as in the trial valuation appended the greatest error in any year does not amount to unity.

The trial valuation was made by the O^{M} Table at 3 per-cent interest and included only policies effected within the five years preceding the date of valuation. The premiums were assumed to be due eight months after the valuation date, so that the values of these were $P_{\frac{2}{3}|}\mathbf{a}_{x\overline{n}|}$, or, under Herr Altenburger's method, $P_{\frac{2}{3}|}\mathbf{a}_{x} - \dfrac{P\mathbb{N}_{x+n+\frac{2}{3}}}{D_x}$. The policies were tabulated according to age nearest birthday at 31 December, and the constants were obtained by multiplying the premiums and loadings by the $O^{M(5)}$ values of $\mathbb{N}_{x+n+\frac{2}{3}}$, $(x+n)$ being the age nearest birthday at the end of the calendar year of last payment.

To deal with the premiums only, the sums of these and of the corresponding constants were obtained at each age, and the sums of the constants were divided by the $O^{M(5)}$ D_x, giving the values of the deferred premiums. Each of these values was in turn divided by the corresponding sum of the premiums, which gave the average deferred annuities ${}_{n+\frac{2}{3}|}\mathbf{a}_x$, and these subtracted from ${}_{\frac{2}{3}|}\mathbf{a}_x$ gave the $O^{M(5)}$ temporary annuities, ${}_{\frac{2}{3}|}\mathbf{a}_{x\overline{n}|}$, by which the premiums might have been multiplied for an $O^{M(5)}$ valuation. The values of n were found by interpolation from a table of the $O^{M(5)}$ values of ${}_{\frac{2}{3}|}\mathbf{a}_{x\overline{n}|}$, and these same values of n were used in obtaining, from a similar table of the O^{M} values of ${}_{\frac{2}{3}|}\mathbf{a}_{x\overline{n}|}$, the annuities by the latter table of mortality by which the premiums were finally multiplied. The two interpolations necessary were taken by first differences to three decimal places. A separate calculation requires to be made for the values of n in the case of the loadings, as the incidence is generally different.

I have taken the case where the constants are calculated by the $O^{M(5)}$ Table, but the contrary will obviously hold, and the method affords a ready means of finding the increased reserve required for an O^{M} and $O^{M(5)}$ valuation, where an office makes an O^{M} valuation, and has the constants already calculated by that table.

Yours faithfully,

ALEX. FRASER.

Scottish Life Assurance Co.,
19, *St. Andrew Square, Edinburgh,*
7 *November* 1906.

Valuation of Premiums.

Age	Premiums	n	Approximate Value	Exact Value	Error
48	383·4	17·744	4519·5	4519·8	−·3
43	238·6	15·334	2658·5	2658·5	0
38	135·4	14·573	1486·0	1486·8	−·8
33	291·5	20·430	4093·2	4093·3	−·1
28	261·2	19·475	3612·7	3613·0	−·3

Valuation of Loadings.

Age	Loadings	n	Approximate Value	Exact Value	Error
48	64·9	17·916	769·4	769·4	0
43	46·5	15·318	517·7	517·7	0
38	26·3	15·328	299·2	299·5	−·3
33	66·9	20·681	946·6	946·5	+·1
28	64·3	19·617	893·6	894·1	−·5