TOPOLOGICAL SOLUTIONS FOR COLLIMATED MHD FLOWS

K. Tsinganos[(1)], E. Trussoni[(2)]

[(1)] *Department of Physics, University of Crete, Heraklion, Greece*
[(2)] *Istituto di Cosmo-geofisica del C.N.R., Torino, Italy*

Different astrophysical contexts (e.g. extragalactic jets, star forming regions) are related with collimated ouflows. An analysis of this phenomenology in principle would require a fully numerical treatment, however analytical investigations are also possible by assuming a suitable behaviour of streamlines. In this framework we investigate solutions of the hydrodynamic and MHD equations describing helicoidal collimated outflows from a central gravitating object.
a) *Collimated hydrodynamic flows.* To construct an analytical solution of the Navier-Stokes equations

$$\nabla \cdot (\rho \mathbf{v}) = 0 , \qquad \rho(\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla P - \frac{\rho G M}{r^2}\mathbf{r} \tag{1}$$

that could represent a collimated outflow with small mathematical complexity, we shall assume that the radial speed V_R, the density ρ, the azimuthal speed V_ϕ and the pressure P are separable in the radial ($R = r/r_o$) and latitudinal (θ) coordinates:

$$V_R(R,\theta) = V_o Y(R)\frac{\cos\theta}{g(\omega,\theta)}, \quad \rho(R,\theta) = \frac{\rho_o}{Y(R)R^2} g^2(\omega,\theta), \quad V_\phi(R,\theta) = \frac{V_1}{R}\frac{\sin\theta}{g(\omega,\theta)}$$

$$P(R,\theta) = \frac{1}{2}\rho_o V_o^2 [Q_o(R) + Q_1(R)\sin^2\theta], \qquad g(\omega,\theta) = [1+\omega\sin^2\theta]^{1/2} \tag{2}$$

Note that for large ω the flow is strongly collimated around the polar axis ($\theta = 0$). Substituting Eqs.(2) in the R and θ components of the momentum balance equation, the θ dependence is decoupled and the radial dependence of the radial flow speed is the solution to the following nonlinear equation (Tsinganos and Trussoni 1989),

$$\frac{dY}{dR} = \frac{Y}{R}\frac{\omega\nu^2 R - 6\lambda^2}{2Y^2R^2 + \lambda^2}, \qquad \lambda = \frac{V_1}{V_o}, \qquad \nu = \frac{V_{esc}}{V_o} \tag{3}$$

with the asymptotic behavior, for $R \to \infty$, $Y(R) \to [Y(R)_\infty - 1/R]^{1/2}$. In the region $R > 1$ $Y$is monotonically increasing: a strong acceleration is found above the base $R \approx 1$, while the asymptotic speed V_∞ increases with the collimation parameter ω. The topologies of the Mach number, $M(R,\theta) = V_R/V_S$, $V_S = (5P/3\rho)^{1/2}$, show

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three families of curves: (i) the wind Mach curve, with $M(R=1) \ll 1$ that crosses the line $M = 1$ and increases as $R^{1/2}$ at large R (the pressure $\rightarrow 0$ like $1/R$ at $R \rightarrow \infty$); (ii) the breeze curves, that start similarly with $M(R=1) \ll 1$ but for large R they drop as $1/R$ (with the pressure dropping to an asymptotic finite value); (iii) the terminated Mach curves, that reach asymptotically up to a distance R_{max} (where the pressure is zero). The topologies are very different from the classical Parker solution: the sonic point is not a singularity of the equations, and the critical point is shifted to infinity (Fig.1). This behaviour is related with the heating of the plasma consistent with the streamline pattern, as can be seen by analysing the radial dependence of the polytropic index $\Gamma = \partial(\ln P)/\partial(\ln \rho)$ at constant θ (Fig. 2). Near the base $\Gamma \leq 1$ and the temperature ($\propto V_S^2$) increases with R: here the heating is so strong that part of the energy goes to expansion and the remaining rises the temperature of the gas. For larger R the temperature declines, after a maximum where $\Gamma = 1$, and for $R \rightarrow \infty$ asymtotically $\Gamma \rightarrow 3/2$.
b) *Collimated hydromagnetic flows.* In oder to solve the MHD equations

$$\nabla \cdot (\rho \mathbf{v}) = \nabla \cdot \mathbf{B} = \nabla \times (\mathbf{v} \times \mathbf{B}) = 0$$

$$\rho(\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla P - \frac{\rho G M}{r^2}\mathbf{r} + \frac{1}{4\pi}(\nabla \times \mathbf{B}) \times \mathbf{B} \qquad (4)$$

the radial flow speed and density are always given by Eqs. (2), the radial component of the magnetic field is $B_R = B_o/(R^2 \cos\theta)$, and the azimuthal components of $\mathbf{B}$ and $\mathbf{v}$ are given by:

$$B_\phi(R,\theta) = -\lambda B_o \frac{\sin\theta}{R} \frac{1 - R^2/R_*^2}{1 - M_a^2}, \qquad V_\phi(R,\theta) = -\lambda V_o \frac{R\sin\theta}{g(\omega,\theta)} \frac{Y_* - Y}{1 - M_a^2} \qquad (5)$$

where M_a is the radial Alfven Mach number, $Y_* = Y(R_*)$, and where R_*, the Alfvenic critical point ($M_a(R_*) = 1$), is a singular point of expressions (5). By substituting Eqs. (2) and (5) in Eqs. (4) we obtain a single equation for $Y(R)$,

$$\frac{dY}{dR} = \frac{f(Y, R, R_*, \omega\nu, \lambda, \eta)}{g(Y, R, R_*, \omega\nu, \lambda, \eta)}, \qquad \eta = \frac{V_o}{V_a} \qquad (7)$$

that has two critical points: at $R = R_*$ and $R = R_X > R_*$. The Alfvenic point at R_* is mathematically a sink point wherein many solutions with different slopes pass through, while the second critical point at R_X is an ordinary X-type critical point (Fig 3). There is a unique solution (for fixed parameters) that starts at the base with $Y(R=1) = 1$ and low V_o, crosses smoothly both critical points at R_* and R_X, and finally reaches infinity with a high terminal speed and zero pressure. The topologies appear quite differerent with respect to previous studies: only two critical points are found in spite of three, corresponding to the Alfven, Fast and Slow MHD waves velocities (Weber and Davis 1967). Again this is related with the energy heating of the plasma.

The main consequence of this analytical investigation is that, both in the hydrodynamic and MHD cases, the collimation is strictly related with the energy processes which accelerate the flow.

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REFERENCES

Tsinganos, K. and Trussoni, E.: 1989, *Astron. Astrophys.*, submitted
Weber, E.J. and Davis, L.: 1967, *Astrophys. J.* **148**, 217.

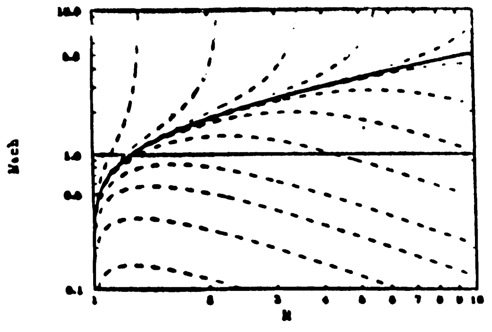

Figure 1. Topology of the Mach number for $\lambda = 0.5$, $\nu = 120$, $\omega = 4$ and $\theta = 0$.

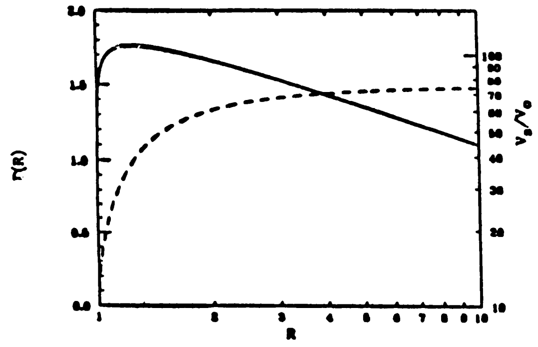

Figure 2. Radial dependence of Γ (dotted line) and V_S (full line) for $\theta = 0$

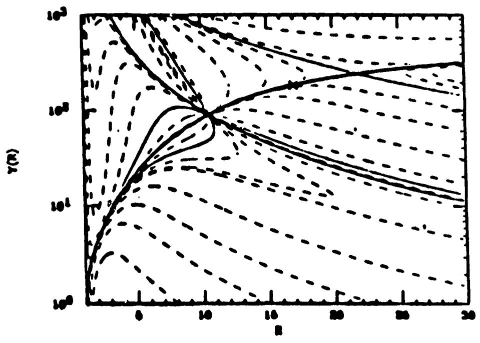

Figure 3. Topology of the radial velocity for the same parameters of Fig. 1, with $\eta = 0.01$. The cross indicates the position of the sonic point.

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PUDRITZ: Could you further clarify your treatment: (1) how does your ω relate to the Weber–Davis parameter (1–D flows) which characterizes the focussing of flows, (2) again using Weber–Davis language, are your hydromagnetic polytrope models slow or fast magnetic rotators?

TSINGANOS: The Weber–Davis (1967, Astrophys. J. **148**, 217) analysis applies to the equatorial plane and assumes a polytropic relationship with constant γ. Our study has $\gamma \neq$ constant and applies to $0 < \theta < \pi/2$. Some of the results in Weber–Davis (1967) are recovered, for example the relative contribution of the fluid and magnetic components to the total angular momentum carried away by the star. We are presently investigating the range of values of the parameters for which we obtain wind-type solutions. This is the case for example with $\lambda = V_\phi(R=1)/V_R(R=1)$.

LOW: I like to make a clarifying comment on the issue of the critical points of the steady wind equations. I believe the speaker has closed his MHD equations by prescribing the magnetic field in place of an energy equation like the polytropic law. Energy conservation is then imposed by introducing externally added heat, defined *a posteriori*, to balance energy everywhere. This procedure does not introduce critical points into the governing equations explicitly.

TSINGANOS: Let me clarify three points. First, we prescribe the angular dependence of the field lines, while the radial dependence comes out of the conservation laws. Second, we obtain the Alfvénic critical, but not the fast and slow mode critical points. These sonic points exist, but they are *not* critical, i.e. singularities in the equations. And third, the solution is self-consistent, i.e. satisfying all conservation laws, something unavailable with previous studies. For example, you have force balance both across and along the stream lines.

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