BOOK REVIEWS

WLOKA, J. T., ROWLEY, B. and LAWRUK, B. *Boundary value problems for elliptic systems* (Cambridge University Press, Cambridge, 1995), xiv + 641 pp., 0 521 43011 9 (hardback), £60 (US$89.95).

The study of boundary value problems for elliptic systems is one of the central subjects in mathematics. Because of the variety and importance of their applications, in particular to mathematics and physics, boundary value problems caused developments in various areas of mathematics. Nowadays the theory of boundary value problems for elliptic systems invokes modern techniques from algebra, geometry and analysis. The present book demonstrates this fact very clearly.

The main aim of the authors is to discuss the index theory for boundary value problems and a substantial part of the book is devoted to the introduction of the necessary mathematical tools. A self-contained account of the relevant topics from differential geometry and the theory of pseudo-differential operators is given. But while these ingredients are fairly standard in the study of boundary value problems, the book distinguishes itself by the subsequent implementation of the spectral theory of matrix polynomials

$$L(\lambda) := \sum_{j=0}^{l} \lambda^j A_j, \qquad A_j \in GL(\mathbb{C}^p, \mathbb{C}^p).$$

This problem generalises the common spectral theory for $\lambda \mathbb{I} - A$. The detailed introduction to this subject in the beginning of the book is without doubt of independent interest. In application to pseudo-differential operators it provides a suitable tool for manipulations with the principal symbols and leads to simplifications in the proofs of the basic results.

The main part of the book deals with the reduction of boundary value problems to elliptic problems on the boundary. The L-condition is discussed in detail and the structure of pseudo-differential operators near a boundary are studied. Finally the index formula for boundary value problems in the plane is discussed.

The presentation of the material is clear and concise. In particular, students and non-specialists will value the material included on differential geometry and on pseudo-differential operators, which justifies the considerable length of the text. Although it cannot replace other textbooks on these topics, the book gains considerably from this inclusion. Each chapter starts with a short outline of its plan, which helps the reader to concentrate on the essentials while reading the techniques. Although all theorems are proved in detail, the authors' principal concern is the communication of the main ideas. So, for instance, in order to give a more elementary presentation the authors avoid the direct application of the Atiyah–Singer theorem. Examples and exercises help the reader to understand the material. A keyword index, a list of symbols and the clear structure make the text easily accessible. The book can be recommended both as a textbook for graduate students and as a handbook for researchers.

T. WEIDL