MEAN-FIELD MODELS OF GALACTIC DYNAMOS
ADMITTING AXISYMMETRIC AND NON-AXISYMMETRIC MAGNETIC FIELD STRUCTURES

K.-H. Rädler and E. Wiedemann
Central Institute of Astrophysics
DDR-1561 Potsdam, GDR

ABSTRACT. A concept of mean-field models of galactic dynamos is explained in which α-effect and differential rotation act in an infinite slab of electrically conducting fluid surrounded by free space or by conducting matter at rest. Some results for α^2-dynamos are presented. In the limiting case in which the α-coefficient does not depend on the radius, axisymmetric and non-axisymmetric magnetic fields can be maintained with equal ease. For reasonable α-distributions a preference of axisymmetric fields is suggested as long as an isotropic α-effect is considered but it may disappear with anisotropic α-effect. The effect of differential rotation, which allows $\alpha\omega$-dynamos, is briefly discussed.

1. INTRODUCTION

There are good reasons to assume that the large scale magnetic fields observed in galaxies are due to dynamo processes. The construction of appropriate dynamo models is, however, a difficult mathematical task. One promising approach to the understanding of such processes is the investigation of mean-field models in which the induction effects, in particular α-effect and differential rotation, are confined to a slab of infinite radial extent. If moreover the induction effects and the magnetic fields generated are localized in a finite region around the rotation axis, the infinite radial extent is no longer important, and the models can indeed reflect features of dynamo processes in galaxies or other flat objects. Here we explain this approach in some more detail and present a few initial results.

2. THE MODELS AND THE MATHEMATICAL METHOD

Let us consider an infinitely extended slab of electrically conducting fluid possessing internal motions with non-zero helicity. Relative to cylindrical coordinates r, φ, z, the slab is defined by $-d \leq z \leq d$ where d is its half-thickness. There the mean magnetic flux density, **B**, is assumed to obey the equations

$$\eta \Delta \mathbf{B} + \mathrm{curl}(\mathbf{u} \times \mathbf{B} + \boldsymbol{\mathcal{E}}) - \partial \mathbf{B}/\partial t = 0 \,, \quad \mathrm{div}\, \mathbf{B} = 0. \qquad (1)$$

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The magnetic diffusivity η is assumed to be constant. The velocity u of the mean motion and the mean electromotive force $\mathcal{E}$ due to fluctuations are defined by

$$u = \omega r \hat{\varphi}, \quad \mathcal{E} = -\alpha_1 (B + a(\hat{z} \cdot B)\hat{z}), \qquad (2)$$

which describe the differential rotation and the α-effect. The angular velocity ω and the α-effect coefficient α_1 are supposed to depend on r and z but not on φ, and to be symmetric or antisymmetric about the equatorial plane z = 0, respectively. $\hat{\varphi}$ and $\hat{z}$ are unit vectors in φ and z-direction. Some anisotropy of the α-effect, defined by the constant a, is admitted.

The sourroundings of the slab, $|z| > d$, are supposed to be either free space or homogeneous conducting matter at rest. Accordingly, B in this external region must be an irrotational solenoidal field or obey equations of type (1) with η replaced by another constant, and u and $\mathcal{E}$ equal to zero. Furthermore it is required that B as well as the tangential components of the mean electric field are continuous across the boundaries $|z| = d$, and B vanishes as $|z| \to \infty$.

The solutions B to our problem are B-modes of the form

$$B = \mathrm{Re}\ (\hat{B} \exp(im\varphi + \lambda t)) , \qquad (3)$$

or superpositions of them. $\hat{B}$ is a complex axisymmetric steady vector field which is either antisymmetric or symmetric about the equatorial plane, indicated by A or S in the following, m is a non-negative integer, and λ a complex constant, the real part of which is the growth rate of the mode. As usual we speak of Am or Sm modes. Clearly m = 0 corresponds to axisymmetric and m = 1 to bisymmetric field structures.

It is useful to represent B in the form

$$B = -\,\mathrm{curl}(\hat{z} \times \nabla S) - \hat{z} \times \nabla T$$

$$(S,T) = \int_0^\infty (s_m(\varkappa,\zeta),\ t_m(\varkappa,\zeta))\ J_m(\varkappa\varrho)\ \varkappa d\varkappa \qquad (4)$$

$$\times \exp(im\varphi + \lambda t),$$

where J_m is a Bessel function of the first kind, $\varrho = r/d$ and $\zeta = z/d$. In this way the equations governing B can be reduced to a set of two second-order ordinary differential equations for s_m and t_m with respect to ζ which are, however, at the same time integral equations with respect to $\varkappa$. These equations, together with the boundary conditions, pose an eigenvalue problem for λ, which has to be solved numerically.

In all the examples which follow, α_1 is specified by

$$\alpha_1 = \alpha_0 f(\varrho) \sin(\pi\zeta) \qquad (5)$$

where α_0 is a positive constant and f will be fixed later. The dimensionless measure C_α of the magnitude of the α-effect is defined by

$$C_\alpha = \alpha_0 d/\eta . \qquad (6)$$

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3. α^2-MODELS WITH α INDEPENDENT OF RADIUS

It is instructive to study first the simple case in which there is no mean motion and α_1 does not depend on the radius. In this limit, s_m and t_m have a delta-like dependence on $\varkappa$. The integration in (4) can be omitted and $\varkappa$ fixed arbitrarily. The equations for s_m and t_m no longer contain integrals. The B-modes show a wave-like radial variation; $\varkappa$ is the dimensionless wave number. Since B only decays slowly with growing radius the energy of each individual mode is infinite.

As can easily be seen from the formulation of the eigenvalue problem for this limit, λ does not depend on m. That is, in addition to an axisymmetric B-mode, m = 0, there are always a bisymmetric mode, m = 1, and modes with higher m which have the same excitation conditions and growth rates.

A number of different models has been investigated numerically. For the examples mentioned here (5) with f=1 has been used. Fig.1 shows dependencies of the marginal C_α, that is, those with $Re(\lambda) = 0$, on $\varkappa$.

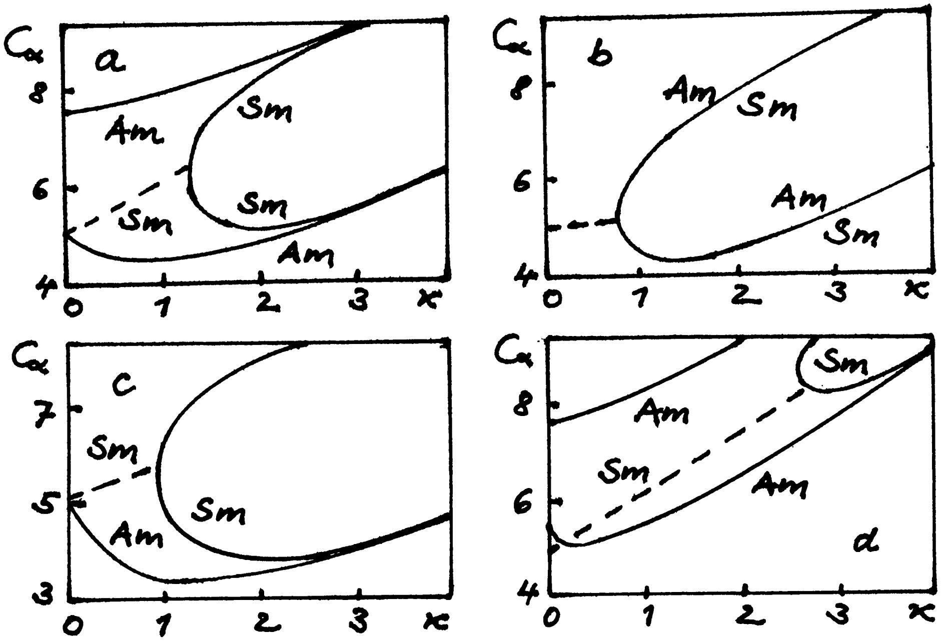

Fig. 1. Marginal values of C_α versus $\varkappa$ for several α^2-models.
(a) isotropic α-effect, i.e. a=0, surroundings free space.
(b) isotropic α-effect, i.e. a=0, surroundings conducting matter with the same magnetic diffusivity as in the slab; the Am and Sm curves lie close together and are not distinguishable in the drawing.
(c) and (d) anisotropic α-effect, a=1 and a=-0.75, respectively, surroundings free space. Solid lines correspond to non-oscillatory modes, i.e. $I_m(\lambda)=0$, broken lines to oscillatory modes, i.e. $I_m(\lambda)=0$.

The most easily excitable modes are those with the smallest marginal values of C_α. In the majority of the models investigated so far Am-modes play this preferred part. Contrary to a case with discontinuous α_1-distribution investigated earlier by Rädler and Wiedemann (1990) the minimum of the marginal C_α for a preferred mode generally occurs not with $\varkappa=0$ but with $\varkappa$ in the order of unity. Then the characteristic radial length scale of the field structure is comparable to the slab thickness.

Our results are of some interest in view of the "local approximation" used in investigating slab models, e.g., by Ruzmaikin et al. (1980) and Zeldovich et al. (1983). As was shown by Rädler and Bräuer (1987) this approximation ignores important modes. Significantly, amongst the "forgotten modes" are also modes which are preferred in the above sense. Moreover the "local approximation" is questionable in so far as it starts from modes with $\varkappa = 0$ although modes with $\varkappa \neq 0$ can be more readily excitable.

4. α^2-MODELS WITH α DEPENDING ON RADIUS

Let us now proceed to the case in which there is again no mean motion but α_1 depends on the radius and vanishes at infinity. Then λ is no longer independent of m. Numerical investigations have been carried out with models in which α_1 is specified by (5) and $f = \exp(-(\varrho/\varrho_0)^2)$ where ϱ_0 is a constant. Tab. 1 gives, for models with isotropic α-effect, marginal values of C_α for modes with m = 0. In these models the modes with m = 0 are presumably preferred to those with $m \neq 0$. For $m \neq 0$ no marginal values of C_α smaller than those for m = 0 have been found. There are, however, reasons to assume that anisotropies of the α-effect may lead to a preference of modes with $m \neq 0$.

Table 1. Marginal values of C_α for models with isotropic α-effect, i.e. a = 0, and various ϱ_0, surroundings free space. Note that $\varrho_0 = \infty$ means f = 1.

ϱ_0	C_α AO	C_α SO
∞	4,5	5,0
10	9,5	10,4
5	9,7	11,3

The case considered before in which α_1 is independent of the radius corresponds to $\varrho_0 = \infty$. The results for this case suggest that the field structures for $\varrho_0 \gg 1$ show a radial variation with a characteristic scale given by the thickness of the slab rather than the radial scale of the α-distribution, that is, a number of reversals along the radius is to be expected. In our preliminary field presentations for $\varrho_0 = 10$ one such reversal is indicated.

5. REMARKS CONCERNING $\alpha\omega$- MODELS

Starting from α^2-models and admitting differential rotation we arrive

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at $\alpha\omega$-models. As for the angular velocity ω we focus our attention on its dependence on r. With axisymmetric B-modes it can be easily deduced that the differential rotation in general favours A0-modes if the signs of α_1 and $\partial\omega/\partial r$ for z>0 differ, and S0-modes if they coincide. It is an open question under which conditions non-axisymmetric fields are preferred, if at all. A sufficiently strong differential rotation always favours axisymmetric fields.

There are some reasons to believe that $\alpha\omega$-models constructed by modifying α^2-models as considered above with ρ_0>>1 can reflect typical features of the observed galactic magnetic fields. Start from an α^2-model of that kind and consider, e.g., an axisymmetric field. Suppose then, thinking of the observed rotation profiles, an angular velocity which varies only slightly in the neighbourhood of the rotation axis up to a radius of the order of the slab thickness and decays after that with growing radius. Then a modified field is to be expected which has a noticeable poloidal part near the axis but is predominantly toroidal in the outer regions. This would correspond to the observations of strong poloidal fields in the galactic centers. There is, of course, also the possibility of reversals along the radius in this outer region, which would then suggest an explanation of the reversals found in the Milky Way galaxy.

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DEINZER: Of what kind are your excited dynamo modes: oscillating or non-oscillating?

RÄDLER: In the simple α^2 models in which α does not depend on radius both oscillatory and non-oscillatory modes have been found. Oscillatory behaviour, however, occurs only with small radial wave numbers, that is, with large radial scales of the fields. As a rule, the most easily excitable model of a given symmetry type is non-oscillatory. For models in which α depends on radius only axisymmetric modes have been investigated so far. They proved to be non-oscillatory, too. I expect that the non-axisymmetric modes in such models are oscillatory, which simply means that the fields rotate about the disk axis.

SHUKUROV: How sensitive are the properties of the "forgotten modes" to the boundary conditions at the disk edge, and which particular boundary conditions have you used?

RÄDLER: I discovered the "forgotten modes" in a simple model of an α^2 dynamo in an infinite slab with an α coefficient independent of the radius. The space outside the slab was supposed to be vacuum. In this model the "local approximation" does not provide the most easily excitable mode, which is a non-oscillatory A- (antisymmetric) mode, and suggests that it is an oscillatory S- (symmetric) mode which plays the preferred role (see Rädler and Bräuer, 1987, Astr. Nachr. **308**, 101). I want to stress, however, that the problem of "forgotten modes" is not restricted to this example. It is a general shortcoming of all approaches to disk models starting from solutions of the dynamo equations in which the radial derivatives are ignored, that is, from solutions with zero radial dynamo number. In addition to the fact that such approaches ignore important modes you should also bear in mind that the most easily excitable modes not necessarily correspond to very small radial wave numbers. In most of the examples I mentioned in my talk they are wave numbers in the order of unity that play a preferred role.

SOKOLOFF: Galaxies are not exactly flat slabs, but they are thicker in the outer part. Sometimes this effect is important. Is it possible or difficult to introduce such details in your computer investigations?

RÄDLER: The mathematical method I used, which is in a sense analogous to the Bullard-Gellman formalism in the case of spherical models, works very well with slab models. Its application to models with other geometry is possible but rather complicated. I would prefer another approach to such models.

DEINZER: If it is possible to excite non-axisymmetric dynamo modes before axisymmetric modes for a uniform background (without density waves), then magnetic fields alone would be able to explain the spiral structure of galaxies. Is this feasible?

RÄDLER: In mean-field dynamo models which are completely axisymmetric with respect to the mean motion, the α parameter, etc., both axisymmetric and non-axisymmetric mean magnetic fields are possible. In the case of spherical models we know many examples in which non-axisymmetric modes are even easier excitable than the axisymmetric ones, and for disk models first examples of that kind have been given. Of course, if non-axisymmetric magnetic fields grow in an axisymmetric body in the above sense, they will disturb its symmetry. In particular, bisymmetric magnetic field structures may lead to spiral structures in the density distribution. This, however, does not mean that the spiral structure of the galaxies is a consequence of their magnetic fields.