Take OC along the t-axis $= \dfrac{t_0}{n(1-\epsilon^{-1/n})}$.

Beginning with C, mark off a number of points C_1, C_2, C_3, etc., at intervals of t/n.

AC cuts the ordinate B_1 in A_1.

A_1C_1 cuts the ordinate B_2 in A_2.

A_2C_2 cuts the ordinate B_3 in A_3, and so on.

A_1, A_2, A_3, etc., are points on the curve.

The following are values of the function OC/t_0

n	3	5	10	∞
$\{n(1-\epsilon^{-1/n})\}^{-1}$	1·176	1·105	1·051	1.

A convenient number to take for n is 5, for which OC may be taken with sufficient accuracy for most purposes as 1·1. This fits in very easily with the use of squared paper. The case $n=\infty$ gives the well-known property of the subtangent which has often been applied to a geometrical construction, but fails because of the necessity of working with a finite number of ordinates.

Obvious modifications make the construction applicable to the curve $y=a(1-\epsilon^{-t/t_0})$.

DAVID ROBERTSON.

Integration by Parts. A Failing Case.—Integrate $\tan x$ by parts as follows:—

$$\int \tan x dx = \int \frac{\sin x}{\cos x} dx = -\frac{\cos x}{\cos x} + \int \tan x dx,$$

i.e. $0 = -1$, which is absurd.

It might be conjectured at first sight that the above result is due to a disregarded constant of integration, but this is not so, because we can imagine all the integrals taken between the same limits, in which case there is no constant. The true explanation is to be found by considering the proof of the theorem of integration by parts.

Let u and v be functions of x, then the following is an identity

$$uv = \frac{du}{dx}\int vdx + uv - \frac{du}{dx}\int vdx.$$

Re-write as,

$$uv = \frac{d}{dx}\left(u\int vdx\right) - \frac{du}{dx}\int vdx \qquad \text{.....................(1)}$$

Integrate

$$\int uvdx = u\int vdx - \int\left\{\frac{du}{dx}\int vdx\right\}dx \qquad (2)$$

which is the usual formula.

If, however, in (1) $u\int vdx$ be a constant its differential coefficient is zero, and on integration in (2) *it remains zero.* In other words the formula for integration by parts no longer holds true, if the parts be so chosen, that the first term of the result, which is usually memorised as "the one part into the integral of the other," is a constant. This exception is not noted in the text-books; and the error to which it gives rise is not in all cases so evident, as in the example chosen above.

J. R. MILNE.

Vectors as applied to Problems in Velocity.—A velocity is a vector quantity, for it possesses magnitude and direction. It is, of course, necessary to make clear in every case what given point (moving or not) is considered as fixed for the time being, to which the velocity of the moving point (so-called, though it may only be relatively moving) is referred.

If P is the point considered fixed, and the point Q is moving relatively to P, we may denote the velocity of Q relative to P by the symbol $V_{Q/P}$.

The velocity of P relative to Q will be expressed by $V_{P/Q}$. Obviously $\overline{V}_{P/Q} = -\overline{V}_{Q/P}$.

In kinematics two kinds of problem present themselves. In the first and simplest it is required to find the resultant of a number of mutually independent velocities, all of which are referred to the same fixed point, usually the Earth.

Suppose we have n velocities, each referred to the same point E, viz., $V_{1/E}$, $V_{2/E}$, ..., $V_{n/E}$. Then if $V_{0/E}$ represents the resultant velocity we have $\overline{V}_{0/E} = \overline{V}_{1/E} + \overline{V}_{2/E} + \ldots + \overline{V}_{n/E}$, the summation being carried out by the method of the Vector Polygon.

Such problems are so simple that they hardly demand any special notation. But let us now consider the case when the velocities involved are mutually dependent.