

$$(\alpha) \sum_{t=1}^{2^n} \frac{X_t^{a.s.}}{n} \rightarrow 0 \qquad (\beta) \sum_{t=1}^n \frac{X_t^{2^P}}{n} \rightarrow \sigma_0^2$$

In this, as usual, $a.s.$ \rightarrow signifies almost sure convergence, \rightarrow convergence in probability.

In the last part of the paper the concept of exchangeability is further amplified to include stochastic processes. A process with exchangeable increments is said to occur when the variables

$$Y_{2^n, k} = X_{t_{2^n, k}} - X_{t_{2^{n-1}, k-1}}$$

(the index range of X is the interval $[a, b]$ and

$$t_{2^n, k} = a + [(b-a)k/2^n], k = 0, 1, \dots, 2^n$$

for all n are terms of an infinite series of exchangeable variables. After discussing the fundamental characteristics of such processes, the author points out the connection with the class of infinitely divisible distribution laws. Of special interest are processes, the sample functions of which are either continuous or discrete (step-functions with constant height). In the first case, we get weighted normal distributions as probability laws of the processes. They are not weighted normal if, and only if, the increments are independent. Similar statements hold good for sample functions of the second type. Here weighted normal distributions have to be substituted by weighted Poisson distributions; the characterization of independence can be derived in an analogous way.

It is to be hoped that this interesting publication receives the attention it deserves and that it gives inducement for further investigations. Bühlmann's treatment of the subject is from a pure theoretical point of view, and a certain acquaintance with higher algebra and functional analysis is indispensable to the understanding. Only the future can show how far the results will prove useful for practical problems.

J. Kupper

The Objectives of an Insurance Company by KARL BORCH. Skandinavisk Aktuarietidskrift 1962 (p. 162).

In continuation of some of his previous papers the author defines the objectives pursued by an insurance company by a mathematical function. This function to be maximized is usually referred to as utility function and defined by the risk situation of the company. Starting from the so-called Bernoulli hypothesis and assuming a rational behaviour two major decisions to be reached by a company are discussed. It is obvious that the decisions are reached so as to give the highest utility. The first decision concerns the determination of premiums to be offered to the public and the amounts to be spent for the promotion of sales, the second the companies reinsurance decisions.

A Contribution to the Theory of Reinsurance Markets by KARL BÜHL. Skandinavisk Aktuarietidskrift 1962 (p. 186).

In a previous paper the author has shown that there exists no market price in a reinsurance treaty which—when applied to all transactions—will lead

to a Pareto optimal arrangement. This completely negative result which implies that there are no reinsurance transactions made in a perfectly competitive market is again confirmed to a certain extent in this paper by an illustrative numerical example.

M. Derron

Application of Mathematical Models in General Insurance by PAUL JOHANSEN. Forsikringsaktieselskabet Nye Danske af 1864. Centenary 19th April 1964.

This slender publication, written on the occasion of the Centenary of the Nye Danske af 1864, presents an excellent survey of the current state of the mathematical foundation of non-life insurance in which there is currently rapid development. The book is purposely written in non-mathematical language and those acquainted with the subject will find that the fluent style and lucid arrangement make it delightful reading. Above all, actuaries whose interest have been mainly in life business, will find the book an agreeable and beneficial introduction to non-life insurance.

As there is no authoritative text book in English on the mathematics of non-life insurance, and the current rate of development would soon render such a text book out of date, the would-be student has to refer to papers scattered in various journals. With no systematic instruction it is a matter of considerable difficulty to become conversant with the subject. This lacuna is largely filled by the present work in which, in spite of the inherent difficulty of the subject matter, the significant fields of enquiry within the scope of non-life mathematics are succinctly examined in an easily comprehensible manner and in a clear, systematic form.

The paper begins with a consideration of mathematical models and, especially, of the methods employed in life insurance, and then proceeds to the fundamental questions of non-life insurance mathematics. In particular, not only the problems of tariff rating, but also the ascertainment of security reserves and the questions relating to reinsurance are expounded. Elements of the risk theory of non-life insurance mathematics, with special reference to the collective risk theory evolved in Scandinavia and its projected elaboration in recent years through generalised models, are then subjected to thorough scrutiny. Numerical examples contribute to a deeper perception of the matter.

Both the learned author as well as Nye Danske af 1864 are to be congratulated on the successful work. It represents, as it were, a record, accessible even to the public at large, of the latest state of the mathematics of non-life insurance.

Hans Ammeter