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ARTICLE

How to be absolutely fair Part II: Philosophy meets economics

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Abstract
In the article 'How to be absolutely fair, Part I: the Fairness formula', we presented the first theory of comparative and absolute fairness. Here, we relate the implications of our Fairness formula to economic theories of fair division. Our analysis makes contributions to both philosophy and economics: to the philosophical literature, we add an axiomatic discussion of proportionality and fairness. To the economic literature, we add an appealing normative theory of absolute and comparative fairness that can be used to evaluate axioms and division rules. Also, we provide a novel definition and characterization of the *absolute priority rule*.

Keywords: fairness; (weighted) bankruptcy problems; claims; proportionality

1. Introduction

Fairness theories in philosophy and economics have hitherto developed in relative isolation from each other. It is thus all the more intriguing that there is significant overlap in their outlook and methods which has, by and large, gone unnoticed – or at the very least, not been well-documented and rarely discussed. Fairness theories from both disciplines analyse similar fair division problems, in which a scarce estate is to be divided fairly between claimants. For illustration, take the following fair division problem. It is exemplary for canonical problems of fair division analysed in *both* philosophy and economics.

Owing Money. Romeo owes 20 to Abram and 60 to Benvolio but has only 40 left. How, in order to be fair, should Romeo divide the 40?

In philosophy, Broomean theories of fairness analyse fair division problems such as Owing Money by applying the *Broomean formula*. It says that fairness requires that *claims* should be satisfied in proportion to their strength. Claims are a specific type of reason as to why a person should receive a good. They are 'duties owed to the

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person herself', as Broome puts it.[1] The Broomean fairness literature has generated a thriving debate in philosophy in the last years.[2] According to the Broomean formula, fairness is a *strictly comparative* notion as it only requires the proportional satisfaction of individual claims, not their satisfaction as such. Thus, *any* allocation in which Benvolio receives three times as much as Abram is fair. This includes the intuitively fair allocation $(10,\ 30)$, but also, for example, $(5,\ 15)$. Further principles need to be invoked to motivate the allocation $(10,\ 30)$. Many contributors to the Broomean fairness literature agree that a key principle to realize this allocation is that of *absolute fairness*, which demands the satisfaction of claims as such. Combined with the Broomean formula, this principle requires that, in order to be fair, Romeo should realize allocation $(10,\ 30)$.[3] We concur with this analysis and have formulated a theory of fairness that accommodates both comparative and absolute fairness. The cornerstone of our two-dimensional theory of fairness is the *Fairness formula* (FF).

Fairness formula (FF). Fairness requires one: (i) to satisfy absolute claims (of individuals and groups) to as large an extent as possible, subject to the constraint that no one receives more than they have a claim to; (ii) to satisfy (absolute and notional) individual claims in proportion to their strength; (iii) to prioritize requirement (i) over (ii) whenever these two conflict, but in such a way that one does as much as possible to respect (ii).

We introduced and justified the FF in our article 'How to be absolutely fair, Part I: the Fairness formula'. There, we observed that the requirements of absolute and comparative fairness – FF(i) and FF(ii) – may be incompatible, which explains the third clause in the Fairness formula. Also, we presented the *absolute priority rule* which implements the FF and makes precise its content, in particular clause (iii).

In this article, we relate the implications of our *Fairness formula* (FF) to economic theories of fair division. The starting point of our analysis is the astonishing similarity of fairness frameworks in philosophy and economics. Previously, we have shown that key concepts which figure in philosophical discussions about fairness can be mapped onto mathematical structures that we baptized *Broomean problems* $\mathcal{B} = (E, N, a, s)$, where the *E*state is to be divided amongst the individuals of N whose *claim amounts* and *claim strengths* are described by a and s respectively. The parallels to fair division theories in economics

[1]Broome contrasts claims with *teleological reasons* and *side-constraints* but does not offer a detailed account of the nature of claims. Hence, in this sense his theory of fairness is incomplete. However, as Piller (2017: 216) observes, 'this incompleteness might not matter … because we understand talk of claims pre-theoretically'. We concur with Piller and will, in this paper, rely on this intuitive understanding of a claim. However, a full-blown theory of fairness should come with a theory of the sources or grounds for claims. We will take up this issue in future work.

[2]See Hooker (2005), Saunders (2010), Tomlin (2012), Curtis (2014), Lazenby (2014), Kirkpatrick and Eastwood (2015), Paseau and Saunders (2015), Vong (2015, 2018, 2020), Sharadin (2016), Heilmann and Wintein (2017), Piller (2017), Wintein and Heilmann (2018, 2020, 2021). Broome presents partial articulations of his theory before his canonical (1990) contribution in e.g. Broome (1988) and Broome (1984).

[3]In contrast, Broome himself does not endorse a criterion of absolute fairness. He agrees that Romeo should realize $(10,\ 30)$ but not as a matter of fairness.

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are striking. An important literature in economics studies *bankruptcy problems* $\mathfrak{B} = (E, N, a)$, where E and N are interpreted as they are in Broomean problems and where a is loosely interpreted as 'claims', wants or demands. Moreover, Casas-Méndez *et al.* (2011) study so called *weighted bankruptcy problems* (E, N, a, s), which extend bankruptcy problems with 'weights' s and which are formally equivalent to our Broomean problems. Indeed, we will show that a Broomean problem is a weighted bankruptcy problem when a and s are interpreted as *claim amounts* and *claims strengths* respectively. And so, in order to harness these parallels between philosophy and economics, we apply the general FF developed previously to Broomean problems, for which the FF translates into the following.[4]

Fairness formula for $\mathcal{B}$roomean problems: FF$\mathcal{B}$. For a Broomean problem $\mathcal{B} = (E, N, a, s)$, fairness requires one to realize an allocation x which is:

- (i) (a) Efficient, i.e x allocates the entire estate.
  - (b) Claims-respecting, i.e. x does not award anyone more than their claim.
- (ii) Satisfies claims in proportion to their strength.
- (iii) Whenever (i) and (ii) conflict: x should respect (i) and satisfy claims in proportion to their strength to as large a degree as possible.

On the basis of FF$\mathcal{B}$, we derive four results.

First, we characterize the conditions under which FF$\mathcal{B}$(i) and FF$\mathcal{B}$(ii) are compatible. We do so in terms of the *weighted proportional rule P* which divides the Estate in proportion to the strength-weighted amounts $s_i a_i$ of the individuals. Although P is not normatively appealing, it paves the way for deriving our further results.

Second, we offer a novel characterization of the absolute priority rule $P^\dagger$: we show that $P^\dagger$ is the only division rule which is *efficient*, *fully proportionally reimbursing* and which satisfies *partially reimbursed claims in proportion to their strength*. On the basis of this characterization, we argue that $P^\dagger$ operationalizes FF$\mathcal{B}$. In particular, $P^\dagger$ selects an allocation which respects both FF$\mathcal{B}$(i) and FF$\mathcal{B}$(ii) whenever doing so is possible; when it is not, $P^\dagger$ selects an efficient and claims-respecting allocation which, as we'll argue, respects FF$\mathcal{B}$(iii).

Third, we show that the algorithmic definition of the absolute priority rule $P^\dagger$ that we provided in our previous article is equivalent to the definition of the *weighted constrained proportional rule* $\mathbb{P}^w$ due to Casas-Méndez *et al.* (2011). We contrast and compare our characterization of $P^\dagger$ to the characterization by Casas-Méndez *et al.* (2011) of $\mathbb{P}^w$ in terms of a 'weighted version of strategy-proofness'.

Fourth, we introduce and characterize the *comparative priority rule* $P^\ddagger$, which is the counterpart of $P^\dagger$: in case of a conflict between comparative and absolute fairness, the comparative priority rule $P^\ddagger$ prioritizes the former over the latter while it does 'as much as possible to respect the requirements of absolute fairness'. Although we do not think that $P^\ddagger$ is normatively appealing, studying $P^\ddagger$ sheds light on the philosophical debate on comparative and absolute fairness.

[4]In the Appendix, we explain how one obtains FF$\mathcal{B}$ from the Fairness formula.

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In general, together with our previous article 'How to be absolutely fair, Part I: the Fairness formula', we present a comprehensive two-dimensional theory of fairness which tells us what it *means* to be fair and how to *realize* fair divisions. Our theory draws on and exploits hitherto un(der)appreciated differences and complementarities between philosophy and economics fairness research. The present article contributes to both literatures. To the philosophical literature, we add an axiomatic discussion of proportionality and fairness. Specifically, the discussion of the different 'proportionality rules' P, $P^{\dagger}$ and $P^{\ddagger}$ and the connections we make to the economic literature on (weighted) bankruptcy problems offer fruitful resources for the further development of Broomean fairness. To the economic literature, we add an appealing normative theory of absolute and comparative fairness that can be used to justify axioms, such as efficiency, and division rules. Specifically, we provide a novel (algorithmic) definition and novel characterization of the weighted constrained proportional rule and a new interpretation of weighted bankruptcy problems.

The paper is structured as follows. In section 2 we introduce Broomean problems in a formal framework and illustrate that FF$\mathcal{B}$(i) and FF$\mathcal{B}$(ii) may be incompatible. In section 3 we discuss division rules for Broomean problems and we present our four results. In section 4 we discuss (weighted) bankruptcy problems and their interpretation in the economics literature. In section 5 we conclude.

2. The Fairness Formula in a Formal Framework

2.1. Broomean problems and their allocations

In this section, we present the core elements of our theory in a formal framework.

A *Broomean problem* is a structure

$$\mathcal{B} = (E, N, a, s),$$

where the estate $E > 0$ specifies the amount of the good-to-be-divided amongst the individuals in $N = \{1, \ldots, n\}$. An individual $i \in N$ has a claim (a_i, s_i) with amount $a_i \geq 0$ and strength $s_i > 0$, as specified by amounts-vector a and strengths-vector s, the amounts-vector being such that $\sum_{i \in N} a_i \geq E$: the sum of claims (weakly) exceeds the estate.[5] As claim-strengths are strictly comparative, they are only determined up to an arbitrary positive multiplicative constant ρ: if $s = \rho \cdot s'$ then vectors s and s' determine the same claim-strengths. However, for sake of definiteness we will typically[6] normalize claim strengths and assume that $\sum_{i \in N} s_i = 1$. As an example of a Broomean problem, consider the representation of $\mathcal{O}$wing Money:

$$\mathcal{O} = \left(40, \{A, B\}, (20,\ 60), \left(\frac{1}{2}, \frac{1}{2}\right)\right)$$

[5]In the article 'How to be absolutely fair, Part I: the Fairness formula' we explicitly discuss the FF in relation to cases of abundant good. Although such cases are conceptually interesting, we do not discuss them here for sake of simplicity – formally, their treatment is straightforward.

[6]Not always though: in section 3.2 it is convenient to skip this convention in some places – nothing hinges on this.

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Indeed, the claims of A(bram) and B(envolio) in Owing Money have different amounts (20 and 60) but they are equally strong.

An *allocation* x *for* $\mathcal{B}$ allots an amount $x_i \geq 0$ to each individual $i \in N$ and respects the estate: $\sum_{i \in N} x_i \leq E$. With slight abuse of notation, we will write $x \in \mathcal{B}$ to indicate that x is an allocation for $\mathcal{B}$. We say that:

Efficiency: $x \in \mathcal{B}$ is *efficient* when $\sum_{i \in N} x_i = E$.
Claims-respecting: $x \in \mathcal{B}$ is *claims-respecting* when $x_i \leq a_i$ for each $i \in N$.

When an agent has a claim with an amount of a_i and receives x_i, the satisfaction of that claim may be expressed as:

$$\mathrm{Sat}(x_i, a_i) = \min\left\{\frac{x_i}{a_i},\ 1\right\} \cdot 100\%$$

That is, claim satisfaction is a *constrained* (by the amount of the claim) and *linear* function of receipt.[7] An allocation x for a Broomean problem is said to *satisfy claims in proportion to their strength* just in case, for any two individuals i and j: if i's claims is ρ times as strong as j's claim, then i's claim receives ρ times as much satisfaction. That is:

Satisfies claims in proportion to their strength: $x \in \mathcal{B}$ *satisfies claims in proportion to their strength* when $\mathrm{Sat}(x_i, a_i) = \frac{s_i}{s_j} \cdot \mathrm{Sat}(x_j, a_j)$ for all $i,\ j \in N$.

2.2. The Fairness formula for Broomean problems

We study the implications of applying the Fairness formula (FF) to Broomean problems.[8] For Broomean problems, as we demonstrate in the Appendix, the requirements of the FF afford the following concise and simple presentation:

Fairness formula for $\mathcal{B}$roomean problems: FF$\mathcal{B}$. For a Broomean problem $\mathcal{B} = (E, N, a, s)$, fairness requires one to realize an allocation x which is:

(i) (a) Efficient, i.e x allocates the entire estate.
  (b) Claims-respecting, i.e. x does not award anyone more than their claim.
(ii) Satisfies claims in proportion to their strength.
(iii) Whenever (i) and (ii) conflict: x should respect (i) and satisfy claims in proportion to their strength to as large a degree as possible.

[7]While in this article, we *only* consider fair division problems in which claim satisfaction is linear, we do not commit to the view that claim satisfaction is linear *tout court*, i.e. that claim satisfaction is linear in all fair division problems. For a detailed discussion of this aspect, see our article 'How to be absolutely fair, Part I'.

[8]Importantly, the Fairness formula (FF) is general and thus not restricted to one specific way of modelling fair division problems, i.e. not to one type of *fairness structures*, as we explain in our article 'How to be Absolutely Fair, Part I'. In this article, owing to the close parallels between Broomean problems and the mathematical structures used in the bankruptcy literature, we apply the Fairness formula (FF) just to those. Other examples of fairness structures include apportionment problems (cf. Balinski and Young 2001; Wintein and Heilmann 2018) and cooperative games (cf. Aumann and Maschler 1985; Wintein and Heilmann 2020).

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Whereas FF$\mathcal{B}$(i) and FF$\mathcal{B}$(ii) unambiguously define properties for allocations, the meaning of FF$\mathcal{B}$(iii) is underspecified. We will elaborate on and make precise the content of FF$\mathcal{B}$(iii) in section 3, where we define the absolute priority rule.

For $\mathcal{O}$wing Money we do not need to rely on FF$\mathcal{B}$(iii): allocation $(10,\ 30)$ satisfies FF$\mathcal{B}$(i) and FF$\mathcal{B}$(ii) so that there is no conflict between absolute and comparative fairness for $\mathcal{O}$wing Money. However, not all problems are like that. To see that it may be impossible to simultaneously respect FF$\mathcal{B}$(i) and FF$\mathcal{B}$(ii), consider the following problem.

Needing Owed Money. Romeo owes 20 to Abram and 60 to Benvolio and has 80 left. Abram needs his money twice as strongly as Benvolio. Romeo is bound to care for the needs of Abram and Benvolio, such that Romeo's reason for reimbursing Abram is twice as strong as his reason for repaying Benvolio. How, in order to be fair, should Romeo divide the 80?

$\mathcal{N}$eeding Owed Money is represented as follows:

$$\mathcal{N} = \left(80, \{A, B\}, (20,\ 60), \left(\frac{2}{3}, \frac{1}{3}\right)\right)$$

It is readily verified that $(20,\ 60)$ is the *only* allocation for $\mathcal{N}$ which respects FF$\mathcal{B}$(i). As claims are not satisfied in proportion to their strength in $(20,\ 60)$, $\mathcal{N}$ illustrates that FF$\mathcal{B}$(i) and FF$\mathcal{B}$(ii) may conflict. But also, as $(20,\ 60)$ is the *only* allocation for $\mathcal{N}$ which respects FF$\mathcal{B}$(i), any theory which seeks to resolve this conflict by prioritizing absolute fairness must recommend $(20,\ 60)$ for $\mathcal{N}$.

However, when there is a conflict between absolute and comparative fairness, there are typically *many* allocations which satisfy FF$\mathcal{B}$(i). For instance, consider the following Broomean problem $\mathcal{M}$, to which we will refer later on as the $\mathcal{M}$*ore money problem*:

$$\mathcal{M} = \left(80, \{A, B, C\}, (20,\ 60,\ 40), \left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right)\right)$$

Indeed, as the reader may care to verify, and as a direct consequence of Proposition 2 below, there is no allocation for $\mathcal{M}$ which respects both FF$\mathcal{B}$(i) and FF$\mathcal{B}$(ii). Now there are many allocations for $\mathcal{M}$ which respect FF$\mathcal{B}$(i), such as $(0,\ 60,\ 20)$, $(20,\ 20,\ 40)$ or $(20,\ 36,\ 24)$. Owing to its underspecified meaning, it is not clear which of these allocations is recommended by FF$\mathcal{B}$(iii). In the next section we will introduce the *absolute priority rule* $P^\dagger$, a division rule for Broomean problems whose properties makes precise the content of FF$\mathcal{B}$(iii). For $\mathcal{M}$, $P^\dagger$ recommends $(20,\ 36,\ 24)$.

3. Division Rules

3.1. Division rules and their properties

A *division rule* f maps any Broomean problem $\mathcal{B}$ to an allocation $f(\mathcal{B})$. In section 2.1, we defined what it means for an allocation x to be efficient, claims-respecting and to satisfy claims in proportion to their strength. Each of these three properties gives rise to a corresponding property of a division rule: a division rule

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f is *efficient/claims-respecting/satisfies claims in proportion to their strength* when, for any Broomean problem $\mathcal{B}$, $f(\mathcal{B})$ is efficient/claims-respecting/satisfies claims in proportion to their strength.

Our discussion of allocations for $\mathcal{N}$eeding Owed Money readily translates into the following impossibility result for division rules.

Proposition 1 There is no division rule which is efficient, claims-respecting and which satisfies claims in proportion to their strength.
Proof: This directly follows from our discussion of $\mathcal{N}$eeding Owed Money. □

It will be useful to define the *weighted proportional rule P*, which divides the estate proportional to the *strength-weighted amounts*, $s_i a_i$, of the individuals.

$$P(\mathcal{B})_i = \frac{s_i a_i}{\sum_{j \in N} s_j a_j} E \tag{1}$$

Alternatively, we can define P as follows:

$$P(\mathcal{B})_i = \lambda s_i a_i \text{ with } \lambda > 0 \text{ s.t.} \sum_{i \in N} P(\mathcal{B})_i = E \tag{2}$$

To see that (1) and (2) are, indeed, equivalent, observe that the value of λ which solves (2), call it λ^P, is equal to $\frac{E}{\sum_{j \in N} s_j a_j}$ so that each individual i is awarded $\lambda^P s_i a_i$ according to both definitions of the weighted proportional rule.

Although P is efficient, it is neither claims-respecting nor does it satisfy claims in proportion to their strengths, as its recommendation $P(\mathcal{N}) = (32,\ 48)$ for $\mathcal{N}$eeding Owed Money testifies. Hence, an allocation that is recommended by P may violate both FF$\mathcal{B}$(i) and FF$\mathcal{B}$(ii). Although we do not want to recommend P as a rule of fair division, P conveniently allows us to characterize the conditions for which there *are* allocations which simultaneously satisfy FF$\mathcal{B}$(i) and FF$\mathcal{B}$(ii):

Proposition 2 *For any Broomean problem $\mathcal{B}$, we have:*

(i) *There is an $x \in \mathcal{B}$ which is efficient, claims-respecting and which satisfies claims in proportion to their strength if and only if $P(\mathcal{B})_i \leq a_i$ for all $i \in N$.*
(ii) *If $x \in \mathcal{B}$ is efficient, claims-respecting and satisfies claims in proportion to their strength, then $x = P(\mathcal{B})$*

Proof: Let $\mathcal{B}$ be Broomean problem and let $x \in \mathcal{B}$ be an efficient and claims-respecting allocation. We claim that the following two statements are equivalent:

(a) x satisfies claims in proportion to their strength.
(b) For all $i \in N$: $x_i = \lambda s_i a_i$ for some $\lambda > 0$.

To see that (a) implies (b) note that as x is claims-respecting it follows that $Sat(x_i, a_i) = \frac{x_i}{a_i}$ for all $i \in N$. Thus when x satisfies claims in proportion to their strength it follows that $\frac{x_i}{a_i} = \frac{s_i}{s_j} \frac{x_j}{a_j}$ so that $x_j = \frac{s_j a_j}{s_i a_i} x_i$. So then, as x is efficient, we have that $\sum_{j \in N} \frac{s_j a_j}{s_i \cdot a_i} x_i = E$, so that for all $i \in N$:

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$$x_i = \lambda^P s_i a_i, \text{ with } \lambda^P = \frac{E}{\sum_{j \in N} s_j a_j} > 0$$

Thus, (a) implies (b).

Now let x be an efficient claims-respecting allocation for which (b) holds. It then follows that $Sat(x_i, a_i) = \frac{x_i}{a_i} = \lambda \cdot s_i$ for all $i \in N$. From $Sat(x_i, a_i) = \lambda \cdot s_i$ and $Sat(x_j, a_j) = \lambda \cdot s_j$ it follows that $Sat(x_i, a_i) = \frac{s_i}{s_j} \cdot Sat(x_j, a_j)$ so that x satisfies claims in proportion to their strength. Thus, (b) implies (a).

The left-to-right direction of (i) now follows from the proof that (a) implies (b): an allocation with $\lambda^P s_i a_i$ for all $i \in N$ is equal to $P(\mathcal{B})$. The right-to-left direction of (i) is immediate: if $P(\mathcal{B})_i \leq a_i$ for all $i \in N$ then $P(\mathcal{B})$ is efficient, claims-respecting and satisfies claims in proportion to their strength. Claim (ii) follows from (i).

So, Proposition 2 tells us that it is perfectly fine to use P whenever $P(\mathcal{B})_i \leq a_i$ for all individuals i. The question, which we will answer in section 3.2, thus becomes how P should be extended to Broomean problems with individuals for which $P(\mathcal{B})_i > a_i$.

Now, although P does not satisfy claims in proportion to their strength, it does satisfy all claims that are *partially reimbursed* in proportion to their strength. We say that an allocation $x \in \mathcal{B}$ *satisfies partially reimbursed claims in proportion to their strength* when:

$$\text{Sat}(x_i, a_i) = \frac{s_i}{s_j} \cdot \text{Sat}(x_j, a_j) \text{ for all } i, j \text{ such that } x_i < a_i \text{ and } x_j < a_j$$

The corresponding property for division rules then reads as follows.

Satisfies partially reimbursed claims in proportion to their strength. A division rule f *satisfies partially reimbursed claims in proportion to their strength* when, for any Broomean problem $\mathcal{B}$, $f(\mathcal{B})$ satisfies partially reimbursed claims in proportion to their strength.

Indeed, as an immediate consequence of its definition, P satisfies partially reimbursed claims in proportion to their strength. Another division rule which does so is the *absolute priority rule* $P^\dagger$, to which we will turn next.

3.2 The absolute priority rule $P^\dagger$

The absolute priority rule $P^\dagger$ is defined as follows[9]:

$$P^\dagger(\mathcal{B})_i = \min\{\lambda s_i a_i, a_i\} \text{ with } \lambda > 0 \text{ s.t. } \sum_{i \in N} P^\dagger(\mathcal{B})_i = E \tag{3}$$

It is an immediate consequence of definition (3) that $P^\dagger$ is efficient and claims-respecting. Moreover, it readily follows that $P^\dagger$ recommends an efficient and claims-respecting allocation which satisfies claims in proportion to their strength whenever such an allocation exists:

[9]This definition of $P^\dagger$ is identical to the definition of the weighted constrained proportional rule by Casas-Méndez *et al.* (2011).

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Proposition 3 Let $\mathcal{B}$ be a Broomean problem. If there is an $x \in \mathcal{B}$ which is efficient, claims-respecting and which satisfies claims in proportion to their strength, then $x = P^{\dagger}(\mathcal{B})$.

Proof: Proposition 2 says that an allocation for $\mathcal{B}$ with the three mentioned properties exists iff $P(\mathcal{B}) \leq a_i$ for each $i \in N$ and also, that an allocation $x \in \mathcal{B}$ has the three mentioned properties iff $x = P(\mathcal{B})$. Now it readily follows from the definitions of P and $P^{\dagger}$ that when $P(\mathcal{B}) \leq a_i$ for each $i \in N$, $P^{\dagger}$ and P coincide on $\mathcal{B}$, from which proposition 3 follows. □

Thus, the upshot of proposition 3 is that $P^{\dagger}$ respects both FF$\mathcal{B}$(i) and FF$\mathcal{B}$(ii) whenever doing so is possible. Let us now turn to the sense in which $P^{\dagger}$ gives substance to FF$\mathcal{B}$(iii). In order to do so, let us define, for any Broomean problem $\mathcal{B} = (E, N, a, s)$ and subset $J \subseteq N$ of individuals, the *remainder problem* $\mathcal{B}^J$ as follows:

$$\mathcal{B}^J = \left(E - \sum_{j \in J} a_j,\ N \backslash J,\ a_{N \backslash J},\ s_{N \backslash J}\right)$$

Thus $\mathcal{B}^J$ is the problem that results from $\mathcal{B}$ when the individuals in J leave with their claim amounts so that, per definition, $\mathcal{B}^{\emptyset} = \mathcal{B}$. In remainder problem $\mathcal{B}^J$, the remaining estate $E - \sum_{j \in J} a_j$ has to be divided amongst the remaining individuals in $N \backslash J$ whose claim amounts and strengths are specified by the restrictions of, respectively, a and s to $N \backslash J$. As an example, with respect to $\mathcal{M}$ore money as defined in section 2.2, we have:

$$\mathcal{M}^{\{A\}} = \left(60,\ \{B, C\}, (60, 40), \left(\frac{1}{4}, \frac{1}{4}\right)\right)$$

We now follow Casas-Méndez *et al.* (2011) and define the set $R_{\mathcal{B}}$ of *fully proportionally reimbursable individuals* in a Broomean problem $\mathcal{B}$.

Fully proportionally reimbursable individuals. Let $\mathcal{B}$ be a Broomean problem. The set $R_{\mathcal{B}}$ of fully proportionally reimbursable individuals in $\mathcal{B}$ is defined recursively, by repeated applications of P to $\mathcal{B}$ and its remainder problems $\mathcal{B}^{J_k}$, where:

$$J_0 = \emptyset, \quad J_{k+1} = J_n \cup \{i \in N \backslash J_n | P(\mathcal{B}^{J_k})_i \geq a_i\}$$

$$R_{\mathcal{B}} = J_{k^\star}, \text{where } k^\star \geq 0 \text{ is such that } J_{k^\star} = J_{k^\star + 1}$$

To illustrate the definition of $R_{\mathcal{B}}$ we consider $\mathcal{M}$ore money. Applying P to $\mathcal{M}^{J_0} = \mathcal{M}$ yields $\left(22\frac{6}{7}, 34\frac{2}{7}, 22\frac{6}{7}\right)$ in which only A is allotted more than his claim amount so that $J_1 = J_0 \cup \{A\} = \{A\}$. Applying P to $\mathcal{M}^{J_1} = \mathcal{M}^{\{A\}}$ yields $(36, 24)$ in which no individual gets more than their claim amount. Hence $J_2 = \{A\} \cup \emptyset = \{A\}$ as well, so that $R_{\mathcal{M}} = \{A\}$.

The definition of $R_{\mathcal{B}}$ allows for the following alternative definition of $P^{\dagger}$ which, as we prove below, is equivalent to definition (3).

$$P^{\dagger}(\mathcal{B})_i = \begin{cases} a_i & \text{if } i \in R_{\mathcal{B}} \\ P(\mathcal{B}^{R_{\mathcal{B}}})_i & \text{if } i \in N \backslash R_{\mathcal{B}} \end{cases} \tag{4}$$

As $R_{\mathcal{B}}$ is obtained by repeated applications of P to $\mathcal{B}$ and its remainder problems $\mathcal{B}^J$, an inspection of definition (4) reveals that $P^{\dagger}(\mathcal{B})$ can also be obtained as such.

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Indeed, definition (4) give rise to the following algorithm for computing the $P^{\dagger}$ allocation:

Absolute Priority Rule Algorithm. Award each individual their *P*roportional division of the estate—unless at least one individual's *P*roportional division is larger than their claim, in which case award such individuals their entire claim, remove them from the set of individuals under consideration and their claim from the estate, and repeat.

Let us illustrate this algorithm, i.e. definition (4), by applying it to the following problem:

$$\mathcal{S} = \left(50, \{A, B, C\}, (20,\ 15,\ 30), \left(\frac{1}{2}, \frac{7}{20}, \frac{3}{20}\right)\right)$$

First we apply P to $\mathcal{S}$, which yields $(25.32,\ 13.29,\ 11.39)$. As P allots more to A than his claim amount, A is fully reimbursed, i.e. allotted 20 and removed, resulting in the following remainder problem:

$$\mathcal{S}^{\{A\}} = \left(30, \{B, C\}, (15,\ 30), \left(\frac{7}{20}, \frac{3}{20}\right)\right)$$

Next, apply P to $\mathcal{S}^{\{A\}}$, which yields $(16.15,\ 13.85)$. Now P allots more to B than his claim amount of 15 so that B is fully reimbursed and removed, resulting in remainder problem:

$$\mathcal{S}^{\{A,B\}} = \left(5, \{C\}, 30, \frac{3}{20}\right)$$

Finally, applying P to $\mathcal{S}^{\{A,B\}}$ yields 5 for C, which does not exceed his claim amount. Hence, $P^{\dagger}(\mathcal{S}) = (20,\ 15, 5)$ and $R_{\mathcal{S}} = \{A, B\}$.

Let us now prove, as promised, that definition (3) and (4) are equivalent.

Proposition 4 Definitions (3) and (4) of $P^{\dagger}$ are equivalent.
Proof: Let $\mathcal{B} = (E, N, a, s)$ be a Broomean problem. For each $k \geq 0$, let $J_k \subseteq N$ be defined as in the definition of the set of fully proportionally reimbursable individuals $R_{\mathcal{B}}$. For each J_k, we define λ_k as the value of λ for which:

$$\sum_{j \in J_k} a_j + \sum_{j \in N \setminus J_k} \lambda s_j a_j = E$$

It readily follows from the definition of J_k and λ_k that:

(i) For all $k \geq 0$, for all $j \in N \setminus J_k : \lambda_k s_j a_j = P(\mathcal{B}^{J_k})_j$
(ii) For all $k \geq 1$: $J_k = \{j \in N | \lambda_{k-1} s_j a_j \geq a_j\}$

Let $k^{\star}$ the (smallest) value of k for which $J_{k^{\star}} = J_{k^{\star}+1}$ and remember that $J_{k^{\star}} = R_{\mathcal{B}}$, the set of fully proportionally reimbursable individuals in $\mathcal{B}$. We claim that:

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$$\sum_{i \in N} \min\{\lambda_{k^\star} s_i a_i, a_i\} = \sum_{i \in J_{k^\star}} a_i + \sum_{i \in N \backslash J_{k^\star}} \lambda_{k^\star} s_i a_i = E$$

The first equality of the claim follows from the fact that $J_k^\star = \{j \in N | \lambda_{k^\star} s_j a_j \geq a_j\}$, which directly follows from (ii) and the definition of $\lambda_{k^\star}$. The second equality follows from the definition of $\lambda_{k^\star}$. Hence $\lambda_{k^\star}$ is the value of λ for which $\sum_{i \in N} \min\{\lambda s_i a_i, a_i\} = E$ so that, according to definition (3), each individual i receives $\min\{\lambda_{k^\star} s_i a_i, a_i\}$. As $J_{k^\star} = R_{\mathcal{B}}$, it follows from (i), (ii) and the definition of $\lambda^\star$, that:

$$\min\{\lambda_{k^\star} s_i a_i, a_i\} = \begin{cases} a_i & \text{if } i \in R_{\mathcal{B}} \\ P(\mathcal{B}^{R_{\mathcal{B}}})_i & \text{if } i \in N \backslash R_{\mathcal{B}} \end{cases}$$

Thus, definitions (3) and (4) of $P^\dagger$ are equivalent. □

It is immediately clear from definition (4) that $P^\dagger$ is *fully proportionally reimbursing*, a property which Casas-Méndez *et al.* (2011) define as follows.

Fully proportionally reimbursing. A division rule f is *fully proportionally reimbursing* when, for each Broomean problem $\mathcal{B}$, $f(\mathcal{B})$ is fully proportionally reimbursing: $f(\mathcal{B})_i = a_i$ for each $i \in R_{\mathcal{B}}$.

Conjointly with *efficiency* and *satisfying partially reimbursed claims in proportion to their strength*, the *fully proportionally reimbursing* property can be used to characterize $P^\dagger$, as attested by the following proposition.

Proposition 5 A division rule f is efficient, fully proportionally reimbursing and satisfies partially reimbursed claims in proportion to their strength if and only if f is the absolute priority rule $P^\dagger$.

Proof: It is obvious that $P^\dagger$ has the three properties. Conversely, let f be a division rule which has the three properties. f allots all individuals in $R_{\mathcal{B}}$ their full claim amount as f is fully proportionally reimbursing. As f is efficient, it has to allot all of the remaining $E - \sum_{j \in R_{\mathcal{B}}} a_j$ to the individuals in $N \backslash R_{\mathcal{B}}$. There is a unique way in which this can be done in such a manner that the claims of all (and only) individuals in $N \backslash R_{\mathcal{B}}$ are (partially reimbursed and) satisfied in proportion to their strength and that is by applying P to $\mathcal{B}^{R_{\mathcal{B}}}$. Hence f is the absolute priority rule $P^\dagger$. □

The properties of proposition 5 are *logically independent*, where a set Γ of properties for division rules is said to be logically independent if for each property γ in Γ, there is a division rule which violates γ but which satisfies all other properties in Γ. To see that efficiency, fully proportionally reimbursing and satisfying partially reimbursed claims in proportion to their strength are logically independent, observe that:

- Allocation $(20,\ 24,\ 16)$ for $\mathcal{M} = \left(80, \{A, B, C\}, (20,\ 60,\ 40), \left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right)\right)$ is fully proportionally reimbursing, satisfies partially reimbursed claims in proportion to their strength, but is not efficient. Hence, as $P^\dagger$ is efficient, claims-respecting and satisfies partially reimbursed claims in proportion to their strength, division rule g is claims-respecting and satisfies partially reimbursed claims in proportion to their strength but is not efficient:

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$$g(\mathcal{B}) = \begin{cases} (20, 24, 16) & \text{if } \mathcal{B} = \mathcal{M} \\ P^{\dagger}(\mathcal{B}) & \text{otherwise.} \end{cases}$$

Thus, g establishes that efficiency is logically independent of being fully proportionally reimbursing and satisfying partially reimbursed claims in proportion to their strength.

- Similarly, to see that being fully proportionally reimbursing is independent of efficiency and satisfying partially reimbursed claims in proportion to their strength, observe that (10, 60, 10) for $\mathcal{M}$ is efficient, satisfies partially reimbursed claims in proportion to their strength, but is not fully proportionally reimbursing (as $R_{\mathcal{M}} = \{A\}$ and as A does not get fully reimbursed).
- Similarly, to see that satisfying partially reimbursed claims in proportion to their strength is independent of being fully proportionally reimbursing and efficiency, observe that (20, 30, 30) for $\mathcal{M}$ is fully proportionally reimbursing and efficient but does not satisfy partially reimbursed claims in proportion to their strength.

The characterization of $P^{\dagger}$ that is provided by proposition 5 is normatively appealing: it is in virtue of proposition 5 and proposition 3 that we claim that $P^{\dagger}$ captures and makes precise the content of FF$\mathcal{B}$. Proposition 3 tells us that $P^{\dagger}$ selects an allocation which respects both FF$\mathcal{B}$(i) and FF$\mathcal{B}$(ii) whenever doing so is possible. Whenever such is not possible, FF$\mathcal{B}$(iii) says that fairness requires to select an allocation which respects the requirements of absolute fairness, i.e. FF$\mathcal{B}$(i), and which 'satisfies claims in proportion to their strength to as large a degree as possible'. By selecting an efficient allocation which fully reimburses all fully proportionally reimbursable individuals and in which the claims of all other individuals are satisfied in proportion to their strength, the absolute priority $P^{\dagger}$ does exactly that: the properties of $P^{\dagger}$ make precise the content of the Fairness formula for $\mathcal{B}$roomean problems.

3.3. The comparative priority rule $P^{\ddagger}$

In this section, we will define the *comparative priority rule* $P^{\ddagger}$. Although we do not think that $P^{\ddagger}$ is normatively appealing, it is interesting to study $P^{\ddagger}$ as it is the counterpart of the absolute priority rule $P^{\dagger}$: in case of a conflict between comparative and absolute fairness, the comparative priority rule $P^{\ddagger}$ prioritizes the former over the latter while it does 'as much as possible to respect the requirements of absolute fairness'.

According to the *comparative priority rule* $P^{\ddagger}$ an individual i receives:

$$P^{\ddagger}(\mathcal{B})_i = \lambda^{\ddagger} s_i a_i, \text{ with } \lambda^{\ddagger} \text{the solution to the following problem :} \tag{5}$$

$$\begin{array}{llll} & \max \quad \lambda & & \\ \text{subject to :} & \lambda s_i a_i \leq a_i & \text{for all } i \in N, & (\textit{amount constraint}) \\ & \sum_{i \in N} \lambda s_i a_i \leq E & & (\textit{estate constraint}) \end{array}$$

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It readily follows from its definition that $P^{\ddagger}$ is maximally claims-respecting proportional where:

Maximally claims-respecting proportional. A division rule f is *maximally claims-respecting proportional* when, for any Broomean problem $\mathcal{B}$, $f(\mathcal{B})$ is maximally claims-respecting proportional, i.e.:

(i) $f(\mathcal{B})$ satisfies claims in proportion to their strength, and
(ii) there is no $y \in \mathcal{B}$ which is claims-respecting, which satisfies claims in proportion to their strength and which is such that $\sum_{i \in N} y_i > \sum_{i \in N} f(\mathcal{B})_i$.

In fact, the maximally claims-respecting proportional property by itself characterizes $P^{\ddagger}$, as recorded by the following proposition.

Proposition 6 *A division rule f is maximally claims-respecting proportional if and only if f is the comparative priority rule $P^{\ddagger}$.*

Proof: For any Broomean problem, there is a unique allocation which is maximally claims-respecting proportional and it is clear from the definition of $P^{\ddagger}$ in terms of constrained optimization problem (5) that, for any Broomean problem, $P^{\ddagger}$ selects this allocation. □

Although $P^{\ddagger}$ always recommends maximally claims-respecting proportional allocations, $P^{\ddagger}$ does not always recommend *maximally proportional allocations*, where an allocation $x \in \mathcal{B}$ is maximally proportional just in case:

(i) x satisfies claims in proportion to their strength, and
(ii) there is no $y \in \mathcal{B}$ which satisfies claim in proportion to their strength and which is such that $\sum_{i \in N} y_i > \sum_{i \in N} x_i$.

To see that $P^{\ddagger}$ need not yield maximally proportional allocations, consider Broomean problem $\mathcal{T}$:

$$\mathcal{T} = \left(14, \{A, B\}, (10,\ 6), \left(\frac{2}{3}, \frac{1}{3}\right)\right)$$

Note that allocation $(11,\ 3)$ for $\mathcal{T}$ is maximally proportional: the claims of A and B receive 100% and 50% satisfaction respectively so that $(11,\ 3)$ satisfies claims in proportion to their strength and, as $(11,\ 3)$ is efficient, is maximally proportional as well. But $(11,\ 3)$ is not a claims-respecting allocation and thus not recommended by $P^{\ddagger}$. Indeed, $(10,\ 3)$ is maximally claims-respecting proportional (but not maximally proportional) and thus recommended by $P^{\ddagger}$.

Our characterization of $P^{\ddagger}$ in terms of a single property is basically a restatement of $P^{\ddagger}$'s definition as given by (5). This definition requires, in order to determine $P^{\ddagger}(\mathcal{B})$, to solve a constrained optimization problem. However, $P^{\ddagger}$ also allows for an alternative, computationally much simpler definition that we will present next. In order to do so remember that λ^{P} is the value of λ which solves equation (4) so

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that, per definition, $P(\mathcal{B})_i = \lambda^P s_i a_i$. Also, let $s^{max} = \max\{s_1, \ldots, s_n\}$ be the maximal claim-strength in $\mathcal{B}$. Then, an alternative definition of $P^{\ddagger}$ is as follows:

$$P^{\ddagger}(\mathcal{B})_i = \lambda^{\ddagger} s_i a_i, \text{with } \lambda^{\ddagger} = \min\left\{\frac{1}{s^{max}}, \lambda^P\right\} \tag{6}$$

Proposition 7 *Definitions (5) and (6) of $P^{\ddagger}$ are equivalent.*

Proof: Consider the constrained optimization problem of definition (5). Now, for $\lambda = 0$ the amount and estate constraint are clearly respected but equally clearly, 0 is not the largest value of λ for which these constraints are respected. To obtain the largest value of λ which respects the constraints, start with $\lambda = 0$ and then increase λ until one of the constraints becomes *binding*, i.e. until λ reaches a value for which one of the constraints is respected with equality. The value of λ for which a constraint becomes binding in this manner, i.e. the largest value of λ which respects all the constraints, we call $\lambda^{\ddagger}$. There are two relevant cases pertaining to the type of the constraint that becomes binding:

(i) One of the amount constraints becomes binding., i.e. $\lambda^{\ddagger} s_i a_i = a_i$ so that $\lambda^{\ddagger} s_i = 1$ for some $i \in N$. If so, it clearly has to be the constraint of the individual with the largest claim-strength that is binding, so that $\lambda^{\ddagger} = \frac{1}{s^{max}}$.
(ii) The estate constraint becomes binding, i.e. $\sum_{i \in N} \lambda^{\ddagger} s_i a_i = E$. In this case we have $\lambda^{\ddagger} = \lambda^P$.

Clearly then, when $\frac{1}{s^{max}} \leq \lambda^P$ we are in case (i) and when $\lambda^P \leq \frac{1}{s^{max}}$ we are in case (ii), so that $\lambda^{\ddagger}$ is the minimum of $\frac{1}{s^{max}}$ and λ^P, which is what definition (6) says. □

As we remarked above, we do not think that $P^{\ddagger}$ is normatively appealing. However, in the philosophical debate on absolute and comparative fairness, Hooker (2005: 341) seems to suggest otherwise when he writes that 'fairness requires the greatest possible proportionate satisfaction of claims'. Now to say that an allocation should realize 'the greatest possible proportionate satisfaction of claims' seems to be tantamount to saying that an allocation should be *maximally claims-respecting proportional*. Hence, $P^{\ddagger}$ arguably makes precise Hooker's intuitive sketch of an account of absolute and comparative fairness. Or, if not, $P^{\ddagger}$ forces Hooker to re-articulate his account in different terms.

3.4. Three proportionality rules

As the reader may have observed already, the three 'proportionality rules' P, $P^{\dagger}$ and $P^{\ddagger}$ allow for a uniform presentation. Indeed, with λ^P, $\lambda^{\dagger}$ and $\lambda^{\ddagger}$ the values of λ which solve, respectively, (2), (3) and (5), we have:

$$P(\mathcal{B})_i = \lambda_i^P s_i a_i, \quad P^{\dagger}(\mathcal{B})_i = min\left\{\lambda^{\dagger} s_i a_i, a_i\right\}, \quad P^{\ddagger}(\mathcal{B})_i = \lambda_i^{\ddagger} s_i a_i$$

These 'λ-definitions' of the proportionality rules may be helpful for understanding the relations between P, $P^{\dagger}$ and $P^{\ddagger}$. In particular, one readily verifies that:

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Table 1. Allocations for $\mathcal{N}$eeding Owed Money as a function of λ

λ	$(\lambda \cdot \frac{2}{3} \cdot 20,\ \lambda \cdot \frac{1}{3} \cdot 60)$	$(min\{\lambda \cdot \frac{2}{3} \cdot 20,\ 20\}, min\{\lambda \cdot \frac{1}{3} \cdot 60, 60\})$
1.2	$(16,\ 24)$	$(16,\ 24)$
$\lambda^{\ddagger} = 1.5$	$P^{\ddagger}(\mathcal{N}) = (20,\ 30)$	$(20,\ 30)$
1.8	$(24,\ 36)$	$(20,\ 36)$
2.1	$(28,\ 42)$	$(20,\ 42)$
$\lambda^{P} = 2.4$	$P(\mathcal{N}) = (32,\ 48)$	$(20,\ 48)$
2.7	–	$(20,\ 54)$
$\lambda^{\dagger} = 3.0$	–	$P^{\dagger}(\mathcal{N}) = (20,\ 60)$

- For any Broomean problem $\mathcal{B}$: $\lambda^{\ddagger} \leq \lambda^{P} \leq \lambda^{\dagger}$.
- If there is no $x \in \mathcal{B}$ which is efficient, claims-respecting and which satisfies claims in proportion to their strength, then $\lambda^{\ddagger} < \lambda^{P} < \lambda^{\dagger}$.

The second claim is illustrated by Table 1, which records the recommendations of the three proportionality rules for $\mathcal{N}$eeding Owed Money:

$$\mathcal{N} = \left(80, \{A, B\}, (20,\ 60), \left(\frac{2}{3}, \frac{1}{3}\right)\right)$$

4. Fair Division in Economics

4.1. Bankruptcy problems

Fair division problems such as Owing Money are paradigmatic for the mathematical and economic literature (Thomson 2019) on so-called *bankruptcy problems*. A bankruptcy problem is a triple

$$\mathfrak{B} = (E, N, a)$$

where $E > 0$, $N = \{1, \ldots, n\}$, $a_i \geq 0$ and $\sum_{i \in N} a_i \geq E$. Indeed, there are important parallels between these fairness structures: a bankruptcy problem is, formally, a Broomean problem without claim strengths.[10]

As for its interpretation, the individuals in a bankruptcy problem are sometimes referred to as 'claimants' and the entries in a as representing their 'claims'. Indeed, it is not uncommon in the literature to refer to bankruptcy problems as 'claims problems'. However, the notion of a 'claim' is basically a primitive term here, with a much broader meaning than it has in the philosophical literature and often times the a vector is interpreted as specifying demands or wants.

[10]As we explain in detail in our article 'How to be absolutely fair, Part I: the Fairness formula', verbal descriptions of fair division problems such as Owing Money can be modelled in different *fairness structures* (such as Broomean problems, or bankruptcy problems). Fairness structures take the available information from verbal descriptions of fair division problems and organize it in a framework that allows to make recommendations for how to divide fairly.

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Nevertheless, fair division problems which involve (Broomean) *claims that are all of equal strength*, such as Owing Money, are conveniently represented as bankruptcy problems. Indeed, we can either represent Owing Money as Broomean problem $\mathcal{O}$, as we did before, or as a bankruptcy problem $\mathfrak{O}$.

$$\mathcal{O} = \left(40, \{A, B\}, (20,\ 60), \left(\frac{1}{2}, \frac{1}{2}\right)\right), \quad \mathfrak{O} = (40, \{A, B\}, (20,\ 60))$$

The *proportional rule* $\mathbb{P}$ divides the estate in any bankruptcy problem proportional to claims:

$$\mathbb{P}(\mathfrak{B})_i = \frac{a_i}{\sum_{j \in N} a_j} \cdot E$$

For Owing Money, we have that $\mathbb{P}(\mathfrak{O}) = P^{\dagger}(\mathcal{O})$: the recommendation of $\mathbb{P}$ coincides with that of the absolute priority rule $P^{\dagger}$ which is induced by the FF. It readily follows, as recorded by the following proposition, that this result holds for any problem in which all claims are equally strong.

Proposition 8 *Let* $\mathfrak{B} = (E, N, a)$ *be a bankruptcy problem, let* $\bar{s}_i = \frac{1}{n}$ *for all* $i \in N$ *and let* $\mathcal{B} = (E, N, a, \bar{s})$ *be the equal-strength Broomean problem induced by* $\mathfrak{B}$. *Then:* $\mathbb{P}(\mathfrak{B}) = \mathbb{P}(\mathcal{B}) = \mathbb{P}^{\dagger}(\mathcal{B}) = \mathbb{P}^{\ddagger}(\mathcal{B})$.
Proof: Immediate from definitions. □

Now, the proportional rule $\mathbb{P}$ is but one of the many bankruptcy rules that are studied in the economic literature (Herrero and Villar 2001). Consider Table 2, which displays allocations for $\mathfrak{O}$wing Money for three different allocation rules.

Whereas $\mathbb{P}$ awards shares proportional to claims, *CEA* equalizes awards as much as possible, without giving any agent more than their claim. *CEL* first calculates the difference between the sum of all claims (80) and the estate (40) to determine the joint loss L (40), which is then equally shared between all individuals without awarding any individual a negative amount. More generally, *CEA*, *CEL* and $\mathbb{P}$ can be defined as follows:

$$CEA(\mathfrak{B})_i = \min\{a_i, \lambda\} \text{ with } \lambda > 0 \text{ s.t.} \sum_{i \in N} CEA(\mathfrak{B})_i = E$$

$$CEL(\mathfrak{B})_i = \max\{a_i - \lambda, 0\} \text{ with } \lambda > 0 \text{ s.t.} \sum_{i \in N} CEL(\mathfrak{B})_i = E$$

$$\mathbb{P}(\mathfrak{B})_i = \lambda a_i \text{ with } \lambda > 0 \text{ s.t.} \sum_{i \in N} \mathbb{P}(\mathfrak{B}) \ = E$$

Apart from $\mathbb{P}$, *CEA* and *CEL*, a multitude of other bankruptcy rules have been proposed in the literature. Indeed, proportionality is, in contrast to the philosophical literature, not afforded a special normative status:

> The best-known rule is the proportional rule, which chooses awards proportional to claims. Proportionality is often taken as the definition of fairness [. . .], but we will challenge this position and start from more elementary considerations. (Thomson 2003: 250)

Table 2. Allocations for 𝔒wing Money, in different bankruptcy rules

Division rules	Abram	Benvolio
Proportional rule $\mathbb{P}$	10	30
Constrained Equal Awards (*CEA*)	20	20
Constrained Equal Losses (*CEL*)	0	40

Indeed, in the literature on bankruptcy problems, alternative rules are studied and compared on the basis of their elementary properties or *axioms*. An important aspect of the study of bankruptcy problems characterize bankruptcy rules, i.e. to single out a bankruptcy rule as the only one satisfying a set set of axioms. Preferably, the axioms occurring in a characterization are logically independent from one another and plausible in the sense that they embody clear ethical or operational criteria. Indeed, we have adopted this approach in section 3.2, where we characterized the absolute priority rule $P^\dagger$ in terms of three logically independent properties, which are plausible in the sense that they embody the principles of absolute and comparative fairness as articulated by FF$\mathcal{B}$.

As such, the literature on bankruptcy problems offers little help for understanding the behaviour and properties of $P^\dagger$, or other division rules, that are defined for *all* Broomean problems – where claims may vary in amounts *and* strengths. However, an underdeveloped extension of the basic model of bankruptcy problems assigns *weights* to claimants. By doing so, one obtains *weighted bankruptcy problems*, which are formal equivalents of our Broomean problems and which we discuss in the next section.

4.2. Weighted bankruptcy problems

A weighted bankruptcy problem is a tuple $\mathcal{B} = (E, N, a, s)$ where $E > 0$, $N = \{1, \ldots, n\}$, $a_i \geq 0$, $\sum_{i \in N} a_i \geq E$ and $s_i > 0$. The interpretation of E, N and a carries over from the standard bankruptcy model whereas s indicates a vector of *weights*, which …

> indicate the relative importance that should be given to claimants […], with the convention that a relatively larger weight assigned to a claimant is to be interpreted as a desired more favourable treatment for that claimant. (Thomson 2019: 82)

This interpretation of the 'weights' s is rather broad, but subsumes our interpretation of s as recording claim *strengths*, indicating the relative strength of the reason we have for satisfying the claim of a particular individual. Indeed, a Broomean problem just is a weighted bankruptcy problem under a specific interpretation of the *a*mounts and *s*trengths of *claims*.

Whereas there is a vast literature on bankruptcy problems, their weighted versions have received considerably less attention. One of the few exceptions is Casas-Méndez *et al.* (2011:161), who 'consider a relevant topic on the subject of

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bankruptcy which has not to date received much attention: weighted bankruptcy problems'. The authors define, and characterize, weighted versions of the $\mathbb{P}$, *CEA* and *CEL* bankruptcy rules:

$$\mathbb{P}^w(\mathcal{B})_i = \min\{\lambda s_i a_i, a_i\} \text{ with } \lambda > 0 \text{ s.t.} \sum_{i \in N} \mathbb{P}^w(\mathcal{B})_i = E$$

$$CEA^w(\mathcal{B})_i = \min\{a_i, \lambda s_i\} \text{ with } \lambda > 0 \text{ s.t.} \sum_{i \in N} CEA^w(\mathcal{B})_i = E$$

$$CEL^w(\mathcal{B})_i = \max\{a_i - \lambda s_i, 0\} \text{ with } \lambda > 0 \text{ s.t.} \sum_{i \in N} CEL^w(\mathcal{B})_i = E$$

Indeed, the *weighted constrained proportional rule* $\mathbb{P}^w$ of Casas-Méndez *et al.* is identical to the absolute priority rule $P^\dagger$.

Now, the main motivation that Casas-Méndez *et al.* (2011: 161) give for defining and studying $\mathbb{P}^w$, CEA^w or CEL^w is that 'some of the most important bankruptcy rules have not been studied in the weighted framework'. In particular, they do little to motivate $\mathbb{P}^w$ and do not argue that $\mathbb{P}^w$ is preferable to the other two rules.

Also, Casas-Méndez *et al.* simply call $\mathbb{P}^w = P^\dagger$ *the* extension of $\mathbb{P}$ to the weighted framework and do not consider P or $P^\ddagger$ as candidates. The latter is, at least from a formal perspective, understandable. For Casas-Méndez *et al. define* a division rule to be efficient and claims-respecting. In fact, the latter convention is common in the economics literature. Yet, according to this convention, neither P nor $P^\ddagger$ qualify as division rule. We think our discussion shows that it is theoretically fruitful to adopt a more liberal definition of a division rule (one that does not presuppose that a rule is efficient or claims-respecting). Indeed, by doing so we can contrast and compare $P^\dagger$ with P and $P^\ddagger$, which fosters the study of (absolute and comparative) fairness and proportionality.

In addition to defining $\mathbb{P}^w$, CEA^w and CEL^w, Casas-Méndez *et al.* (2011) characterize these weighted bankruptcy rules. Translated to the framework of this paper, in which division rules are neither assumed to be efficient nor claims-respecting, they provide the following characterization of $P^\dagger$.

Proposition 9 $P^\dagger$ is the only division rule which is efficient, fully proportionally reimbursing, strategy-proof for amounts and strategy-proof for strengths.
Proof: See Casas-Méndez *et al.* (2011). □

Thus whereas proposition 5 characterizes $P^\dagger$ in terms of efficiency, fully proportionally reimbursing and satisfying partially reimbursed claims in proportion to their strength, the characterization of proposition 9 trades in our last axiom for two axioms of strategy-proofness. Roughly, a division rule is *strategy-proof for amounts* when no group of individuals K *whose claims all have the same strength* σ can benefit from aggregating their claims into a single claim with amount $\sum_{\in K} a_j$ and strength σ. Conversely, a division rule is *strategy-proof for strengths* when no group of individuals K *whose claims all have the same amount* α can benefit from aggregating their claims into a single claim with amount α and strength $\sum_{j \in K} s_j$. Formally, the two axioms of strategy-proofness are defined as follows.

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Strategy-proof for amounts. Let $\mathcal{B} = (E, N, a, s)$ be a Broomean problem and let $\mathcal{B}' = (E, N', a', s')$ be obtained from $\mathcal{B}$ by replacing an individual $i \in N$ with a set $K = \{i_1, \ldots, i_k\}$ of individuals with claims whose amounts sum to a_i and whose strengths are all equal to s_i.[11] A division rule f is *strategy-proof for amounts* if for any $\mathcal{B}$ and $\mathcal{B}'$ that are related as such, we have $f(\mathcal{B})_j = f(\mathcal{B}')_j$ for all $j \in N - \{i\}$.

Strategy-proof for strengths. Let $\mathcal{B} = (E, N, a, s)$ be a Broomean problem and let $\mathcal{B}' = (E, N', a', s')$ be obtained from $\mathcal{B}$ by replacing an individual $i \in N$ with a set $K = \{i_1, \ldots, i_k\}$ of individuals with claims whose strengths sum to s_i and whose amounts are all equal to a_i.[12] A division rule f is *strategy-proof for strengths* if for any $\mathcal{B}$ and $\mathcal{B}'$ that are related as such, we have $f(\mathcal{B})_j = f(\mathcal{B}')_j$ for all $j \in N - \{i\}$.

The strategy-proof for amounts and the strategy-proof for strengths axiom generalize the strategy-proofness axiom[13] for bankruptcy rules (cf. O'Neill (1982)) to the weighted framework. The proof of proposition 9 is an adaptation of a proof due to Curiel *et al.* (1987), who characterize bankruptcy rule $\mathbb{P}$ in terms of efficiency, equal treatment of equals and strategy-proofness[14]. Just as for their definition of $\mathbb{P}^w$, Casas-Méndez *et al.* hardly motivate or justify (the properties occurring in) their characterization of $\mathbb{P}^w$. We do not doubt that, on some occasions, it may be desirable to have a strategy-proof division rule at one's disposal. However, desirable as they may be, it is hard to see how the strategy-proofness axioms can be justified in terms of *fairness*. In sharp contrast, the axioms used in the characterization given by proposition 5 can all be justified by appealing to the account of absolute and comparative fairness that we develop in our article 'How to be absolutely fair, Part I'.

At the same time, our characterization of $P^\dagger$ is less informative than that of Casas-Méndez *et al.* in the sense that the properties used in our characterization can be more or less 'read off' of the definition (4) of $P^\dagger$. In contrast, to find out that $P^\dagger$ satisfies the two strategy-proofness axioms is not something that can easily be deduced from an inspection of its definition. There is a further difference between the two characterizations. We exploit three properties to characterize $P^\dagger$ that are *punctual*, while the strategy-proofness axioms of Casas-Méndez *et al.* are *relational*, where:

> A punctual axiom applies to each problem separately and a relational axiom relates choices made across problems that are related in certain ways.
>
> (Thomson 2012: 391)

For example, efficiency is a punctual axiom: it specifies that *for each problem* (separately) an allocation which allots all of the estate has to be selected. Similarly, fully proportionally reimbursing and satisfying partially reimbursed

[11]Thus $(a'_j, s'_j) = (a_j, s_j)$ for all $j \in N - \{i\}$, $\sum_{j \in K} a'_j = a_i$ and $s_j = s_i$ for all $j \in K$.

[12]Thus $(a'_j, s'_j) = (a_j, s_j)$ for all $j \in N - \{i\}$, $\sum_{j \in K} s'_j = s_i$ and $a_j = a_i$ for all $j \in K$.

[13]Let $\mathfrak{B} = (E, N, a)$ be a bankruptcy problem and let $\mathfrak{B}' = (E, N', a')$ be obtained from $\mathfrak{B}$ by replacing an individual $i \in N$ with a set $K = \{i_1, \ldots, i_k\}$ of individuals whose claims sum to a_i. A bankruptcy rule f is *strategy-proof* if for any $\mathfrak{B}$ and $\mathfrak{B}'$ that are related as such, we have $f(\mathcal{B})_j = f(\mathcal{B}')_j$ for all $j \in N - \{i\}$.

[14]Let $\mathfrak{B} = (E, N, a)$ be a bankruptcy problem and let $\mathfrak{B}' = (E, N', a')$ be obtained from $\mathfrak{B}$ by replacing an individual $i \in N$ with a set $K = \{i_1, \ldots, i_k\}$ of individuals whose claims sum to a_i. A bankruptcy rule f is *strategy-proof* if for any $\mathfrak{B}$ and $\mathfrak{B}'$ that are related as such, we have $f(\mathcal{B})_j = f(\mathcal{B}')_j$ for all $j \in N - \{i\}$.

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claims in proportion to their strength are punctual axioms. In contrast, strategy-proofness for amounts (and strengths) is a relational axiom and, as such, makes a conditional statement: *if* two problems $\mathcal{B}$ and $\mathcal{B}'$ are *related* in a certain way, *then* the recommended allocations for $\mathcal{B}$ and $\mathcal{B}'$ should also be related in a certain way – in the case of strategy-proofness the recommended allocations should be identical.

5. Conclusion

Fairness has an absolute and a comparative dimension. The requirements of both dimensions of fairness are captured in general terms by the Fairness formula (FF). In this article, we applied the FF to Broomean problems which yielded FF$\mathcal{B}$. We used FF$\mathcal{B}$ to study associated division rules and argued that FF$\mathcal{B}$ singles out the absolute priority rule $P^{\dagger}$ as the rule of *fair* division. We observed that a Broomean problem is formally equivalent to a *weighted* bankruptcy problem, a recent extension of the *bankruptcy problems* that has received considerable attention in the economic literature.

Apart from the specific results we obtained in this article, the more encompassing message is this: the philosophical literature on fairness and the economic literature on (weighted) bankruptcy problems allow for fruitful interdisciplinary research on fairness and fair division. In particular, the philosophical literature on (Broomean) fairness offers a conceptually rigorous analysis of the notion of 'claims' and 'fairness'. Philosophical fairness theories are hence geared towards expressing precisely *what it means* to be (absolutely) fair. By contrast, in the economic literature, concepts such as claims and fairness are typically employed with relative conceptual liberty. Yet, the mathematical precision of the economic frameworks affords to operationalize them and facilitates axiomatic study of a wide variety of division rules. This literature is hence geared towards characterizing with great precision *how* to be (absolutely) fair. That is, conjointly, the philosophical and economic literatures harbour the resources needed for the development of a theory of fairness which tells us what fairness *is* and how it should be *realized*.

Together with our 'How to be absolutely fair, Part I: the Fairness formula', we have presented an outline of a two-dimensional theory of fairness which tells us what it *means* to be fair and how to *realize* fair divisions. Our theory draws on and exploits hitherto un(der)appreciated differences and complementarities between philosophy and economics fairness research. In future work, we hope to apply the Fairness formula to fairness structures beyond Broomean problems, including structures where the estate is an indivisible good (such as seats in a parliament) or structures where claims cannot be unambiguously ascribed to individuals.[15] By doing so, we hope to contribute to the development of a comprehensive theory of fairness. For now, we hope that our two articles motivate authors from both philosophy and economics to join us in developing such a theory. Fairness has not only two dimensions, comparative and absolute, but there are also two disciplines which are both key to understand it better.

[15]As explained in Heilmann and Wintein (2017) cooperative game theory yield fairness structures that allows one to model such situations. To apply the Fairness formula to cooperative games is work in progress.

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Appendix

From the Fairness formula to FF$\mathcal{B}$

Here is the Fairness formula:

Fairness formula (FF). Fairness requires one: (i) to satisfy absolute claims (of individuals and groups) to as large an extent as possible, subject to the constraint that no one receives more than they have a claim to; (ii) to satisfy (absolute and notional) individual claims in proportion to their strength; (iii) to prioritize requirement (i) over (ii) whenever these two conflict, but in such a way that one does as much as possible to respect (ii).

In this appendix, we will briefly explain the constituent notions of the FF and indicate how one derives FF$\mathcal{B}$ from the Fairness formula. To foster our discussion, we will contrast and compare the representations, as Broomean problems, of Owing Money and Investing Time:

Investing Time. Anna and Beta have invested time in realizing a joint project. For a certain period of time, Anna has spent one day a week on the project, whereas Beta has spent three days a week on it. After some time, Anna and Beta split apart and their fiduciary, Rachel, is responsible for the division of the value of their project, which is 20. How, in order to be fair, should Rachel divide the 20?

Here is how $\mathcal{O}$wing Money and $\mathcal{I}$nvesting Time are represented as Broomean problems:

$$\mathcal{O} = \left(40, \{A, B\}, (20,\ 60), \left(\frac{1}{2}, \frac{1}{2}\right)\right), \quad \mathcal{I} = \left(20, \{A, B\}, (20,\ 20), \left(\frac{1}{4}, \frac{3}{4}\right)\right)$$

In Owing Money, Romeo owes it to Abram to allot him *all of* the 20, *irrespective of what claims other agents may have*. Hence, we say that Abram has an *absolute claim* with an amount of 20 and similarly Benvolio has an *absolute claim* with an amount of 60. The *strength* of a claim specifies how strong the reason is, *as compared to the reasons for satisfying the claims of the other agents*, for satisfying that particular claim. That is, claim-strength is a *strictly comparative* notion. As Romeo has just as much reason to reimburse Abram as he has to reimburse Benvolio, the claims of Abram and Benvolio – to get reimbursed by Romeo – are equally strong. Hence, $\mathcal{O}$wing Money can represented as Broomean problem $\mathcal{O}$ *of type* **I**:

$$(E, N, a, s) \text{ is of type } \mathbf{I}\text{: each } i \in N \text{ has an absolute claim to } a_i$$

Now absolute fairness, as specified by FF(i), requires that absolute claims 'are satisfied to as great a degree as possible'. For Owing Money, and other type **I** problems, this comes down to the requirement that all of the estate is allocated in such a way than no claimant receives more than the amount of their claim, exactly as FF$\mathcal{B}$(i) has it.

But not all claims are absolute. To see this, consider $\mathcal{I}$nvesting Time. Anna has contributed to the realization of the joint project, in virtue of which she has a claim to its value of 20. However, her claim is not absolute as she should get (only) a part of the 20, a part which is determined by comparing her claim with that of others. Claims which are not absolute we call *notional*. So both Anna and Beta have a notional claim with an amount of 20. However, as Beta spent three times as much time on the project as Anna did, her claim is ('all else being equal') three times as strong as that of Anna. Hence, $\mathcal{I}$ represents the amounts and strengths of the notional claims in $\mathcal{I}$nvesting Time. So far so good.

But now a puzzle arises. For, as absolute fairness requires the satisfaction of *absolute* claims and as neither Ann nor Bob has an absolute claim in Investing Time, how can the Fairness formula be invoked to justify FF-C(i) for Investing Time? Here is how. Although neither Anna nor Beta has an absolute claim, this is not to say that no agent has an absolute claim in Investing Time. As Anna and Beta *jointly* realized the value of 20, Rachel owes it to *Anna and Bob together* to allot the 20, and all of the 20, to *them*. That is, the *group* consisting of Anna and Beta has an absolute claim with an amount of 20. That is, $\mathcal{I}$nvesting Time is a Broomean problem *of type* **II**:

$$(E, N, a, s), \text{is of type } \mathbf{II} : \begin{cases} \text{group } N \text{ has an absolute claim to } E \\ \text{each} i \in N \text{ has a notional claim to } a_i = E \end{cases}$$

In How to be absolutely fair, Part I, we further specify what it means to say that one allocation satisfies (individual or group) claims to a larger extent than another:

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- Allocation satisfies the claims of the individuals to a larger extent than allocation y when the satisfaction afforded by x to *each* of the individuals is greater than or equal to that afforded by y and strictly greater for *at least one* individual.[16]
- Allocation x satisfies the claim of a group of individuals to a larger extent than y when the sum-total allotted these individuals by x satisfies their group claim to a larger extent than the sum-total allotted to them by y.[17]

We then stipulate that:

- Allocation x *satisfies claims to as large an extent as possible* just in case there is no allocation y available which satisfies claims to a larger extent than x does.

With these definitions in place, it then readily follows that, for Broomean problems of type **I** and **II**, FF(i) requires the realization of an efficient and claims-respecting allocation, which is exactly what FF$\mathcal{B}$(i) says. Also, for Broomean problems of type **I** and **II**, FF(ii) clearly implies FF$\mathcal{B}$(ii). In a nutshell, for Broomean problems of type **I** and **II**, FF implies FF$\mathcal{B}$. So indeed, our justification of FF$\mathcal{B}$ in terms of the Fairness formula is conditional on the assumption, for which we have not argued and will not argue, that any Broomean problem is of type **I** or **II**.

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[16]That is: $Sat(x_i, a_i) \geq Sat(y_i, a_i)$ for all i and $Sat(x_i, a_i) > Sat(y_i, a_i)$ for some i in N.
[17]So when group N has a claim with amount E the condition is that $Sat\left(\sum_{i \in N} x_i, E\right) > Sat\left(\sum_{i \in N} y_i, E\right)$.

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