IIB. THE UNDERLYING STARS: THEORY

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ROTATING STELLAR ATMOSPHERES

(Review Paper)

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Abstract. The effects of rapid rotation on the emergent energy distribution, line profiles, atmospheric motions and polarization are discussed. A simplified explanation of some of the effects is presented. Results of detailed radiation transfer calculations are briefly reviewed. The rotation can lead to circulation and turbulent motions in the photospheric layers which could affect the outflow from the Be stars. The rotation rates actually observed in the Be stars are sufficiently below the critical rate that many of the effects predicted by the plane parallel atmosphere calculations should be small. Nevertheless, the models are useful and necessary for estimating rotation speeds from lines that are widely separated in wavelength. The rapidly rotating photospheric models predict far too small an infrared excess, as well as too small an intrinsic polarization. The explanation of these observations requires that geometrically extended envelopes be considered. Theoretical models for the intrinsic polarization are critically discussed. It is stressed that polarization is a powerful diagnostic for determining the asymmetrical structure of the outer atmospheres of the Be stars.

ROTATING STELLAR PHOTOSPHERES

Let us consider some of the interesting effects that we could observe if we could spin-up a main sequence star to a very rapid rotation rate. Model calculations tell us that following would occur as the rotation rate increases beyond about 80 per cent of the critical speed.

—The overall shape of the star becomes flattened.
—The equatorial regions become cooler relative to those near the pole.
—The emergent flux thus tends to become "flatter ", because of the development of an excess of infrared radiation (Collins & Sonneborn 1978).
—The observable energy distribution becomes dependent on both the rotation rate and on the angle at which the star is viewed. This has an odd effect on the predicted location of a star on the Color Magnitude diagram. The star can occupy any point in a "rotational displacement fan" that spreads from the main sequence to the giant branch for the B stars (Collins & Harrington 1966; Collins 1970; Maeder & Peytremann 1972).
—Different lines in the same part of the stellar spectrum can be broadened differently, so line strength ratios can change. This can lead to a change in the spectral class that is derived for the star from standard classification line ratios (Collins 1974).
—The Ultraviolet lines tend to be broadened less than those at optical wavelengths (Hutchings & Stoeckley 1977), making it difficult to derive an unambiguous value for

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V sin i for the star. However, the wavelength dependence of the broadening makes it possible, in principal, to derive the value for the star's inclination angle, i (Hutchings, Nemec, & Cassidy 1979; Carpenter, Slettebak & Sonneborn 1984).
—Motions are induced in the atmosphere of the star by the rotation. These can be circulatory, or can be in the form of nearly sonic turbulence because of shear flow instabilities (Smith 1970; Moss and Smith 1981).
—The emergent flux becomes polarized. The magnitude and even the direction of this polarization can depend on the frequency at which it is measured (Sonneborn 1982).

These are just a few of the strange results of rotation that have been investigated using model atmosphere theory in the past 20 years.

Simplified Model

It is not possible here to discuss all of the papers in which these phenomena have been predicted or explained, but some of the results can be at least partially understood using the simple picture for a rotationally distorted star shown in Figure 1. Let us discuss the figure and see what it implies. The structure of a rotating stellar atmosphere can be derived from a modified hydrostatic equilibrium equation. Following the discussion in Clayton (1968), the hydrostatic equilibrium equation takes the form:

$$1/\rho\ \nabla P = -\nabla(\Phi_G + \Phi_\Omega) = -\nabla\Phi \tag{1}$$

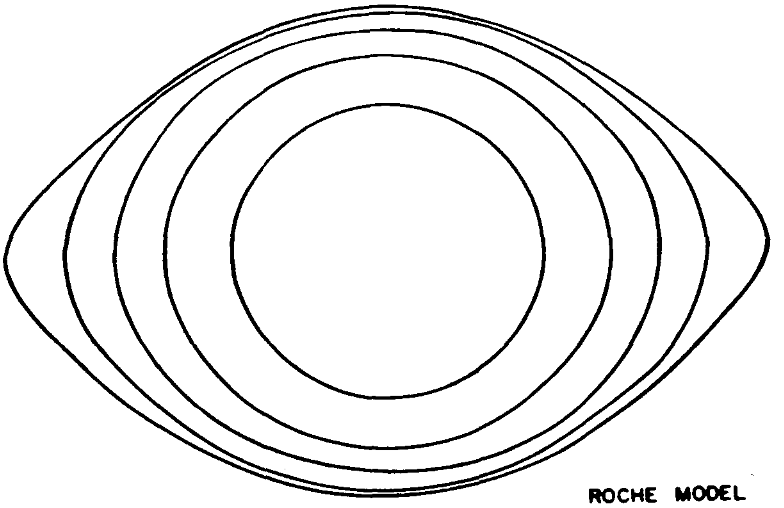
where Φ is the sum of the gravitational potential Φ_G plus centrifugal potential, Φ_Ω, which for solid body rotation is $\Phi_\Omega = -0.5\,\Omega^2 r^2 \sin^2\theta$. From equation (1), the surfaces of equipotential are seen to be equi-pressure surfaces. Taking the curl of equation (1) leads to the result that the equipotential surfaces are also ones of equal density. Then from the perfect gas law, $P = \rho R T$, those surfaces are also equi-temperature ones. This last statement allows us to discuss the radiation field from a rotating star, because it means that the surfaces shown in Figure 1 are also surfaces of equal Planck function. The radiative flux equation in the diffusion approximation can be written as

$$\mathbf{H} = -\frac{1}{3\kappa\rho}\frac{dB}{d\Phi}\nabla\Phi \tag{2}$$

This says that the locally emergent flux is proportional to the local gravity ($g_{eff} = \nabla\Phi$) Thus we get the von Zeipel result stating that the local effective temperature is proportional to the local effective gravity. This result can be explained pictorially by considering the equipotential surfaces shown in Figure 1. Note that the surfaces are very close to one another in the polar region. Thus the gradient of the temperature (and therefore gradient of the Plank function) is very steep at the pole.

We know from basic stellar atmosphere theory that a steep gradient in $dB/d\tau$ has two major effects. First it gives rise to a large radiative flux, as we have noted. Second, a steep Planck function gradient leads to a steeper "limb darkening law", $I(\mu)$ versus μ. This means that the radiation field is "sharply peaked in the outward direction". This has interesting consequences in regards to the polarization expected from the star. Consider observing the star "equator-on", i.e. the observer is in the equatorial plane of the star. Assume also that there are free electrons in the atmosphere. The electrons in the polar

Figure 1. Projected equipotential surfaces in a rigidly rotating Roche model of a star (from Cassinelli and Haisch 1974).



region will be induced to oscillate primarily in directions that are nearly perpendicular to the polar axis. Scattered light observed with a polarimeter will have a larger intensity for the case in which the electric vector is perpendicular to the polar axis than for the case in which the which the electric vector is parallel to the polar axis. From Figure 1 we can deduce that the gradient $dB/d\tau$ is relatively small in the equatorial zone, indicating that the forward peaking of the radiation field from that zone will not be very strong. So in the equatorial zone the radiation field will be much more uniformly distributed in the outward direction, and thus the direction of the oscillations of the electrons will not have a strong tendency to be parallel to the polar direction. Under the circumstances just described, the star observed as a whole will display a net polarization, because the polarized intensity from the polar regions is significantly larger than that from the equatorial regions. The detailed models of Harrington and Collins (1978), Poeckert and Marlborough (1978), and Cassinelli and Haisch (1974) show this effect. A distant observer should detect a net residual polarization that is perpendicular to the polar axis of the star. Incidently, the direction of this polarization is rather peculiar, because it is perpendicular to the polarization expected from a star with a equatorially enhanced extended envelope. The polarization expected from plane parallel model atmospheres has the opposite sign from that expected from the classic Be disk model !

Before leaving this discussion of the effects of the forward peaking of the radiation field in a rotationally distorted stellar atmosphere it is interesting to note some wavelength dependent effects. The direction of the polarization that we have just derived is wavelength dependent, for example . The wavelength dependence arises because the equatorial regions are cooler than those at the pole, so at ultraviolet wavelengths the radiation field from the equator is further towards the Wien region of the Planck distribution. In the Wien region, the monochromatic Planck function, B_ν, is a very steep function of T. Therefore there can be a very steep gradient in the thermal source function in the equatorial zones relative to that at the pole. This enhanced forward peaking tends to enhance the polarization of the intensity from the equatorial zone. Furthermore, the geometrical distortion effect of rotation increases the area of the equatorial latitudes relative to those at the pole. The combination of the enhanced

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equatorial polarization plus surface area effect can cause the equatorial zones to contribute more to the ultrviolet polarization than does the polar region. A change in the sign of the polarization can therefore occur as one scans from optical wavelengths to shorter wavelenths. Be stars are especially interesting in this regard because the peak of the radiation field of stars with temperatures in the range 10000 K to 30000 K falls in the observable range of 1000 to 3000 Angstroms. This discussion of the directionality of the polarization of the emergent polarization illustrates the considerations involved in the derivation and interpretation of the emergent fluxes from rapidly rotating stars.

There is a second wavelength dependent effect that we can discuss in the context of our simple picture that is based on Figure 1. The ultraviolet continuum of a star tends to come predominantly from the regions near the pole because of the higher effective temperature there. The ultraviolet spectral lines will also therefore have a larger contribution to their equivalent width arising from the polar regions. The regions near the pole are also not rotating with as high a linear speed as are the equatorial regions (because $V_\phi = \Omega \times r$). This means that the ultraviolet lines should undergo less rotational broadening that do optical lines. This difference in the widths of ultraviolet lines was observed by Morton *et al.* (1972) in early Copernicus spectra of the rapidly rotating star ζ Ophiuchi, as will be discussed further later on.

Model Photospheres

Actual model atmospheres for rapidly rotating stars are of course much more sophisticated than the simple picture that we have been using thus far. The history of detailed model atmospheres goes back to the paper of Collins (1963). Important advances in the subject were made in the papers by Collins & Harrington (1966,1968), Sonneborn and Collins (1977), Collins and Smith (1985), Hardorp and Strittmatter (1968), Maeder and Peytremann (1972), Stoeckley and Mihalas (1973). The models are calculated as follows: 1. Given a stellar mass and rotation rate, Ω, interior theoretical models for rotating stars (eg. those of Sackman & Anand, 1971) provide the underlying shape of the star, $R(\theta)$, the luminosity, and the local effective gravity, g. The proportionality of the radiative flux and gravity, from von Zeipel's theorem, provides the local effective temperature. 2. The surface of the star can then be " tiled " by a grid of model atmospheres. For each grid region or tile, plane parallel models provide the monochromic intensity, $I_\nu(\mu)$, as a function of angle from the local normal. The rotating photosphere problem is complicated in that we have several new geometry factors to contend with; the inclination angle of line of sight to the observer relative to the polar axis, i, and the generally non-circular projected area of the distorted star. 3. The appropriate intensity from each tile must be used in a surface integration to find the Specific Integrated Luminosity that was explained by Collins in his talk at this meeting. 4. Finally, most of the papers mentioned above convolve the emergent fluxes with observational response functions to derive observables such as U,B,V,u,b,v,y magnitudes, $H\beta$ indices, and perhaps line profiles as a function of Ω and inclination angle i, so that V sin i can be derived. The calculation of observables is much more commonly done in the field of rotating stellar atmospheres, than for for stellar atmospheres in general. This is because concepts that we are all familiar with, such as effective temperature and surface gravity, become ambiguous in the case of rotating stars where each tile on the surface of the star is characterized by a different T_{eff} and g.

Let us now consider the results of the various model calculations. The results that I will show do not form a fully consistant picture, unfortunately. The figures were chosen primarily because they illustrate some interesting effectsthat rotation has on the stellar continua--or on spectral line profiles.

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Continuous Energy Distributions

Figure 2 shows the apparent energy distribution of a rapidly rotating star seen pole-on and equator-on as compared to a nonrotating star. The results shown are from the paper by Maeder and Peytremann (1972). As seen pole-on the star would appear to be about a half a magnitude brighter over the entire spectral range 1000 to 10000 A. While seen equator-on the star would appear fainter and cooler. These effects can cause a star to have a range of locations in the Color Magnitude diagram, and occupy any point in what is known as the " rotational displacement fan ". This is illustrated in Figure 3, which also comes from Maeder and Peytremann (1972). At the brightest corner of this fan shaped area is a star that is seen pole-on with a rotational rate very near the maximal critical rate. At the faint red corner of the fan shaped region is the star seen equator on. Given the rotational rate Ω, and the inclination angle, i, the position that the star would occupy can be determined by the proceedures discussed above. However the inverse problem of determining the location that the star would occupy if it were not rotating is in general not possible. Note on Figure 3 that the displacement curve starts to move back toward its origin at the very highest speeds. This indicates that displacements in this region are double valued so that a unique inversion from the observational position on the diagram to M,i, and Ω, would be impossible. Collins and Harrington (1976) noted that this overall spread away from the main sequence is as large as the gap between the main sequence and the giant branch. Thus, in principle, the whole range in the positions of Be stars on H-R diagrams could be explained by rotational effects acting on zero age main sequence stars. As we shall see later, the observed range in the rotational speeds, or V sin i, is small enough that such a large spread does not actually occur.

Figure 2. Energy distributions of rotating stars (V_R = 341 km/s), seen pole-on (i=0°) and equator-on (i = 90°) compared to that of a same mass star at rest with M = 1.4 $M_\odot$. The positions of the filters of the Geneva Observatory photometric system are indicated by horizontal bars (from Maeder and Peytremann 1972).

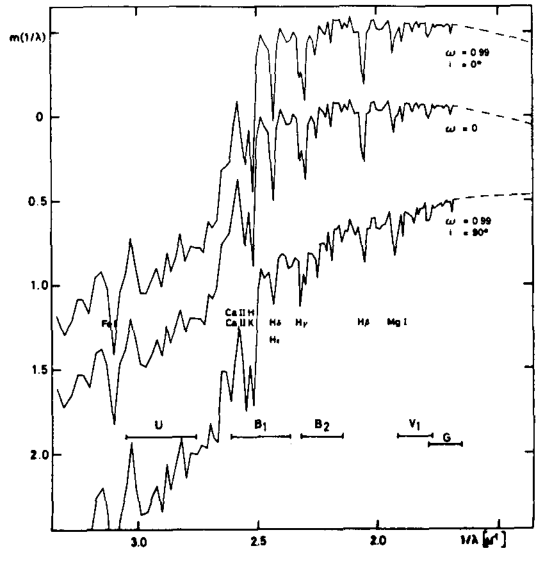

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Figure 3. Colour-magnitude diagram with rotational tracks for various angle i, for models with and without metallic-lines. The small dotted lines indicate the loci of maximum equatorial velocities V_R (from Maeder and Peytremann 1972).

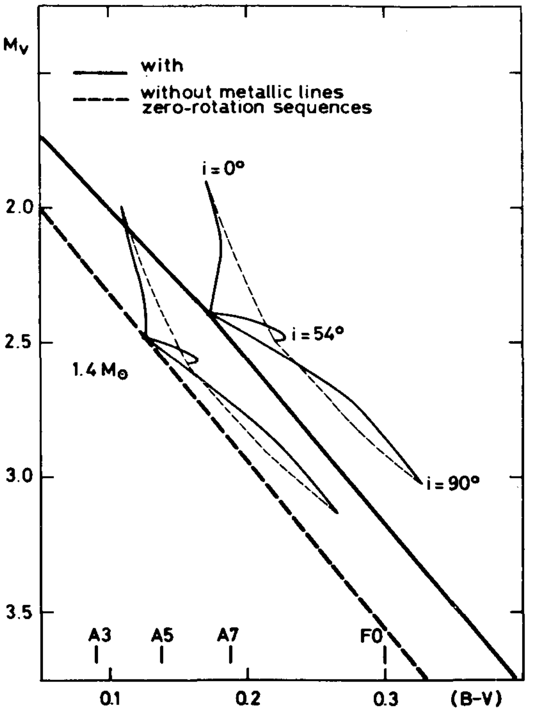
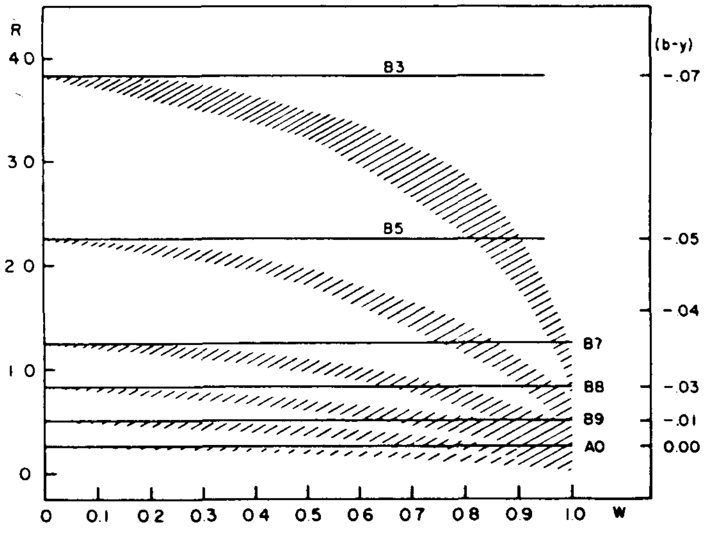


Figure 4. Effects of rotation on the line-strength ratio (R) of He I λ4471 to Mg II λ4481 for models having parameters in the range B3–A0, $\omega = V_\Phi/V_{crit}$ (from Collins 1974).



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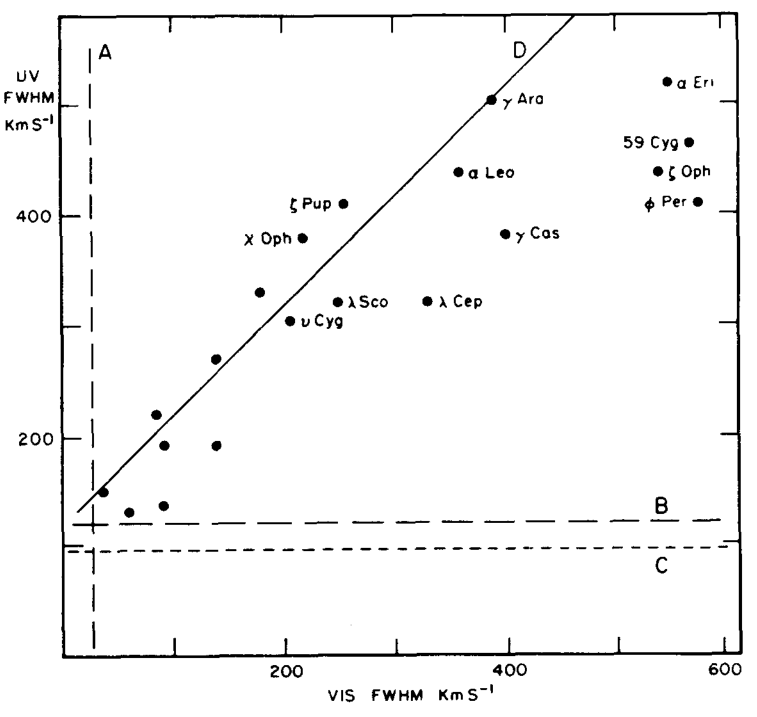
Spectral lines

Let us consider the effects of rotation on the spectral lines. There is rotational broadening, of course, but there are some interesting differences from the classical picture of Shajn and Struve (1929) and Slettebak (1949). Different lines can be broadened differently, for example. Collins (1974) compares the broadening of two lines used in spectral classification of B stars : He II λ4471, and Mg II λ4481. The Helium line is much more sensitive to the local "effective Temperature"; it weakens more towards the equator than does the Mg II line. The change in spectral type that would be inferred from the ratio of the two lines is shown on Figure 4. Note that for rotation speeds above about 0.7 times the maximum, the change in spectral type classification can be large. At speeds near critical, the shift in spectral type covers nearly the full B star spectral range. The profiles computed by Collins (1974) were used by Slettebak et al. (1975) to derive V sin i for a large number of stars.

Another peculiar effect of rotation is the difference in the broadening of the spectral lines at two different wavelength regions. Observations of line widths at ultraviolet wavelengths by Morton et al. (1972) showed them to be narrower than lines in the optical. This effect is clearly illustrated in Figure 5, which is from Hutchings and Stoeckley (1977). Note that for several of the most rapidly rotating stars the full width at half maximum can be nearly 200 km s^{-1} faster at optical wavelengths than in the UV. The reason for this can at least partly be understood from our simple model discussed above.

Figure 5. UV and visual region line broadening. Note that several high rotators fall below the unit slope line (from Hutchings and Stoeckley 1977).



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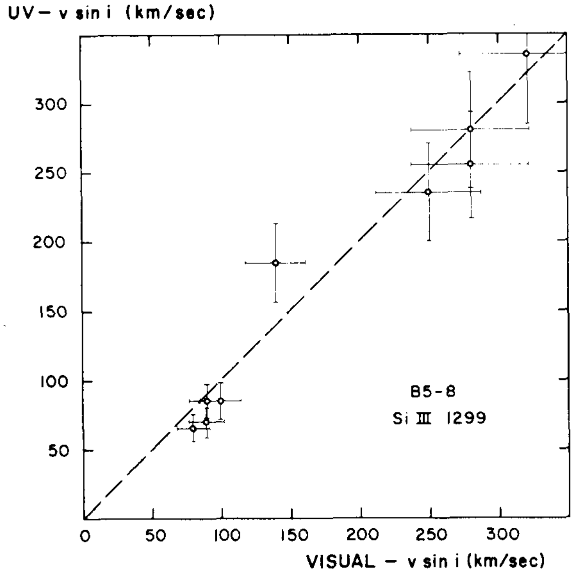
The UV continuum tends to be dominated by the hotter polar regions where the linear speed is not so large, so the lines formed there are narrower. The difference in the line widths has been used by Hutchings et al. (1979) to estimate the incination angle of rotating stars. Several problems with their interpretations have been discussed by Marlborough (1982). Part of the difficulty is that there can be a contribution to the lines from an extended disk about the star.

In spite of the difference in the FWHM of the lines in different spectral regions, it is possible to derive V sin i from lines at either wavelength. This has been shown recently by Carpenter, Slettebak and Sonneborn (1984), and their results are summarized in Figure 6. There is good agreement for the values of V sin i derived from lines at the UV and optical wavelengths. This result illustrates the importance of having available good rotating model atmospheres. The recent studies of V sin i confirm the the conclusion of Slettebak (1956) that the Be stars are not rotating at rates larger than about 0.8 to 0.85 times the critical speed.

Figure 6. visually determined rotational velocities (V sin i) plotted against ultraviolet V sin i values (from Si III λ1299 line profiles) for B5-B8 stars. Error bars of ±15% have been placed on each point. The dashed line is a 45° line representing perfect agreement between the visual and ultraviolet V sin i values (from Carpenter et al. 1984).



Motions in the Photosphere

Rotation not only affects the radiation but the gas dynamics in the star. There can be currents or turbulence produced by the rotation (Moss and Smith 1981). Eddington-Sweet currents arise because in a rotationally distorted star the divergence of the radiative flux is not zero. Consider the expression that we used in equation 2. Taking the divergence of the radiative Flux, we find that the divergence is not zero, but that it depends on $\nabla\Phi$ (see equation 6-74 in Clayton 1968) which is not constant on the equipotential surfaces. As can be seen in Figure 1, $\nabla\Phi$ is largest on the rotational axis

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and smallest inthe equatorial plane. The net result is that gas along the axis of rotation heats up and tends to rise while that at the equatorial plane tends to fall; establishing fluid currents. The topic has been discussed in greatest detail by Smith (1970) and the currents that tend to form are shown in Figure 7. Smith (1970) finds that near the surface, at optical depths less than about 8, the currents would be driven to high, supersonic speed. Shear stresses lead to instabilities that give rise to turbulence in these atmospheric layers. The turbulence can have speeds that are nearly sonic. There are ways to suppress the currents, and the subsequent turbulence, by appealing to certain functional forms for differential rotation. Motions have been ignored in model photosphere calculations thus far because, for example, the energy density in the turbulence would be far smaller than the energy density of the radiation field. We should realize of course that all of the Be phenomena that we are here to discuss, such as winds and high energy phenomena, account for a negligible fraction of the stellar luminosity. The circulation currents and turbulence in rotating stars should be studied to a much greater extent. They could be responsible for many of the Be star properties. Consider the possible effect of initiating a radiatively accelerated wind, for example. Figure 8 shows the interesting figure of Abbott (1978). He showed that for stars earlier than about B1 V the winds could be initiated and accelerated by radiation pressure gradients on lines, while for stars as late as about B 8 V, the radiation forces could sustain a wind if the flow is initiated in some other way. The band in the HR diagram requiring an "extra acceleration mechanism" includes, of course, the stars of interest to us here. An important parameter in the theory of radiation driven winds is the ratio of the outward radiative acceleration to gravity, Γ. We should notice that g_{rad} is proportional to the flux, which for rotating stars is proportional to the latitude dependent g. Therefore, the latitude dependence cancels out. We should expect to first order therefore, that the radiatively accelerated component of the winds of Be stars is independent of latitude. The winds of Be stars are often thought to be of two types; a more or less isotropic low mass loss component, and a much more massive equatorial outflow. From this consideration of Γ, the first of these two components appears to be a radiation sustained wind.

Other interesting phenomena could also be considered as affected by the presence of turbulence in the photosphere: such as magnetic field amplification and mechanical energy generation.

Some Difficulties with Rotating Photosphere Theory

The models for rotationally distorted stellar photospheres have now reached a rather high level of development. They are clearly important, for example in studies to determine the value for V sin i, and perhaps for determining i, independently as we have discussed. However, there are problems facing the subject. Firstly, within the context of plane parallel model atmospheres further consideration must be given to the departures from hydrostatic equilibrium that should occur because of motions in the atmosphere. Secondly, the figures and discussion above show that the dominant effects of the rapid rotation do not occur unless the stars are rotating at speeds greater than about 90 per cent of the critical speed. Observational studies thus far indicate that stars do not rotate faster than about 80 to 85 per cent of the critical rate. More subtle effects of rotation must be searched for. The third problem is that some observations definitely require that the atmosphere be geometrically extended. This is especially evident in comparisons of the model predictions with observations of the infrared continuua and intrinsic polarization.

Figure 7. Circulation pattern in the absence of turbulence. The reversal at $2\pi G\rho = \Omega^2$ was discovered by Gratton (1945). The dead point in the outermost cell is independent of

rotation speed and spectral type; the values for the other dead point, and the outer reversal, are from Smith 1970. The drawing is not to scale (from Smith 1970).

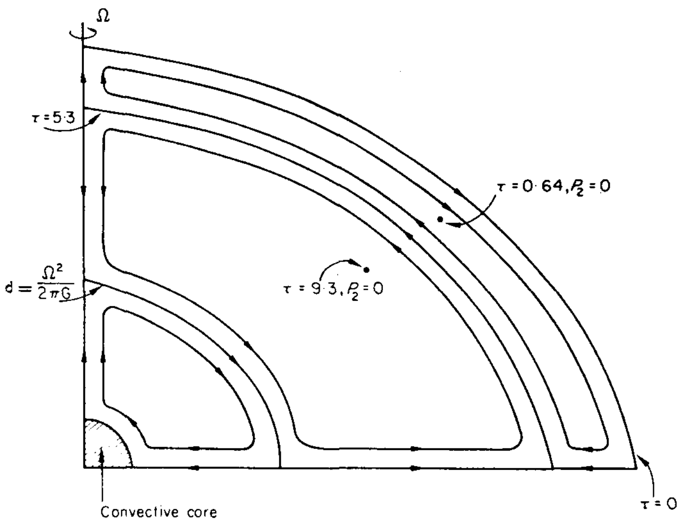
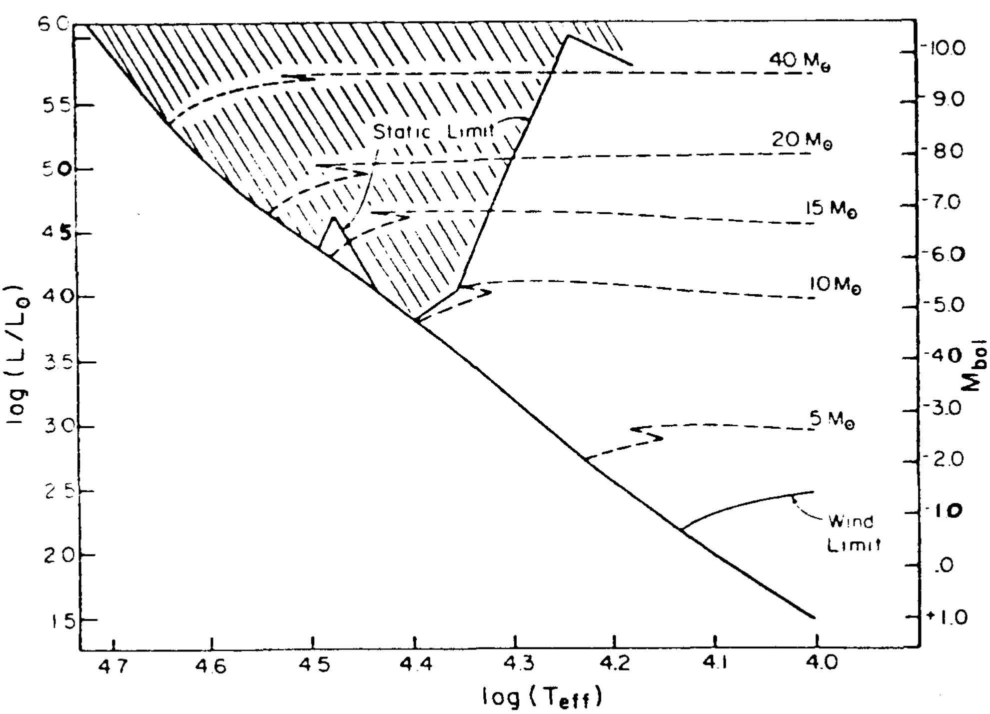


Figure 8. Minimum luminosity required to initiate (static limit) and to sustain (wind limit) a radiatively driven wind.



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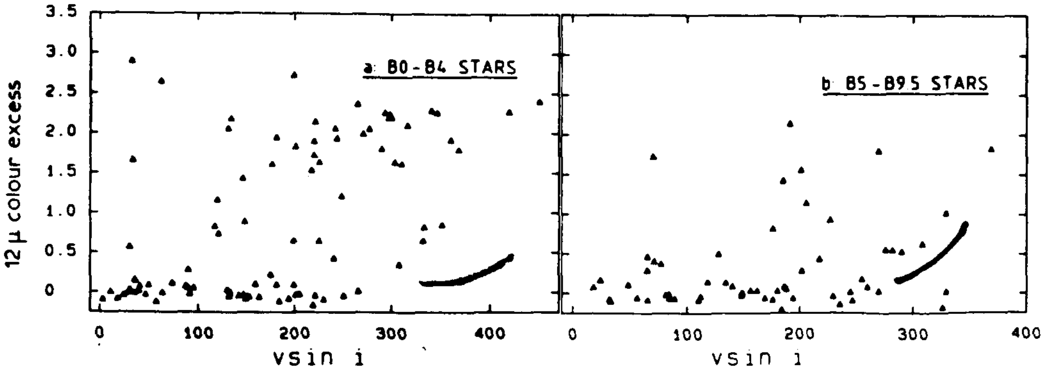
THE EXTENDED STELLAR ATMOSPHERES AND THE INTRINSIC POLARIZATION OF Be STARS

The infrared continuum of Be stars cannot be explained by the plane parallel model atmospheres. This has become especially clear because of the recent IRAS satellite observations of Waters (1986) described by Waters and Lamers elsewhere in these proceedings. Figure 9 shows the infrared excess at 12 microns for Be stars, versus the value of V sin i for the stars. For the B0 to B4 stars that are rotating at speeds greater than about 150 km s^{-1} the stars can have a wide range of IR excesses extending up to very large values of about 2.5 magnitudes. This amount is comparable to that seen in the B supergiant P Cygni which, as we know, has a very massive wind. The results have been explained by Waters using the equatorial disk model of Hartmann (1978). In that model the infrared flux arises from free-free emission by material in the flattened envelope around the star. The rotating photospheric models also predict that there should be a flattening of the energy distribution relative to a nonrotating star, because of the presence of the cool equatorial zones. The results of the continuous energy distributions derived by Collins and Sonneborn (1977), are also plotted as a dark line in Figure 9. Note that even in the extreme case of a star rotating at critical speed the 12 micron excess predicted from the rotating plane-parallel models is too small by a magnitude or more. Figure 9 B shows that the problem of fitting the IR observations for the stars in the spectral range B5 to B9.5 is not so severe; the observed excesses are somewhat smaller, while the models can explain a somewhat larger excess than for the hotter stars.

Figure 9. The 12 μm color excess for (a) B0 to B4 stars and (b) B5 to B9.5 stars. The stars with the high excesses are mostly Be stars, while most of the non Be stars have no excess. The solid curve represents the prediction of the IR excess from the models of Collins and Sonneborn 1977. The IRAS data is from Waters 1986.



The subject of the intrinsic polarization of Be stars is not going to be discussed at length in any other review so I will comment on it. The material in the extended atmosphere region that leads to the IR excess can also give rise to a large intrinsic polarization. This polarization is a particularly important source of information about the rotational distortion

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or asymmetry of the stellar outer atmosphere. Let us consider some of the the basic principles required to understand continuum polarization, and then discuss the current status of polarization modelling.

The intrinsic polarization of a hot star is caused by electron scattering in an asymmetric envelope that is illuminated by a forwardly peaked radiation field. The solar corona is an example of an atmosphere in which the polarization can be very large, as high as about 50 percent! However this is a polarized intensity, refering to a line of sight through the corona. The radiation field in the corona is primarily geometrically diluted photospheric radiation, and it is very strongly forwardly peaked. The electrons are induced to oscillate perpendicular to the radial direction. The largest contribution to the light scattered from the electrons occurs at the point of closest approach to the sun of the line of sight, so the intensity of observed scattered light is strongly polarized in a direction that is parallel to the underlying solar disk. In the case of a plane parallel atmosphere the forward peaking of the radiation at the surface also causes the intensity at the limb to be polarized as we discussed earlier. Chandrasekhar (1950) showed that for a gray model atmosphere the polarization along a line of sight tangential to the limb of a star would be about 11 percent.

For stars other than the sun we do not measure the intensity, but the flux obtained by an integration over the projected disk of the star. Major cancellations occur in that integration. Large polarization of the intensity at one point on the limb tend to be offset by polarized intensities of the opposite sign from a point of the disk in the next quadrant. The intrinsic polarization seen in the net flux in some stars means, therefore, that there is a substantial asymmetry in the geometry of their extended atmospheres.

Several geometries have been suggested, recently, for asymmetries in stellar envelopes and winds. Lamers et al. (1978) have suggested that there are blobs in the winds of late B supergiants such as β Ori. This should lead to a time variable polarization as the blob travels away from the star. In the case of OB supergiants the wind electrons could also be concentrated in one or more " plumes " ; as has been suggested from polarization observations of OB supergiants (Lupie and Nordsieck, as discussed by Cassinelli 1985). The polarization of OB supergiants is observed by Lupie and Nordsieck (1986) to be variable not only in strength, but also in direction. The magnitude of the polarization for the OB supergiants is about 0.3 per cent or less, significantly less than the polarization of the Be stars. For the Be stars there are it seems two main candidates for the asymmetric geometry. The equatorial disk model such as that used in the interpretation of the infrared data discussed above. There is also the possibility that the electron density is enhanced in an asymmetric cocoon or decelerated wind region at roughly 4 to 10 stellar radii (Thomas 1983). The polarization expected for the equatorial disk sort of model has been calculated by Poeckert and Marlborough (1978) as we shall hear in a talk later at this meeting. The polarization from a cocoon model has not been calculated as the model parameters are not yet available.

The basic theory for the calculation of polarization has been developed in a very clear way by Brown and McLean (1977), for any axially symmetric electron envelope about a point source star. They were able to isolate several important factors: (1) the polarization should be proportional to $\sin^2 i$, (2) the polarization is proportional to the integral of n_e radially through the envellope, and (3) the polarization is proportional to a rather simple angular integration. The method has recently been generalized to lift the point source restriction for some cases by Cassinelli, Nordsieck and Murison (1986).

The Polarization of Be Stars

The Be stars show large intrinsic polarization as high as about 1.5 per cent. I want to stress that this amount of net polarization is large, even though it appears rather miniscule in comparison with the 11 and 50 percent polarization of the line of sight intensities referred to above. I have seen several times in the literature recently that the net polarization of Be stars is " small". The source of the confusion can be traced to a misleading statement in the paper of McLean and Brown (1978) stating that " there are no stars with polarizations above 2 percent ··· presumably because extremely oblate envelopes do not occur." This statement was based on the point source approximation. That approximation exagerates the forward peaking of the radiation field and can lead to overestimates of the polarization by factors of two to three. The calculations on which it was based did not account for the major depolarization caused by the presence of absorptive opacity in the envellope. Detailed models that account for a finite disk and for absorptive opacity are hard pushed to reach 1.5 per cent at Visual wavelengths (Cassinelli et al. 1986; Poeckert and Marlborough 1978).

Why do the more detailed polarization models lead to such small values ? The net polarization is the ratio

$$p = \frac{F_{\parallel} - F_{\perp}}{F_{\text{total}}}$$

This tends to be small because most of the contribution to the denominator comes from the circularly symmetric disk underlying the asymmetric envelope. The bulk of the radiation is therefore unpolarized. In order to get polarizations larger than about 2.5 per cent this core must be blocked from view. This can and does happen in the case of dusty envelopes around young stars. In that the central star can be blocked from our view by a circumstellar disk and the polarization of the radiation scattered from material above the poles can be seen. Another reason that the polarization of Be stars is low is that for the B spectral class bound free opacity of hydrogen tends to be larger than the electron scattering opacity. This absorptive opacity gives rise to a strong wavelength dependence of the polarization, with rather large polarizations occuring only at ultraviolet wavelengths just longward of the Lyman limit.

Figure 10 shows the results of a polarization survey of Be stars presented by McLean and Brown (1978). On the figure are shown curves of constant $\sin^2 i$. Note that the polarization is very small for stars with a small V sin i. This provides support for models in which the electrons are concentrated towards an equatorial zone. Note also that the polarization reaches values in the range of 1.0 to 1.5 per cent for the stars seen as rapid rotators.

Figure 11 shows the results from the rotating plane parallel models of Sonneborn (1982). Several interesting features can be seen. The polarization tends to be larger for the earlier B stars because there is less depolarizing caused by the thermal radiation in the atmosphere. That is, scattering tends to be the more dominant source of opacity. There is a large discontinuity in the polarization at the Balmer jump, where again the reason is the depolarizing effect of the thermal emission. At very short wavelengths the polarization shifts direction, (changes sign), from being perpendicular to the polar axis to being parallel to the polar axis. This is caused by the increasing forward peaking of the radiation field of the light in the equatorial regions as these cooler regions shift to the Wien frequency domain. Also the extension of the atmosphere in the equatorial tends to

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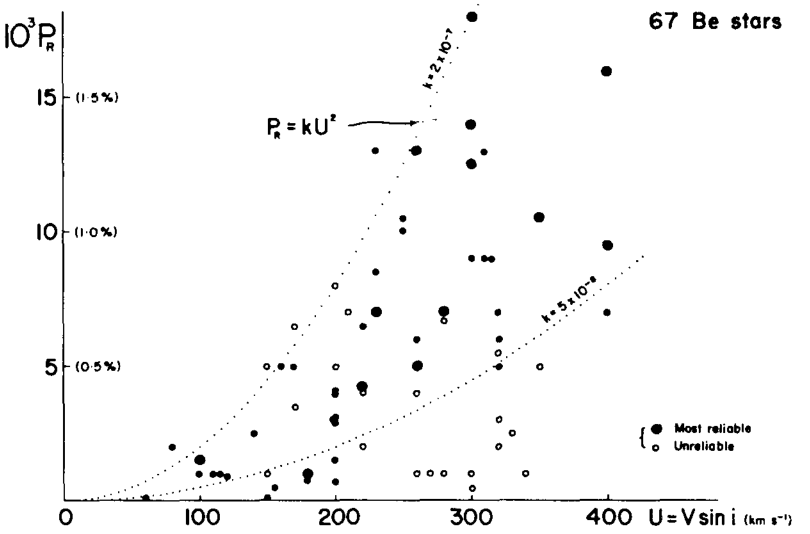
lead to polarization that is parallel to the polar axis. Most importantly we should note in Figure 8 how very small the predicted polarizations are from these plane parallel models...a few hundedths of a percent. Sonneborn has made it very clear that the observed polarization of the Be stars is <u>not</u> produced in the stellar photosphere.

<u>Figure 10.</u> Plot of observed intrinsic polarization versus V sin i. The dotted lines are curves of k $\sin^2$ i with k as shown, indicating agreement with axisymmetric polarization models (from McLean and Brown 1978).



<u>Figure 11.</u> The net linear polarization in the visual and near ultraviolet continua as a function of wavelength predicted from plane parallel model atmospheres for stars of different spectral types, all of which have $v_\phi = .95\ v_{crit}$. Zero polarization is shown by the thin dashed line (from Sonneborn 1982).

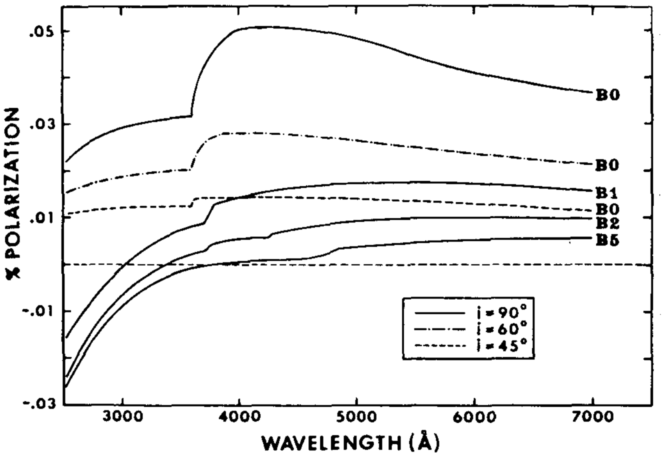
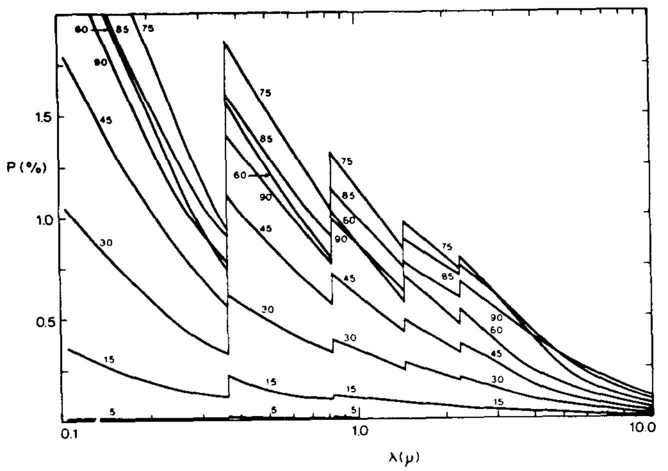


Figure 12 shows the polarization versus wavelength for the disk model of Poeckert and Marlborough (1978), for several inclination angles. The polarization at V does not rise much above 1.5 per cent in the most favorable case. Their model also accounts only for single scattering. Multiple scattering tends to reduce the polarization significantly. The polarization is seen here to rise to very high values at Ultraviolet wavelengths, partly because of the enhanced forward peaking of the radiation field that occurs there, as discussed above. It will be possible soon to measure the polarization at UV wavelengths using the Astro payload that is to be launched by NASA.

Figure 12. The continuous polarization of the standard model of Poeckert and Marlborough 1978. Shown is polarization versus wavelength for different inclinations. At V ($\approx$ 0.5 microns) the maximum polarization is about 1.5% (from Poeckert and Marlborough 1978).



Polarization is a very powerful diagnostic for studying the asymetric geometrical structure of the Be stars. Given an empirical model it is straightforward to calulate the polarization that is to be expected by performing relatively simple integrals over the electron distribution. This should lead to important new constraints on the models now under consideration for explaining Be phenomena.

Summary

Results obtained from a variety of model calculations for rotationally distorted atmospheres have been presented. Some of these can be explained using a simple picture based of the iso-potential contours and applying the diffusion approximation. The effects of rotation for the relevant case of stars rotating at no more than 0.8 V_{crit} are found to be small. The model results are useful, nevertheless, because they are needed to derive V sin i accurately using lines at a wide range of wavelengths. It appears that much more work is needed in the study of motions that are induced in the atmosphere by departures from radiative equilibrium. The predictions of plane parallel model atmospheres for the the infrared infrared excesses and intrinsic polarization have been compared with observations. In both cases it is clear that geometrically extended envelopes are required. The interpretation of the polarization of Be stars should be especially useful for deriving the spatial distribution of the electrons in the outer atmospheres of the stars.

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DISCUSSION FOLLOWING CASSINELLI

Smith (R.C.):

My work on sonic turbulence near the surface assumed uniform rotation. If the rotation is non-uniform then the turbulence could be less (or at least different).

Cassinelli:

Yes, and the turbulence could be larger, depending on the function assumed for the differential rotation.

Baade:

First, regarding turbulence, several observers have pointed out that the visibility of the bumps seen in the spectral lines of Be stars places a limit on the turbulent velocities. Second, although the MgII λ4481 line is the only strong non-hydrogen, non-helium line in the visible spectrum of Be stars, it should be avoided for the analysis of stars with spectral type B3 or earlier, because that line is very often contaminated by circumstellar absorption and/or filled up by emission. Third, nonradial pulsations with a large horizontal amplitude in stellar latitude may perhaps also lead to an altered surface temperature distribution. Fourth, would you expect any back scattering effect by the disk on the equatorial regions of the star?

Cassinelli:

Yes, you are right, the effects of non-radial pulsation on the atmosphere and surface temperature distribution have not been taken into account by the detailed models calculated thus far. Most of the models were developed before the possibility of non- radial pulsation was introduced to the fields of Be stars. As for the effects of back-scattering; I suspect that it is not very important. The radiation from the disk can escape rather easily in a direction normal to the disk and this escaping light will not be emergent on the star. In the spherical case treated by Hummer, the back scattering was much more important because a larger fraction of the envelope radiation would re-enter the "photospheric" regions.

Buscombe:

Since the use of the MgII 4481 Å line for estimating rotational half-width has been called into question, may I remind you that instead of throwing away most of the spectrum, Buscombe and Stoeckley (1975) used all the lines resolved in the spectrum, and also not only the central core but even the wings of the line profiles.

Cassinelli:

I showed the figure with the MgII and the HeII lines because that is perhaps the only example in the model literature in which the effects of rotation on line ratios are affected by the wide range of effective temperature that can be brought on by rotation.

Collins:

Is single scattering assumed in the work of Brown and McClean as well as in the models of Poeckert and Marlborough? Multiple scattering will further reduce the polarization of the model.

Marlborough:

The Poeckert and Marlborough model produced a reasonable Hα line profile but the continuum polarization was too large, although the Balmer jump and wavelength dependence were reasonable. Only single scattering was considered. Multiple scattering would definitely lower the predicted polarization.

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