

This is a marvellous handbook. It is well written, carefully planned, and contains fascinating material. The authors of the chapters are without exception excellent specialists in the topics discussed. Each chapter includes a glossary that provides succinct definitions of most of the terms from that chapter. Individual topics are covered in sections. Each chapter of the book includes a list of references.

The handbook will be a useful source for anyone who is interested in mathematical analysis and, in particular, for those researchers whose primary interest is in metric fixed point theory and its applications. It may not be a book which will be read from cover to cover, but it is very likely that both students and researchers in metric fixed point theory will find the particular information they seek in this detailed and self-contained exposition.

The handbook is an incredible achievement. The editors should certainly be congratulated for managing to organize the work of so many different individuals into such a coherent and usable structure.

A. LATIF

HERTLING, CLAUS *Frobenius manifolds and moduli spaces for singularities* (Cambridge University Press, 2002), 280 pp., 0 521 81296 8 (hardback), £45 (US\$60).

Frobenius manifolds, created by Dubrovin in 1991 from rich theoretical physics material, have been found since in many different fragments of mathematics—quantum cohomology and mirror symmetry, complex geometry, symplectic geometry, singularity theory, integrable systems—raising hopes for unifying them into one picture. It also became clear that the notion of Frobenius manifold is not broad enough to cover *all* objects of the associated working categories; say, on the *B*-side of the mirror symmetry it applies only to extended moduli spaces of *Calabi–Yau* manifolds, the latter forming a rather small subcategory of the category of complex manifolds. In 1998, Hertling and Manin introduced a weaker notion of *F-manifold*, which is, by definition, a pair (M, μ) consisting of a smooth supermanifold M and a smooth \mathcal{O}_M -linear associative graded commutative multiplication on the tangent sheaf, $\mu : \otimes_{\mathcal{O}_M}^2 \mathcal{T}_M \rightarrow \mathcal{T}_M$, satisfying the integrability condition

$$[\mu, \mu] = 0,$$

where the ‘bracket’, $[\mu, \mu] : \otimes_{\mathcal{O}_M}^4 \mathcal{T}_M \rightarrow \mathcal{T}_M$, is given explicitly by

$$\begin{aligned} [\mu, \mu](X, Y, Z, W) := & [\mu(X, Y), \mu(Z, W)] - \mu([\mu(X, Y), Z], W) \\ & - (-1)^{(|X|+|Y|)|Z|} \mu(Z, [\mu(X, Y), W]) - \mu(X, [Y, \mu(Z, W)]) \\ & - \mu(X, [Y, \mu(Z, W)]) - (-1)^{|Y|(|Z|+|W|)} \mu[X, \mu(Z, W)], Y \\ & + (-1)^{|Y||Z|} \mu(X, \mu(Z, [Y, W])) + \mu(X, \mu([Y, Z], W)) \\ & + (-1)^{|Y||Z|} \mu([X, Z], \mu(Y, W)) + (-1)^{|W|(|Y|+|Z|)} \mu([X, W], \mu(Y, Z)). \end{aligned}$$

A non-trivial part of the above definition is an implicit assertion that $[\mu, \mu]$ is a tensor, i.e. \mathcal{O}_M -polylinear in all four inputs. It is here where the assumption that μ is both graded, commutative and associative plays a key role.

The book under review gives a very detailed analysis of the category of *F*-manifolds. In particular, it is shown that any Frobenius manifold is an *F*-manifold; any *F*-manifold with semi-simple product μ can be made into a Frobenius manifold. Most importantly, the author gives a careful and rigorous survey of how *F*-manifolds turn up naturally in singularity theory.

The style of the book is not impressive. Sometimes the author struggles with his English. Nevertheless, the book is clean, rigorous and readable. The researchers in the areas of singularity theory, complex geometry, integrable systems, quantum cohomology, mirror symmetry and

symplectic geometry will find in this book a lot of useful information which has never been given in such detail before.

Here is an outline of the contents.

Part 1. Multiplication on the Tangent Bundle

- (1) Introduction.
- (2) Definition and first properties of F -manifolds (finite-dimensional algebras; vector bundles with multiplication; decomposition of F -manifolds; examples; potentiality).
- (3) Massive F -manifolds and Lagrange maps (Lagrange property; existence of Euler fields; Lyashko–Looijenga maps and graphs of Lagrange maps; miniuniversality).
- (4) Discriminants and modality of F -manifolds (two-dimensional F -manifolds; logarithmic vector fields; discriminants and modality of germs of F -manifolds; analytic spectrum).
- (5) Singularities and Coxeter groups (hypersurface singularities; boundary singularities; Coxeter groups and F -Frobenius manifolds; three-dimensional F -manifolds).

Part 2. Frobenius Manifolds, Gauss–Manin Connections, and Moduli Spaces for Hypersurface Singularities

- (6) Introduction.
- (7) Connections over the punctured plane (flat vector bundles on the punctured plane; (saturated) lattices; Riemann–Hilbert–Birkhoff problem; spectral numbers).
- (8) Meromorphic connections (logarithmic vector fields and differential forms; logarithmic poles along divisors).
- (9) Frobenius manifolds and second structure connections (this chapter reviews some basic properties of Frobenius manifolds).
- (10) Gauss–Manin connections for hypersurface singularities (semi-universal unfoldings and F -manifolds; Gauss–Manin connections; higher residue pairings; polarized mixed Hodge structures and opposite filtrations; the Brieskorn lattice).
- (11) Frobenius manifolds for hypersurface singularities.
- (12) μ -constant stratum (canonical complex structure; period map and infinitesimal Torelli theorem).
- (13) Moduli spaces for singularities (compatibilities, symmetries, and global moduli spaces for singularities).
- (14) Variance of the spectral numbers.

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PELLER, V. V. *Hankel operators and their applications* (Springer, 2003), 784 pp., 0 387 95548 8 (hardback), £63 (US\$99.95).

A Hankel operator on a Hilbert space with orthonormal basis $(e_n)_{n=0}^{\infty}$ is a linear mapping T such that $\langle Te_m, e_n \rangle = a_{m+n}$ for some sequence of numbers $(a_n)_{n=0}^{\infty}$. Thus, if the operator is represented as an infinite matrix, then the entries are constant on stripes going from the bottom left to the top right—this should be contrasted with a Toeplitz matrix, where the stripes go from top left to bottom right.