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ASTEROSEISMOLOGICAL MODELS WITH ROTATION

Doru Marian Suran *and* Gabriela Oprescu
Astronomical Institute of the Romanian Academy
Str.Cutitul de Argint,No 5,75212 Bucharest 28, ROMANIA

Abstract The low frequency **NRP** oscillations are considered in the case of differential rotation. Partial differential equations for adiabatic oscillations are reduced to a system of ordinary equations by means of a truncated spherical-harmonic expansion. The numerical method for solving the system is described. The final model also includes line profile variations (**lpv**) and posttheoretical mass (**pth**) calculations for the complete determinations of the physical, chemical, rotational and seismological properties of the stars.

1. INTRODUCTION

We present a detailed stellar model in which low-frequency non-radial pulsations (**NRP**) are excited by differential rotation (**DR**) in turbulent convection zones. The competitivity between **NRP** and **DR** are responsible for some observational effects such as: line profile variations (**lpv**) and rotational modulation (**RM**).

To this purpose we compile a new stellar model with the following assumptions:

1. Spherical star;
2. Chemical homogeneous star (global or in shells);
3. Inert convection ($\delta L_{conv}=0$), and convective turbulence in rotation (latitude dependent heat transfer **HT**);
4. Turbulent convection in the approximation of anisotropic viscosity **AV**);
5. Stationary ($\partial/\partial=0$), differential,slow rotation :

$$f(r,\theta)=f_0(r)+\varepsilon[f_{10}(r)+f_{12}(r).P_2(\theta)] \qquad (1)$$

6. **NRP** with differential rotation (**NRPDR**).

In this case many types of modes could be excited: *gravity*(g), *pressure ionisation* (f,p), *convective* (c), *quasi-toroidal* (r), *inertial*(i).

The characteristic interactions which could appear in the star are:

$$\text{convection+turbulence} \simeq \text{rotation} \simeq \text{pulsation} \qquad (2)$$

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2. STELLAR MODEL

Basic equations for the model :

$$\partial\rho/\partial t+\nabla.(\rho\mathbf{v})=0$$

$$\rho(\partial/\partial t+\mathbf{v}.\nabla)\mathbf{v}=-\nabla p-\rho\nabla\Phi+R_v$$

$$\rho T(\partial/\partial t+\mathbf{v}.\nabla)S=\rho\varepsilon_N-\nabla.\mathbf{F} \quad (3)$$

$$\nabla^2\Phi=4\pi G\rho \ , \ R_v=\rho\nu[\nabla^2\mathbf{v}+(1/3)\partial.(\partial\mathbf{v})]$$

$$\mathbf{F}=\mathbf{F}_{rad}+\mathbf{F}_{conv}=-(4\pi/3\kappa\rho).\nabla J$$

$$4\pi J=acT^4+TdS/dt$$

are solved by a two-step perturbation model:

$$\mathbf{q} = \mathbf{q}_{eq}{}^{o}+(\mathbf{q}_{conv}{}^{0}+\mathbf{q}_{rot}{}^{0}+\mathbf{q}_{nuc}{}^{0})+\mathbf{q}_{puls}(t) = \mathbf{q}_{eq}{}^{0}(r)+\mathbf{q}_{puls}(t) \quad (4)$$

where $\mathbf{q}_{eq}$- static equilibrium model, $\mathbf{q}_{conv}$-inactive convection; $(\mathbf{q}_{conv}+\mathbf{q}_{rot})$ represents static models with differential rotational in external shells; $(\mathbf{q}_{conv}+\mathbf{q}_{rot}+\mathbf{q}_{nuc})$ represents static models with differential rotation in convective cores; $\mathbf{q}_{puls}(t)$ represents the **NRP** linear adiabatic time perturbation from pulsations.

a. Equilibrium Model ($\partial/\partial t=0$, $\Omega=0$)

The equilibrium model could be defined directly from evolutionary tracks (**SES**) under the assumptions (M=*ass*, t=*ass*), or by a three-zone static model with different equations for atmosphere, envelope and interior. The mode using as input parameters :

$$(M,L,T_e,X_i) \qquad (\textit{atmosph. param.}) \quad (5)$$

calculate all needed physical and thermodinamical parameters in the *"equilibrium"* case:

$$[P_r{}^0,T_r{}^0,M_r{}^0,L_r{}^0,X_{i,r}{}^0] \ , \ [\rho_r,\nabla_r,\kappa_r,\varepsilon_r,\ldots] \quad (6)$$

b. Static Rotational Perturbation

The basic eq.(1) is now solved under the hypothesis of static differential rotation ($\mathbf{v}=\mathbf{r}\times\Omega$). As perturbation we include **AV** and

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HT hypothesis. We perturbs all the physical quantities by a second order Legendre expansion (1).

We have two different cases for the solution of the problem corresponding to envelope (**SURFF**) and/or core (**NUCLEI**) convective zones, where differential rotation acts. Using (6) as input we can derive the new set of rotational parameters (Pidatella *et al* 1986)

$$y=(\psi_{12},\psi'_{12},\psi''_{12}),(P_{10},P_{12},S_{10},S_{12},F_{r,12}),(\omega_{12},\omega_{12},\omega'_{10},\omega'_{12}) \quad (7)$$

c. **NRP** Perturbation

Our second order **NRP** works under the assumptions of linear(', $\delta \propto \exp[i(\omega t+m\varphi)]$), nonadiabatic (for Ω=0) , adiabatic(for $\Omega\neq0$), and low frequency oscillations. The whole set of discrete parameters of the model is (l,m,n,j,i) where the first three are the usuals for **NRP**, the fourth is used in the rotation perturbation: $(\Omega(r,\mu)=\sum_j \Omega_j \mu_j)$; the fifth designed the number of the sheels in the star $(0 \le r_i \le R_{star})$. In our numerical calculations we truncate the **NRP** matrices, and in order to avoid to treat too large matrices, we split the time perturbation equations of (3)into two separate systems corresponding to even and odd modes. The output parameters of the **NRP** calculations are:

$$[(y_{k,i}^{lmn})_{k=1,4}, H_i^{lmn}, T_i^{lmn}], [\omega^{lmn}] \quad (8)$$

where y_i are the usual **NRP** unknowns (Suran 1990).

3. RESULTS

We tested our final model in different cases ,to assure the correctenss of our calculations:

a. **LNAWER+ptm** - models for radial pulsators (Suran 1986);
b. **LNAWER+ptm+lpv** -seismological models for δ Scuties(Suran 1990)
c. **LAWER+lpv**, $\Omega=\Omega(r)$-comparison with previous **NRP** rotational cases studied in the literature (Lee,Saio 1990);
d. **NRP**=0, $\Omega=\Omega(r,\theta)$- **DR** models (Pidatella *et al.* 1986).

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