CORRESPONDENCE.

"ON THE OBJECTIONS TO THE NET-PREMIUM MODE OF VALUATION."

To the Editor of the Journal of the Institute of Actuaries.

SIR,—Since the announcement of Mr. Sprague's recent paper "On the proper method of estimating the liability of an Assurance Company," I have looked forward with much interest to learn the arguments by which the objections which I, in common I believe with many actuaries of higher reputation, feel very strongly to attach to the net-premium method, would be met; and having attentively perused the observations of the members present at the reading of Mr. Sprague's paper, I shall be glad now to be allowed to avail myself of the pages of your influential *Journal* to contribute in my own way my modest quota to the discussion of this important question.

The extravagant language so liberally indulged in by a few extreme adherents of the net-premium school—who seem to have taken silence for consent, and to have supposed their views were approved because not controverted—had convinced me that the time had arrived to make a stand against an abuse which, owing to recent events, was threatening to become mischievous; and I am glad that so eminent and able a man as Mr. Sprague has forestalled me in a task which, however unqualified for its performance, I should certainly, in the absence of a better champion, have felt myself called upon to undertake.

It forms no part of my present design to discuss the merits of the particular method which Mr. Sprague has suggested as a more efficient substitute (for general purposes) for that which forms the object of his attack; and although, in my opinion, there exist other and perhaps scarcely less fatal objections to the net-premium method, I shall confine myself on this occasion to a recapitulation of those already brought forward by Mr. Sprague, and to an examination of the arguments used in opposition.

The first in Mr. Sprague's list of objections is that the net-premium method is in fact open to the charge which proved fatal to the old "re-insurance" plan, as exemplified in the mode of valuation adopted by Offices using the Northampton Table. This objection is that the method fails to accomplish the first and principal object for which a valuation is supposed to be made, viz., to exhibit the actual financial position of the Office to which it is applied.

In illustration of this assertion I will take the case of two Offices, A and B, doing business upon the proprietary or non-participating system,—Office B having started exactly two years after Office A. For simplicity suppose that in Office A the lives all enter at the age of 30 at an annual premium of £2. 7*s.*,* while in Office B they all enter at age 32 at an annual premium of £2. 5*s.** Suppose further that after the expiration of 12 years a "net-premium" actuary is required to say what fund, according to his opinion, Office A should have in hand to enable it to resist an application for a winding-up order in the Court of Chancery. Assume that the valuation is to be made by the old Experience Table at 4 per cent,

* These are actual rates—taken from the table at the end of Jones's work,—and are by no means extreme cases.

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which gives a net premium at age 30 of £1·697 per cent. Under these circumstances the Estimated Liability will be £13·409 per cent.

Now let us further suppose that by a remarkable coincidence the same actuary is called upon at the same time to investigate the condition of Office B. In this case he finds his net-premium (age 32) is £1·804, and the Estimated Liability consequently £11·738 per cent.

So that, putting this and that together:

Office.	Present Age.	Annual Premium payable (per cent).	Estimated Liability (per cent).
A	42	2 7 0	13·409
B	42	2 5 0	11·738

Hence it appears that, to meet precisely the same risk, Office B will be declared solvent if it have in hand £11·738 per cent, while Office A under the same circumstances with the same fund in hand must be pronounced insolvent (if the Actuary has faith in his principles), notwithstanding that in aid of that fund it has to receive in future a higher premium than Office B.

To this it will perhaps be answered that Office A may reasonably be expected to have in hand a larger fund, seeing that it has been receiving a larger premium and for a longer term. This, however, is quite a distinct question,—into which it is not necessary here to enter.* The assertion is, "That the method in question fails to exhibit the *actual* financial position "of the Office to which it is applied," and the adherents of the net-premium school must either admit the charge, or they must assert that, under the circumstances supposed, Office B is better able to meet the claims upon it than Office A. It is quite immaterial to their opponents which horn of the dilemma they may prefer.

I am quite aware that the more moderate of the net-premium school would not venture to declare the Office A in a state of insolvency under the circumstances supposed,—altho' I am by no means sure that others would not do so. But why should they hesitate to carry their reasoning to its only legitimate conclusion? Evidently because they know that altho', *conventionally*, their method is supposed to give a true view of the respective conditions of the two Offices, yet, *as a matter of fact*, it wholly fails to do anything of the kind.

So much then in illustration of Mr. Sprague's first objection, to which apparently not the slightest allusion, direct or indirect, was made in the course of the discussion that followed the reading of the paper.

Mr. Sprague's second objection is that if the valuation be made by a different table from that by which the premiums have been calculated, the use of the formula $1-\frac{1+a_{x+n}}{1+a_x}$ (which is what is invariably understood by a net-premium valuation in such a case) may have the effect of giving credit for a higher premium than is actually receivable.

The only reply made to this objection would appear to be the opinion

* The deficiency might arise in many ways. Say, for instance, that a large portion of the fund had been sunk in investments of too speculative a character.

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expressed in one instance that the same table ought always to be used in the valuation as in computing the premium. The adherents of the net-premium school must elect which ground they prefer to take up. As regards the case supposed by Mr. Sprague it is obvious that if the premium payable should happen to be less than the net premium required by the table used in the valuation (which is quite possible when different tables are used) we shall be committing, without knowing it, the inconceivable absurdity of valuing a higher premium than we are actually entitled to receive. Altho' I think it unlikely that the advocates of the net-premium system, generally, will feel disposed to take up the ground that in selecting your table you take it for better or worse, and must never after change it for another ; yet it is evident, I think, that such a step would at least render their position more logically defensible.

If the number of deaths in a given time, among a limited number of individuals, were a fixed and determinate quantity, instead of being a fluctuating and uncertain one, depending upon imperfectly known and varying causes, the system of Life Assurance finance might have taken quite a different shape from that in which it has actually developed itself. The whole matter would then have resolved itself into a question of ordinary book-keeping ; and the duties of the actuary would have been entirely superseded by those of the accountant. It is a tendency to contemplate the subject from what I may term the accountant's point of view that has given rise to that extraordinary idea which is occasionally ventilated under the denomination of the "retrospective" method; suggested apparently by those popular expositions of the practical working of a Life Assurance Society given by Dr. Farr and others, in which the variation of the amount of the fund from year to year is shown by assuming that the deaths fall out exactly as predicted by the Mortality Table. But absurd as it is, and opposed to the very fundamental principles of the doctrine of probabilities, a "retrospective" method in some form or shape affords, I believe, the one firm footing for those who have abandoned the only true prospective method, which takes into account the facts and probabilities of the *actual* state of affairs,—regarding the *past* only as it tends to throw light on the *future*,—which method Mr. Sprague properly insists upon as the very basis of true actuarial science.

I come now to Mr. Sprague's third objection, viz., that when a Company has been at great expense to procure business and has spent more than the loading of the premiums received, the net-premium method brings out a deficiency, notwithstanding that the expense may have been perfectly wise and justifiable,—and when (I presume Mr. Sprague means) a valuation of the premiums actually payable after making a sufficient reserve for future expenses and omitting negative values would show a surplus sufficient for the safety of the Office.

The answers made to this objection seem to resolve themselves into a denial of the possibility of the conditions with which (somewhat superfluously, I think) Mr. Sprague has qualified his objection. The more moderate of his opponents assert that such an expenditure is under all circumstances unwise, while the more extreme maintain that it is unjustifiable.

That it is unwise to pay for the acquisition of business more than is absolutely necessary to obtain it, or more than it is worth when obtained ; and that it is better that the expense of obtaining it should be spread as

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much as possible over the future premiums; are propositions to which I readily subscribe. But that the entire expense of obtaining an assurance —including, of course, under that head the medical fees, together with a due proportion of the current general expenses of the Office,—cannot consistently with prudence exceed the loading of the first year's premium, is an assertion which cannot be accepted without proof. To those who are in the habit of making their calculations upon true data the determination of the limit of a wise and profitable expenditure for the acquisition of business offers no difficulty whatever. When, therefore, those who tell us that it is unwise to exceed the loading in question can prove by calculation that the Office is better without the assurance than with it if obtained upon such terms, they will not indeed have answered Mr. Sprague's third objection, but they will at all events have convicted that gentleman of having rashly sanctioned an imprudent rate of expenditure.

The assertion that the expenditure is unjustifiable,—that there is in fact an "implied contract" with the assured that the loading only on the premiums received shall be available for expenditure—is easily disposed of. If it mean that there is an implied contract to this effect because it is the proper course, it merely amounts to begging the question at issue—while if it mean more than this it is simply and obviously untrue. It is generally known that the Deeds of Settlement of many Societies lay down rules of procedure quite incompatible with the net-premium mode of valuation. But indeed it is evident that the only possible "implied contract" is, that a sufficient fund shall at all times be maintained for the safety of the Office, and that the profits shall be equitably distributed among those entitled to them. In what manner these important conditions are to be fulfilled the public wisely leaves to the decision of the responsible actuary,—being sufficiently alive to the fact that the most obvious view of a scientific question is frequently the very reverse of the true one, to distrust its own opinion upon a matter in reference to which the most eminent authorities are by no means unanimous. If at the same time it were as careful to ascertain the title that officer has to its confidence, it would seldom have reason to regret having left him unfettered in his judgment.

I hope at some future time to submit to the Institute a description of the method of valuation which seems to me to be the best adapted to accomplish the end in view; and I shall then endeavour to show that, quite independently of the fatal objections urged by Mr. Sprague, the net-premium method, so far from possessing that title to preference which its admirers so unreasonably claim for it, is one which must inevitably disappear with that advance of true actuarial science, which the foundation of the Institute of Actuaries was designed to aid.

I remain, Sir,

Your very obedient servant,

London, 1st June, 1870. W. M. MAKEHAM.

ON HERR WILHELM LAZARUS'S PAPER "ON SOME PROBLEMS IN THE THEORY OF PROBABILITIES."

To the Editor of the Journal of the Institute of Actuaries.

SIR,—In the paper by Herr Wilhelm Lazarus, in the January number of the *Journal*, which treats of an important branch of the theory of proba-