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Superlinear variational boundary value problems and nonuniqueness

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Nonlinear eigenvalues arising in elliptic variational boundary value problems have been studied extensively by F.E. Browder (for this and other references see the author's paper [1]). In strictly non-linear problems there arises the possibility of many qualitatively different solutions corresponding to a single eigenvalue.

In his thesis the author has studied the problems

$$(1) \qquad \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i} \left(a_{ij}(x) \frac{\partial u}{\partial x_j} \right) + c(x)u + b(x, u) = 0$$

$$u = 0 \quad \text{on} \quad \partial\Omega$$

and

$$(2) \qquad \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i} \left(a_{ij}(x) \frac{\partial u}{\partial x_j} \right) + c(x)u - b(x, u) = 0$$

$$u = 0 \quad \text{on} \quad \partial\Omega$$

where the function $b(x, t)$ is odd in t and satisfies the superlinearity condition:

> there exists $\sigma > 1$ such that, for all $x \in \Omega$, $t^{-\sigma}b(x, t)$ is a non-decreasing function in $t > 0$.

Problem (1), considered as an integral equation, has been investigated by Z. Nehari and C.V. Coffman. There exists an infinity of solutions each characterised by a mini-max property. In the case $n = 1$ there is a further characterization in terms of numbers of zeros. The results are

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closely related to the linear eigenvalue problem

$$(3) \qquad \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left(a_{ij}(x)\,\frac{\partial u}{\partial x_j}\right) + \mu c(x)u = 0$$

$$u = 0 \quad \text{on} \quad \partial\Omega .$$

For the problem (2) the author establishes the existence of a finite number of solutions, the multiplicity depending on the number of eigenvalues μ_k of (3) which are less than one. In the case $n = 1$ there is again a characterisation in terms of zeros. For details see [*1*].

Rather than using nonlinear integral equations the author adopts a more direct approach to both problems using the Sobolev space $\dot{W}_2^1(\Omega)$. The problems are easily translated into the nonlinear operator equations

$$Au + Cu \pm Bu = 0$$

in a Hilbert space. The treatment of a constrained variational problem, using Galerkin approximations and a convenient topological invariant, bears resemblance to the work of Browder mentioned above.

Reference

[*1*] J.A. Hempel, "Multiple solutions for a class of nonlinear boundary value problems", *Indiana Univ. Math. J.* 21 (1971-72), 983-996.