DEAR EDITOR,

J. A. Scott's conjecture, at the end of his Note 'On a limit for prime numbers', in the March 2000 *Gazette* (p. 116) is indeed correct.

That is, suppose $a_n \to \infty$ with $a_n^{n/a_n} \to R > 1$. Then $\sum (1/a_n)$ diverges. To see this, note that the hypothesis about R implies that $\frac{n \ln a_n}{a_n} \geqslant k > 0$ for all essentially large n. (*)

Using the fact that $\ln x < x$ in the form $\ln(\sqrt{x}) < \sqrt{x}$ or $\ln x < 2\sqrt{x}$ we deduce from (*) that

$$k \leqslant n.\frac{2\sqrt{a_n}}{a_n} \text{ giving } a_n \leqslant \frac{4n^2}{k^2}.$$

Finally, from (*) again, $\frac{1}{a_n} \geqslant \frac{k}{n \ln a_n} > \frac{k}{n \ln(4n^2/k^2)}$ so that $\sum (1/a_n)$ diverges by comparison with $\sum 1/(n \ln n)$.

Readers may also be interested that there is a very quick derivation of Glaister's formulae for b_n on p. 106 of the same issue.

Consider $f(1 + z) = \frac{z + 1}{z^2 + 2z + 2} = \frac{1}{2}\left[\frac{1}{z - \alpha} + \frac{1}{z - \bar{\alpha}}\right]$ where $\alpha = -1 + i = \sqrt{2}e^{3\pi i/4}$. Binomial expansions give the Taylor coefficients $\frac{f^{(n)}(1)}{n!} = b_n$ as the coefficients of z^n:

$$b_n = -\tfrac{1}{2}\left(\frac{1}{\alpha^{n+1}} + \frac{1}{\bar{\alpha}^{n+1}}\right) = -\operatorname{Re}\left(\frac{1}{\alpha^{n+1}}\right) = \frac{-1}{2^{(n+1)/2}} \cos \tfrac{3\pi}{4}(n + 1).$$

This neatly illustrates Hadamard's dictum that the shortest path to a real variable result may pass through the complex plane!

Yours sincerely,

NICK LORD

Tonbridge School, Kent TN9 1JP

DEAR EDITOR,*

In Note 84.11 (March 2000, pp. 89-90) Alexander J. Gray considers the Fibonacci numbers $f_0 = 0, f_1 = 1, f_n = f_{n-1} + f_{n-2}$ and defines

$$\tau_n = \sum_{k=0}^{n} f_k.$$

He observes and proves that $1 + \tau_n + \tau_{n+1} = \tau_{n+2}$. However, one can also observe that $\tau_n = f_{n+2} - 1$. This observation is easily verified by mathematical induction and then the recurrence $1 + \tau_n + \tau_{n+1} = \tau_{n+2}$ follows immediately from the Fibonacci recurrence.

Yours sincerely,

ROBIN CHAPMAN

School of Mathematical Sciences, University of Exeter EX4 4QE

e-mail *rjc@maths.exeter.ac.uk*

* Editor's Note: Nigel Hodges of Cheltenham wrote making the same point.