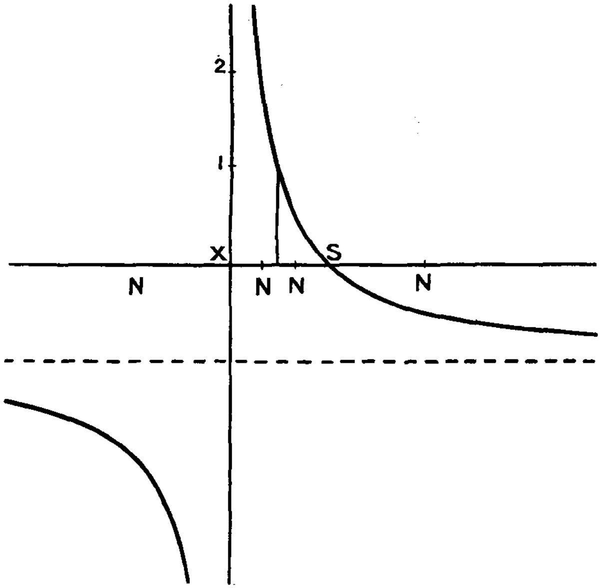
Position-Ratio.—The position-ratio of any point N on the line SX is defined as the ratio $\frac{SN}{NX}$.

The graph in the figure is obtained by drawing through each point on the line a perpendicular equal to its position-ratio, on any convenient scale.



This graph may be used as follows:

I. If the position-ratio is given in magnitude and sign the point is uniquely determined.

II. The graph shows the variation of the position-ratio as N moves along the line.

III. The values of the position-ratios of different points may be compared.

IV. To any point N there corresponds one (and only one) other point whose position-ratio is numerically equal to that of N. (It differs in sign.) Such pairs of points are called harmonic conjugates.

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V. The relative positions of points which are harmonically conjugate may be seen in the figure.

VI. The following theorems, which are important in the usual method of investigating the forms of ellipse, hyperbola and parabola, are at once evident by inspection of the graph.

(i) If A and A′ are the two points whose position-ratios are (numerically) $=e(e<1)$, then for all points between A and A′ the position-ratio is $<e$; for all other points it is $>e$.

(ii) If A and A′ are as before and e is >1, then for all points between A and A′ the position-ratio is $>e$; for all others it is $<e$.

(iii) If O be the point whose position-ratio e is equal to unity, then for all points on one side of O the position-ratio is $<e$, for points on the other side it is $>e$.

Note.—If a position-ratio be taken as $\dfrac{\mathrm{XN}}{\mathrm{NS}}$ the figure is reversed, but the same results follow.

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