On the most Economical Speed to drive a Steamer in relation to the cargo carried and coals consumed.

By W. J. MILLAR, C.E.

The object of this communication is simply to show how a formula may be obtained which will indicate the most economical speed for a steamer in relation to cargo carried and coals consumed on the voyage.

Let V = speed of ship, say in knots.

,, C = total carrying capacity of ship (cargo and coal), say in tons.

,, T = time occupied on voyage, say in hours.

,, S = space traversed, or voyage, say in knots.

Let the power required to drive the ship be supposed to vary as V^3*, then

$$P = mV^3 \quad \ldots \quad \ldots \quad \ldots \quad (1).$$

The *work* done in traversing a given distance will vary as $P \times$ space, hence we may write for total work done on voyage

$$mV^3S. \quad \ldots \quad \ldots \quad \ldots \quad (2).$$

Since, however, this work is obtained from the energy contained in the coal carried, we may express this coal consumpt in tons as

$$K(mV^3S) \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

Hence for the clear cargo capacity we have

$$C - K(mV^3S) \quad \ldots \quad \ldots \quad \ldots \quad (4).$$

Now let the marketable value of the cargo be assumed to vary inversely as the time taken on the voyage, then we may write, as representing this value, $\{C - K(mV^3S)\}/T \quad \ldots \quad \ldots \quad (5).$

But $T = S/V$ hence we have by substitution

$$(CV/S) - K(mV^4) \quad \ldots \quad \ldots \quad (6).$$

Now, since, by the question, this must be a maximum, differentiate and we have

$$(CdV/S) - 4KmV^3dV = 0$$

or

$$C/4KmS = V^3$$

hence

$$V = \sqrt[3]{C/4KmS} \quad \ldots \quad \ldots \quad (7).$$

The most economical speed, therefore, at which the vessel should be driven to fulfil the conditions is that which is equal to the cube root of the total carrying capacity of the ship divided by the product of the space travelled, or voyage completed, and the constants noted.

* Comparisons of power and speed at different speeds show that the exponent of speed is not, however, a constant quantity, but varies itself to some extent with the speed.

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To give an example of application of the rule obtained:—

An Atlantic steamer propelled at an average speed of 16 knots per hour by engines indicating 6000 horse power, with an expenditure of, say, 1000 tons of coal on a voyage of 3000 knots.

The cargo carrying capacity of the vessel may be taken at 3000 tons.

First, to find the value of the constant m, we have $6000 = m16^3$ (see (1)),

hence $$m = 1\tfrac{1}{2} \text{ nearly.}$$

Second, to find the value of the constant K we have

$$1000 = K \times 1\tfrac{1}{2} \times 16^3 \times 3000 \quad \ldots \quad \ldots \quad (3),$$

hence $$K = \text{say } \frac{1}{18,000}.$$

The value of V (see (7)) will therefore be expressed as

$$V = \sqrt[3]{3000 \times C/S}.$$

Now to apply this rule to the case of the same ship or another of same size and power, making a longer voyage, say of 12,000 miles, we have

$$V = \sqrt[3]{(3000 \times 4000/12,000)} = 10 \text{ knots},$$

hence the most economical speed for a voyage of this latter extent becomes 10 knots, as compared with 16 knots for a voyage of 3000 knots.

Note.—It has been assumed, for purposes of simplicity, in the "Paper," that the commercial value of the cargo varies inversely as the time occupied on the voyage.

Clearly this is an assumption which might seldom occur in practice, there being so many conflicting variable elements in connection with such a question viewed purely from a commercial standpoint.

Doubtless, however, there are cases where the assumption cannot be so far from what occurs in practice—*e.g.*, the case of the China tea clipper, or perishable cargoes, such as fruit.

The investigation is simply made with the view of ascertaining the most economical relation between speed, cargo carried, and coal consumed, where certain data are given.

The author has looked at the question more from a mathematical than from a commercial point of view, as it seemed to indicate an investigation where mathematics might be used to point to certain conclusions, which, although not always obtained in practice, yet might be approximated to; the mathematical or physical treatment pointing out the way in which the subject might be treated.

Kötter's Synthetic Geometry of Algebraic Curves—Part III., Involution Nets, and Involutions of 2nd, 3rd, ... Rank.

By Rev. NORMAN FRASER, B.D.

[See Index.]

Sixth Meeting, 12th April 1889.

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On Vortex Motion in a rotating fluid.

By C. CHREE, M.A.

The object of the following paper is to consider the motion of one or more vortices in a compressible fluid, which is rotating as a whole with uniform angular velocity ω about an axis, taken as axis of z. To save space I shall when possible refer for results to a previous paper in the *Proceedings*, distinguishing the equations of that paper, Vol. V., pp. 52-59, by the suffix a.

Formulæ for fluid motion relative to rotating axes are given by Greenhill in the article "Hydrodynamics" in the *Encyclopaedia Britannica*, and by Basset in his "Treatise on Hydrodynamics," Vol. I., § 23. The velocities appearing however in these equations are partly at least absolute velocities, while equations containing only velocities relative to the moving axes seem most suitable for our purpose. Such equations may be obtained shortly as follows, confining our attention to the case when there is no velocity parallel to the axis of rotation.

Let u, v denote the velocity components relative to the moving axes ox, oy in the fluid at the point x, y at the time t, and let u', v'

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