TRANSITION PROBABILITIES FOR FORBIDDEN LINES

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ABSTRACT

A compilation is given of transition probabilities of forbidden lines which occur in the spectra of gaseous nebulae.

The spectra of gaseous nebulae contain both permitted and forbidden lines of many elements in various stages of ionization. These are of great importance because of the information they can provide on the physical conditions and chemical composition of the nebulae. The forbidden lines belong to both non-metals and metals, and indeed for some elements the only observable spectral lines are forbidden lines. Transition probabilities are now known for essentially every forbidden line which has been observed with reasonable certainty in gaseous nebulae, including the Orion Nebula and the very rich-spectrum planetary nebula NGC 7027.

Table 1

Transition probabilities for the $2p^2$ configuration**

Transition	C I	N II	O III	F IV	Ne V
$^1D_2$–$^1S_0$	0·50 8727·4	1·08 5754·6	1·60 4363·2	2·10 3532·2	2·60 2972
$^3P_2$–$^1S_0$	$1·9 \times 10^{-5}$ 4627·3	$1·6 \times 10^{-4}$ 3070·8	$7·1 \times 10^{-4}$ 2331·6	$2·3 \times 10^{-3}$ 1889·3	$6·8 \times 10^{-3}$ 1592·7
$^3P_1$–$^1S_0$	$2·6 \times 10^{-3}$ 4621·5	0·034 3063·0	0·23 2321·1	1·1 1875·5	4·2 1575·2
$^3P_2$–$^1D_2$	$2·3 \times 10^{-4}$ 9849·5	$3·0 \times 10^{-3}$ 6583·4	0·021 5006·8	0·098 4060·2	0·38 3425·9
$^3P_1$–$^1D_2$	$7·8 \times 10^{-5}$ 9823·4	$1·03 \times 10^{-3}$ 6548·1	0·0071 4958·9	0·034 3997·4	0·138 3345·8
$^3P_0$–$^1D_2$	$5·5 \times 10^{-8}$ 9808·9	$4·2 \times 10^{-7}$ 6527·4	$1·9 \times 10^{-6}$ 4931·0	$6·4 \times 10^{-6}$ 3960·7	$1·9 \times 10^{-5}$ 3300·0
$^3P_1$–$^3P_2$	$2·7 \times 10^{-7}$	$7·5 \times 10^{-6}$	$9·8 \times 10^{-5}$	$7·9 \times 10^{-4}$	$4·6 \times 10^{-3}$
$^3P_0$–$^3P_2$	$2·0 \times 10^{-14}$	$1·3 \times 10^{-12}$	$3·5 \times 10^{-11}$	$5·0 \times 10^{-10}$	$5·2 \times 10^{-9}$
$^3P_0$–$^3P_1$	$7·9 \times 10^{-8}$	$2·1 \times 10^{-6}$	$2·6 \times 10^{-5}$	$2·1 \times 10^{-4}$	$1·3 \times 10^{-3}$

** Compiled from Wiese *et al.* (1966), which is based on work by Garstang; Naqvi; and Yamanouchi and Horie.

* Of the National Bureau of Standards and the University of Colorado.

Osterbrock and O'Dell (eds.), Planetary Nebulae, 143–152. © *I.A.U.*

Table 2

Transition probabilities* for the $2p^3$ configuration

Transition	N I	O II	F III	Ne IV	Mg VI
$^2P_{\frac{1}{2}}-{}^2P_{1\frac{1}{2}}$	(very small)	$6{\cdot}0 \times 10^{-11}$	(very small)	$2{\cdot}3 \times 10^{-9}$	$1{\cdot}6 \times 10^{-5}$
$^2D_{2\frac{1}{2}}-{}^2P_{1\frac{1}{2}}$	0·054	0·115	0·18	0·40	2·4
	10395·4	7319·4	5721·2	4714·3	3485·5
$^2D_{1\frac{1}{2}}-{}^2P_{1\frac{1}{2}}$	0·025	0·061	0·114	0·44	3·8
	10404·1	7330·7	5733·0	4724·2	3488·1
$^2D_{2\frac{1}{2}}-{}^2P_{\frac{1}{2}}$	0·031	0·061	0·088	0·11	0·15
	10395·4	7318·6	5721·2	4715·6	3500·4
$^2D_{1\frac{1}{2}}-{}^2P_{\frac{1}{2}}$	0·047	0·100	0·16	0·39	2·5
	10404·1	7329·9	5733·0	4725·6	3503·0
$^4S_{1\frac{1}{2}}-{}^2P_{1\frac{1}{2}}$	$6{\cdot}2 \times 10^{-3}$	0·060	0·26	1·33	13·
	3466·4	2470·4	1939·6	1608·8	
$^4S_{1\frac{1}{2}}-{}^2P_{\frac{1}{2}}$	$2{\cdot}5 \times 10^{-3}$	0·0238	0·10	0·53	5·3
	3466·4	2470·3	1939·6	1609·0	
$^2D_{2\frac{1}{2}}-{}^2D_{1\frac{1}{2}}$	$1{\cdot}3 \times 10^{-8}$	$1{\cdot}3 \times 10^{-7}$	$7{\cdot}6 \times 10^{-7}$	$1{\cdot}4 \times 10^{-6}$	$1{\cdot}5 \times 10^{-7}$
$^4S_{1\frac{1}{2}}-{}^2D_{2\frac{1}{2}}$	$6{\cdot}9 \times 10^{-6}$	$4{\cdot}8 \times 10^{-5}$	$1{\cdot}3 \times 10^{-4}$	$5{\cdot}9 \times 10^{-4}$	0·0054
	5200·4	3728·8	2933·1	2441·3	
$^4S_{1\frac{1}{2}}-{}^2D_{1\frac{1}{2}}$	$1{\cdot}6 \times 10^{-5}$	$1{\cdot}70 \times 10^{-4}$	$1{\cdot}3 \times 10^{-3}$	$5{\cdot}6 \times 10^{-3}$	0·12
	5197·9	3726·0	2930·0	2438·6	

* Compiled for N I, O II, F III and Ne IV from Wiese *et al.* (1966), which is based on work by Ufford and Gilmour; Naqvi; Garstang; and Seaton and Osterbrock. Mg VI recalculated by Garstang for inclusion here.

Table 3

Transition probabilities* for the $2p^4$ configuration

Transition	O I	F II	Ne III	Na IV
$^1D_2-{}^1S_0$	1·34	2·1	2·8	3·5
	5577·4	4157·5	3342·5	2803·3
$^3P_2-{}^1S_0$	$3{\cdot}7 \times 10^{-4}$	$1{\cdot}6 \times 10^{-3}$	$5{\cdot}1 \times 10^{-3}$	0·012
	2958·4	2225·5	1793·8	1503·7
$^3P_1-{}^1S_0$	0·067	0·49	2·2	7·6
	2972·3	2246·6	1814·8	1529·1
$^3P_2-{}^1D_2$	$5{\cdot}1 \times 10^{-3}$	0·038	0·17	0·66
	6300·3	4789·5	3868·8	3241·7
$^3P_1-{}^1D_2$	$1{\cdot}64 \times 10^{-3}$	0·012	0·052	0·20
	6363·8	4869·3	3967·5	3362·2
$^3P_0-{}^1D_2$	$1{\cdot}1 \times 10^{-6}$	$4{\cdot}1 \times 10^{-6}$	$1{\cdot}2 \times 10^{-5}$	$3{\cdot}0 \times 10^{-5}$
	6391·6	4904·8	4012·7	3416·2
$^3P_1-{}^3P_0$	$1{\cdot}7 \times 10^{-5}$	$1{\cdot}8 \times 10^{-4}$	$1{\cdot}2 \times 10^{-3}$	$5{\cdot}5 \times 10^{-3}$
$^3P_2-{}^3P_0$	$1{\cdot}0 \times 10^{-10}$	$1{\cdot}8 \times 10^{-9}$	$2{\cdot}0 \times 10^{-8}$	$1{\cdot}5 \times 10^{-7}$
$^3P_2-{}^3P_1$	$9{\cdot}0 \times 10^{-5}$	$9{\cdot}0 \times 10^{-4}$	$6{\cdot}0 \times 10^{-3}$	0·030

* Compiled for O I, F II and Ne III from Wiese *et al.* (1966), based on work by Garstang; Naqvi; Yamanouchi and Horie and for (O I) Omholt; Stoffregen and Derblom. Na IV from Malville and Berger (1965).

Table 4

Transition probabilities* for the $3p^2$ configuration

Transition	S III	Cl IV	Ar V	K VI	Ca VII
$^1D_2$–$^1S_0$	2·54 6312·1	3·15 5323·3	3·78 4625·5	4·1 4097:	4·3 3688:
$^3P_2$–$^1S_0$	0·016 3796·7	0·038 3203·2	0·081 2784·4	0·14 2471·7	0·25 2226:
$^3P_1$–$^1S_0$	0·85 3721·7	2·6 3118·3	6·8 2691·4	16· 2366·8	34· 2112:
$^3P_2$–$^1D_2$	0·064 9532·1	0·20 8045·6	0·51 7005·7	1·1 6228·4	2·5 5614·7
$^3P_1$–$^1D_2$	0·025 9069·4	0·080 7530·5	0·22 6435·1	0·53 5603·2	1·2 4939:
$^3P_0$–$^1D_2$	$9·1 \times 10^{-6}$ 8831·5	$2·2 \times 10^{-5}$ 7262·3	$4·9 \times 10^{-5}$ 6131·0	$1·1 \times 10^{-4}$ 5269·2	$2·1 \times 10^{-4}$ 4571:
$^3P_1$–$^3P_2$	$2·4 \times 10^{-3}$	$8·2 \times 10^{-3}$	0·027	0·076	0·20
$^3P_0$–$^3P_2$	$4·7 \times 10^{-8}$	$2·8 \times 10^{-7}$	$1·3 \times 10^{-6}$	$5·4 \times 10^{-6}$	$1·9 \times 10^{-5}$
$^2P_0$–$^3P_1$	$4·7 \times 10^{-4}$	$2·1 \times 10^{-3}$	$8·0 \times 10^{-3}$	0·026	0·076

* S III, Cl IV and Ar V from Czyzak and Krueger (1963), K VI and Ca VII from Malville and Berger (1965).

Table 5

Transition probabilities* for the $3p^3$ configuration

Transition	S II	Cl III	Ar IV	K V
$^2P_{\frac{1}{2}}$–$^2P_{1\frac{1}{2}}$	$1·0 \times 10^{-6}$	$7·6 \times 10^{-6}$	$5·2 \times 10^{-5}$	$2·8 \times 10^{-4}$
$^2D_{2\frac{1}{2}}$–$^2P_{1\frac{1}{2}}$	0·21 10320·6	0·36 8481·6	0·67 7237·3	1·5 6317:
$^2D_{1\frac{1}{2}}$–$^2P_{1\frac{1}{2}}$	0·17 10287·1	0·39 8433·7	0·91 7170·6	2·3 6223:
$^2D_{2\frac{1}{2}}$–$^2P_{\frac{1}{2}}$	0·087 10372·6	0·108 8550·5	0·122 7332·0	0·19 6447:
$^2D_{1\frac{1}{2}}$–$^2P_{\frac{1}{2}}$	0·20 10338·8	0·35 8501·8	0·68 7262·8	1·5 6349:
$^4S_{1\frac{1}{2}}$–$^2P_{1\frac{1}{2}}$	0·34 4068·6	0·96 3342·9	2·55 2854·8	6·5 2494·5
$^4S_{1\frac{1}{2}}$–$^2P_{\frac{1}{2}}$	0·134 4076·4	0·37 3353·3	0·97 2869·1	2·4 2514·5
$^2D_{2\frac{1}{2}}$–$^2D_{1\frac{1}{2}}$	$3·3 \times 10^{-7}$	$3·2 \times 10^{-6}$	$2·3 \times 10^{-5}$	$1·4 \times 10^{-4}$
$^4S_{1\frac{1}{2}}$–$^2D_{2\frac{1}{2}}$	$4·7 \times 10^{-5}$ 6716·4	$1·01 \times 10^{-3}$ 5517·2	$2·2 \times 10^{-3}$ 4711·3	$6·9 \times 10^{-3}$ 4122·6
$^4S_{1\frac{1}{2}}$–$^2D_{1\frac{1}{2}}$	$3·0 \times 10^{-4}$ 6730·8	$7·0 \times 10^{-3}$ 5537·7	0·028 4740·2	0·11 4163·3

* From Czyzak and Krueger (1963) for S II, Cl III and Ar IV. K V calculated by Garstang for inclusion here.

Table 6

Transition probabilities* for the 3p⁴ configuration

Transition	S I	Cl II	Ar III	K IV	Ca V
$^1D_2$–$^1S_0$	1·78	2·3	3·1	3·9	4·6
	7724·7	6152·9	5191·8	4510·9	3996·3
$^3P_2$–$^1S_0$	$7·3 \times 10^{-3}$	0·018	0·043	0·086	0·16
	4506·9	3583·0	3005·1	2593·5	2280·0
$^3P_1$–$^1S_0$	0·35	1·3	4·0	10·4	24·
	4589·0	3675·0	3109·0	2711·2	2412·4
$^3P_2$–$^1D_2$	0·028	0·10	0·32	0·83	1·9
	10819·8	8579·5	7135·8	6101·8	5309·2
$^3P_1$–$^1D_2$	$8·0 \times 10^{-3}$	0·029	0·083	0·20	0·43
	11305·8	9125·8	7751·0	6795·8	6086·9
$^3P_0$–$^1D_2$	$5·0 \times 10^{-6}$	$1·2 \times 10^{-5}$	$2·9 \times 10^{-5}$	$6·0 \times 10^{-5}$	$1·1 \times 10^{-4}$
	11540·1	9381·8	8036·4	7110·4	6428·2
$^3P_1$–$^3P_0$	$3·0 \times 10^{-4}$	$1·4 \times 10^{-3}$	$5·1 \times 10^{-3}$	0·015	0·035
$^3P_2$–$^3P_0$	$7·1 \times 10^{-8}$	$4·8 \times 10^{-7}$	$2·7 \times 10^{-6}$	$1·2 \times 10^{-5}$	$4·5 \times 10^{-5}$
$^3P_2$–$^3P_1$	$1·4 \times 10^{-3}$	$7·5 \times 10^{-3}$	0·031	0·10	0·31

* S I, Cl II and Ar III from Czyzak and Krueger (1963), K IV and Ca V from Malville and Berger (1965).

Table 7

Transition probabilities for selected* lines of Mn VI and Fe VII

Transition	J–J′	Mn VI λ	Mn VI A	Fe VII λ	Fe VII A
a^3F–a^1D	2–2	6518·3	0·14	5721·1	0·30
	3–2	6852:	0·23	6086·9	0·49
	4–2	7315:	$9·0 \times 10^{-4}$	6598·8	$1·6 \times 10^{-3}$
a^3F–a^3P	2–0	5622:	0·087	4989:	0·11
	2–1	5536:	0·031	4893·4	0·043
	3–1	5776·4	0·050	5159·0	0·063
	2–2	5367:	$6·3 \times 10^{-3}$	4699·8	0·012
	3–2	5591:	0·030	4944·0	0·065
	4–2	5894·0	0·050	5277·7	0·060
a^3F–a^1G	3–4	4036·8	0·12	3587·8	0·26
	4–4	4193·1	0·17	3760·3	0·37

* From Garstang (1964) for Mn VI and Pasternack (1940) for Fe VII, with the electric quadrupole contributions in Fe VII reduced by the appropriate factor given by Garstang (1964). Results for many additional lines can be found in these references.

Tables 1–11 give the transition probabilities of spontaneous emission in sec^{-1}. The transitions (with one exception specially noted) take place by magnetic dipole radiation, by electric quadrupole radiation, or by both. When both are possible one type usually predominates, but there are some cases where the two types of radiation have comparable probabilities. In our tables we have given the total transition probability; anyone interested in the type of radiation involved in a particular line and in the individual magnetic dipole or electric quadrupole-transition probabilities is referred to the original papers or compilations mentioned in the notes to the tables. The wavelengths are given (in Ångstroms unless microns are indicated) for lines in the observable spectral region and for some infrared and ultraviolet transitions. Many of the

Table 8

Transition probabilities for selected* lines of Mn v and Fe vi

Transition	J–J′	Mn v λ	Mn v A	Fe vi λ	Fe vi A
$a^4F–a^4P$	4½–2½	6393·6	0·041	5677·0	0·048
	3½–2½	6166·2	0·016	5426·6	0·021
	2½–2½	5991:	$4{\cdot}1 \times 10^{-3}$	5233·9	$5{\cdot}9 \times 10^{-3}$
	1½–2½	5868:	$5{\cdot}0 \times 10^{-4}$	5097·5	$7{\cdot}9 \times 10^{-4}$
	3½–1½	6346:	0·031	5630·8	0·036
	2½–1½	6159:	0·026	5423·9	0·032
	1½–1½	6030:	$9{\cdot}3 \times 10^{-3}$	5277·5	0·014
	2½– ½	6218·6	0·026	5484·8	0·031
	1½– ½	6088:	0·044	5335·2	0·055
$a^4F–a^2G$	4½–4½	5891·1	0·24	5176·4	0·56
	3½–4½	5695:	0·096	4967·3	0·22
	2½–4½	5544:	$1{\cdot}7 \times 10^{-6}$	4805·4	$3{\cdot}1 \times 10^{-6}$
	4½–3½	6069:	$5{\cdot}9 \times 10^{-3}$	5370·5	0·012
	3½–3½	5862·3	0·096	5145·8	0·22
	2½–3½	5703:	0·088	4972·1	0·20
	1½–3½	5592:	$5{\cdot}6 \times 10^{-6}$	4849·0	$1{\cdot}1 \times 10^{-5}$

* From Pasternack (1940) with the electric quadrupole contributions reduced by appropriate factors given by Garstang (1964). There are many other lines of these ions for which data can be found in these references.

wavelengths are quoted from Bowen (1955, 1960). Some wavelengths are uncertain by several tenths of an Ångstrom in cases where they have not been directly observed and reliance is upon predictions based on ultraviolet permitted-line spectroscopy.

A compilation of data for all atoms up to neon has been given by Wiese *et al.* (1966). Where the data we quote are the same as theirs, we reference only their book. References may be found in their book to the original papers by Garstang; Naqvi; Yamanouchi and Horie; Ufford and Gilmour; Seaton and Osterbrock; Omholt; and Stoffregen and Derblom upon which their compilation is based. For atoms heavier than

Table 9

Transition probabilities for selected* lines of Fe III and Fe V

Transition	J–J′	Fe III λ	Fe III A	Fe V λ	Fe V A
$a^5D–a^5D$	0–1		$1·4 \times 10^{-4}$		$1·6 \times 10^{-4}$
	1–2		$6·7 \times 10^{-4}$		$1·2 \times 10^{-3}$
	2–3		$1·8 \times 10^{-3}$		$2·6 \times 10^{-3}$
	3–4		$2·8 \times 10^{-3}$		$3·0 \times 10^{-3}$
$a^5D–a^3P$	1–0	4930·5	0·67	4180·9	1·3
	2–0	4884·5	$2·4 \times 10^{-4}$	4229·3	$2·8 \times 10^{-4}$
	0–1	5084·8	0·091	4003·0	0·13
	1–1	5060·5	$1·5 \times 10^{-4}$	4026·4	$2·1 \times 10^{-4}$
	2–1	5011·3	0·53	4071·3	1·1
	3–1	4936·4	$3·2 \times 10^{-5}$	4136·2	$4·1 \times 10^{-5}$
	0–2	5439·9	$1·5 \times 10^{-5}$	3777·2	$4·1 \times 10^{-5}$
	1–2	5412·2	0·038	3798·0	0·036
	2–2	5355·9	$1·1 \times 10^{-4}$	3838·0	$2·0 \times 10^{-4}$
	3–2	5270·3	0·40	3895·5	0·71
	4–2	5151·9	$7·1 \times 10^{-6}$	3970·0	$1·5 \times 10^{-5}$
$a^3P–a^3P$	0–1		$7·5 \times 10^{-3}$		0·014
	1–2		0·047		0·045
$a^5D–a^3H$	4–4	4881·1	$4·8 \times 10^{-3}$	4227·5	$1·1 \times 10^{-3}$
$a^3H–a^3H$	4–5		$1·9 \times 10^{-4}$		$6·5 \times 10^{-4}$
	5–6		$4·1 \times 10^{-4}$		$5·8 \times 10^{-4}$
$a^5D–a^3F$	0–2	4799·4	$9·3 \times 10^{-6}$	3735·7	$2·2 \times 10^{-5}$
	1–2	4777·9	0·049	3756·1	0·10
	2–2	4733·9	0·10	3795·2	0·20
	3–2	4667·0	0·026	3851·4	0·047
	4–2	4573·8	$2·1 \times 10^{-6}$	3924·2	$1·5 \times 10^{-6}$
	1–3	4814·1	$1·1 \times 10^{-5}$	3744·8	$8·6 \times 10^{-6}$
	2–3	4769·6	0·087	3783·6	0·16
	3–3	4701·6	0·27	3839·5	0·40
	4–3	4607·1	0·038	3911·9	0·066
	2–4	4824·2	$7·6 \times 10^{-6}$	3764·4	$8·6 \times 10^{-7}$
	3–4	4754·8	0·081	3819·8	0·16
	4–4	4658·1	0·44	3891·3	0·74
$a^5D–a^3G$	3–3	4046·4	$8·0 \times 10^{-3}$	3445·4	0·017
	4–4	4008·4	0·019	3463·4	0·032
$a^5D–a^3D$	0–1	3366·2	0·13	?	0·22
	1–1	3355·6	0·15	?	0·19
	1–2	3356·6	0·095	?	0·20
	2–2	3334·9	0·11	?	0·18
	3–2	3301·6	0·027	?	0·11
	2–3	3319·3	0·044	?	0·097
	3–3	3286·2	0·047	?	0·089
	4–3	3239·7	0·23	?	0·37

* From Garstang (1957), where results are given for many additional lines.

Table 10

Transition probabilities for selected* lines of Fe II

Transition	J–J′	λ	A
$a^6D–a^6S$	4½–2½	4287·4	1·12
	3½–2½	4359·3	0·82
	2½–2½	4413·8	0·58
	1½–2½	4452·1	0·37
	½–2½	4474·9	0·18
$a^4F–a^4G$	4½–5½	4244·0	0·90
	3½–5½	4346·9	0·21
	4½–4½	4177·2	0·14
	3½–4½	4276·8	0·65
	2½–4½	4352·8	0·31
	4½–3½	4146·6	$8·7 \times 10^{-3}$
	3½–3½	4244·8	0·25
	2½–3½	4319·6	0·53
	1½–3½	4372·4	0·28
	4½–2½	4134·0	$2·0 \times 10^{-4}$
	3½–2½	4231·6	0·024
	2½–2½	4305·9	0·31
	1½–2½	4358·4	0·73
$a^4F–b^4F$	4½–4½	4814·6	0·40
	3½–4½	4947·4	0·050
	2½–4½	5049·3	$7·2 \times 10^{-4}$
	4½–3½	4774·7	0·13
	3½–3½	4905·3	0·22
	2½–3½	5005·5	0·071
	1½–3½	5076·6	$1·6 \times 10^{-5}$
	4½–2½	4745·5	0·013
	3½–2½	4874·5	0·17
	2½–2½	4973·4	0·14
	1½–2½	5043·5	0·065
	3½–1½	4852·7	0·022
	2½–1½	4950·7	0·17
	1½–1½	5020·2	0·18
$a^6D–b^4F$	4½–4½	4416·3	0·46
	3½–4½	4492·6	0·060
	2½–4½	4550·5	$2·6 \times 10^{-7}$
	4½–3½	4382·8	0·055
	3½–3½	4458·0	0·29
	2½–3½	4514·9	0·066
	1½–3½	4555·0	$4·8 \times 10^{-8}$
	4½–2½	4358·1	$1·6 \times 10^{-5}$
	3½–2½	4432·5	0·054
	2½–2½	4488·8	0·15
	1½–2½	4528·4	0·046
	½–2½	4552·0	$2·0 \times 10^{-6}$
	3½–1½	4414·5	$5·9 \times 10^{-6}$
	2½–1½	4470·3	0·029
	1½–1½	4509·6	0·058
	½–1½	4533·0	0·016

* From Garstang (1962), where data for many other lines may be found.

neon references are given to the original papers containing the data we have quoted. In each case we have quoted what we believe to be the best available results.

In a few cases we have given original data calculated for inclusion here. Mg VI and K V were originally calculated by Pasternack (1940). We have recalculated these, using the best available technique. The principal change results from the use of improved quadrupole radial integrals, which we have estimated by extrapolation. The changes

Table 11

Transition probabilities for some miscellaneous lines

Ion	Transition	J–J′	λ	A	Notes
Mg I	$3s^2\ ^1S–3s3p\ ^3P$	0–2	4562·5	$2{\cdot}0 \times 10^{-4}$	1
Ni III	$^3F–^3P$	3–1	6401·5	0·038	2
	$^3F–^3P$	2–0	6682·2	0·046	
	$^3F–^3P$	3–2	6533·7	0.12	
	$^3F–^1G$	3–4	4596·8	0·18	
C II	$2p\ ^2P$	$\frac{1}{2}$–$1\frac{1}{2}$	156 μ	$2{\cdot}4 \times 10^{-6}$	3
N III	$2p\ ^2P$	$\frac{1}{2}$–$1\frac{1}{2}$	57·3 μ	$4{\cdot}8 \times 10^{-5}$	3
O IV	$2p\ ^2P$	$\frac{1}{2}$–$1\frac{1}{2}$	25·9 μ	$5{\cdot}2 \times 10^{-4}$	3
Ne II	$2p^5\ ^2P$	$1\frac{1}{2}$–$\frac{1}{2}$	12·8 μ	0·0086	3
Mg IV	$2p^5\ ^2P$	$1\frac{1}{2}$–$\frac{1}{2}$	4·49 μ	0·20	4
Si II	$3p\ ^2P$	$\frac{1}{2}$–$1\frac{1}{2}$	34·8 μ	$2{\cdot}1 \times 10^{-4}$	4
S IV	$3p\ ^2P$	$\frac{1}{2}$–$1\frac{1}{2}$	10·6 μ	0·0077	4

[1] This line arises partly from magnetic quadrupole radiation and partly from nuclear-spin-induced electric dipole radiation. See Garstang (1967).
[2] Possible identification of 6401·5 by Flather and Osterbrock (1960). Other Ni III lines have not been seen in gaseous nebulae. Transition probabilities from Garstang (1958).
[3] From Wiese *et al.* (1966).
[4] Calculated by Garstang for inclusion here.

in the magnetic dipole results and in the relative electric quadrupole results are fairly small. We have also revised Pasternack's (1940) results for Mn VI, Fe VII, Mn V and Fe VI by introducing revised quadrupole radial integrals as described by Garstang (1964). We included Mg VI because of its possible, as yet unconfirmed, identification by Gauzit (1966). In Table 11 we have included some infrared transitions; other such transitions appear in many of the other tables, or can be found in the references (e.g. for Fe II and other heavy ions).

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DISCUSSION

Garstang: I would like to open the discussion myself by asking the question I am frequently asked – Do I believe the results? The answer is – Yes, I do. While one cannot entirely exclude the possibility that some unsuspected configuration interaction may produce significant perturbations, this seems unlikely for the transitions of interest for forbidden lines. There is now some substantial evidence for the basic correctness of the results. The comparison of [Fe II] lines in η Carinae with theoretical values (Thackeray, *Mon. Not. R. astr. Soc.*, **135**, 1967, 23) shows astonishingly good agreement, confirming the broad overall accuracy of the relative line strengths. Recent experimental work on interference effects in Zeeman components of two lines of mixed magnetic-dipole and electric-quadrupole radiation of [Pb I] and [Pb II] by Hults (*J. opt. Soc. Am.*, **56**, 1966, 1298) shows excellent agreement with the relative contributions of the two kinds of radiation predicted by Garstang (*J. Res. nat. Bur. Stand., Sec. A*, **68**, 1964, 61). Finally, experiments (Husain and Wiesenfeld, *Nature*, **213**, 1967, 1227) on flash photolysis of trifluoro-iodomethane have led to an estimate of the lifetime of the upper state of the lowest doublet in I I within a factor 3 of the theoretical lifetime given by Garstang (*J. Res. nat. Bur. Stand., Sec. A*, **68**, 1964, 61). In view of the experimental difficulties in handling a state whose lifetime is of the order of 0·1 sec this agreement must be considered satisfactory. Taking all these results together I think we must regard the transition probabilities of forbidden lines as reasonably well established.

Menzel: Some mention has been made of high-level transitions in hydrogen. I have derived an asymptotic formula for the f-values of such transitions between levels of quantum numbers n and n' with $n - n' = c$.

The f-value is

$$f_{nn'} = \frac{4n'}{3c^2} J_c(c)\, J_c'(c) = n' M(c),$$

where $J_c(c)$ and $J_c'(c)$ are respectively the Bessel functions of equal argument and order and its derivative. Examples of $M(c)$ follow:

$$M(1) = 1{\cdot}9077 \times 10^{-1}, \quad M(3) = 8{\cdot}1056 \times 10^{-3}$$
$$M(2) = 2{\cdot}6332 \times 10^{-2}, \quad M(4) = 3{\cdot}4917 \times 10^{-3}.$$

Note that for $n' = 100$, $f_{101,100} = 19$. The question might be asked, How can one reconcile such large f-values with the well-known f-sum rule $f = 1$? The chief point is that one must sum over all transitions, both upward and downward, from level n'. The former are counted positive, the latter, negative. Downward f-values can be calculated from upward ones by the formula $f_{n''n'} = -n''^2 f_{n'n''}/n'^2$. With these formulas, the negative contributions nearly cancel the large positive contributions, leaving the exact remainder

$$4 \sum_{c=1}^{\infty} \frac{J_c(c)\, J_c'(c)}{c} = 1$$

a new theorem in the theory of Bessel functions.

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