EVOLUTION OF CENTRAL STARS OF PLANETARY NEBULAE THEORY*

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1. Elementary Evolution Theory

Before discussing observational inputs and actual model calculations, I want to give a very elementary review of the relevant parts of stellar evolution theory. We shall only be dealing with stellar masses M below the Chandrasekhar limiting mass $M_{ch}(\sim 1{\cdot}2\text{–}1{\cdot}45\ M_{\odot}$, depending on chemical composition). The inequality $M < M_{ch}$ implies that relativistic effects are not of overriding importance and I will not mention them further (however, all quantitative model calculations which I will mention later include all the relativistic corrections to the equation of state, including radiation pressure which is also not very important). Let us try to estimate how the central temperature T_c and (total bolometric) luminosity L varies with central density ρ_c or radius $R(\rho_c \sim MR^{-3})$ for a star of fixed mass.

At low density we can surely use the classical perfect gas law for the equation of state and the mean thermal energy per particle $(\sim kT_c)$ is comparable with the mean gravitational potential energy per particle $(\sim GM/R)$, according to the Virial Theorem. T_c thus *increases* with increasing density $(T_c \sim MR^{-1} \sim M^{2/3}\ \rho_c{}^{1/3})$ for small T_c. However, the (non-relativistic) Fermi energy increases more rapidly $(\sim \rho_c{}^{2/3})$ with density, electron degeneracy becomes more important (when $\rho_c \sim \text{const}\ T_c{}^{3/2}$) and the electrons finally become fully degenerate at *zero* temperature when ρ_c approaches a critical value $\rho_{c,\max}$. The evolutionary phase of *decreasing* temperature as the density increases slightly towards its final value represents the white-dwarf phase, of course, but here we are interested in the phase just before.

Since T_c as a function of ρ_c first increases and then decreases it must reach a maximum value $T_{c,\max}$ at an intermediate value of density (a few times smaller than $\rho_{c,\max}$) when the electrons are partially degenerate. From the arguments above one can easily derive that

$$T_{c,\max} \sim M^{4/3};\ \rho_{c,\max} \sim M^2 .$$

Conversely, for a star to be able to heat up beyond a certain central temperature, its mass must exceed a certain minimum value. Thus only stars more massive than

* This work supported in part by the National Science Foundation under NSF grant GP-6928.

some M_{min} can burn some nuclear fuel (we shall be interested in carbon) in its interior (Deinzer and Salpeter, 1964). Other things being equal, the luminosity L of a star increases with its interior temperature, and L will have a maximum value at an evolutionary stage fairly close to (and slightly before) the one at which T_c reaches its maximum. Further, the numerical value of L_{max} is larger for larger masses, as illustrated schematically in Figure 1. In this figure we also see a schematic color-magnitude dia-

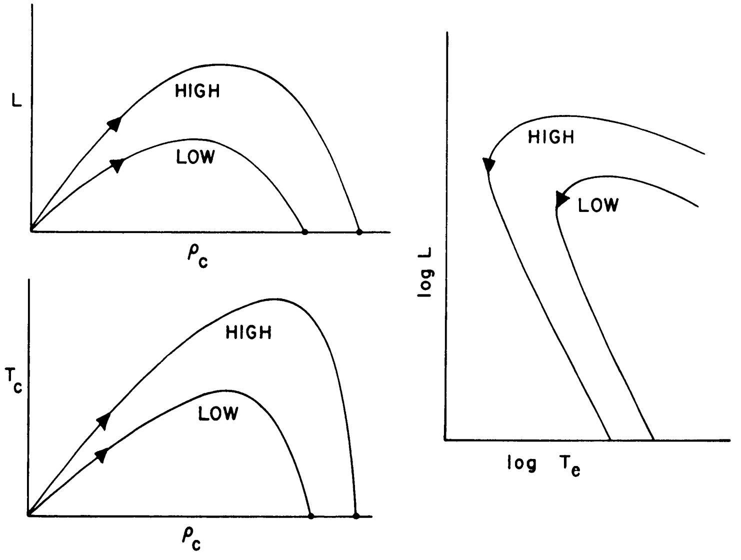

FIG. 1. *Schematic evolutionary tracks of partially degenerate stars.*

gram which illustrates that the effective *surface* temperature T_e also reaches a maximum value soon after the luminosity does, since $T_e^4 \sim LR^{-2}$ and the radius R is already fairly close to its final (smallest) white dwarf value.

2. Mass and Composition

Figure 2 is a schematic color-magnitude diagram, from a review by Stothers (1966), showing the observed positions of the central stars of planetary nebulae as well as the white-dwarf region and the evolutionary track for globular cluster stars. The high values of L and T_e for the central stars and a comparison with Figure 1 make it very plausible that these stars are at an evolutionary phase near their maximum luminosity and interior temperature. This qualitative statement is agreed upon by all, but the quantitative features are still highly controversial. The uncertainties are in a large part due to the fact that we do not have quantitative information on the mass and chemical composition of these stars.

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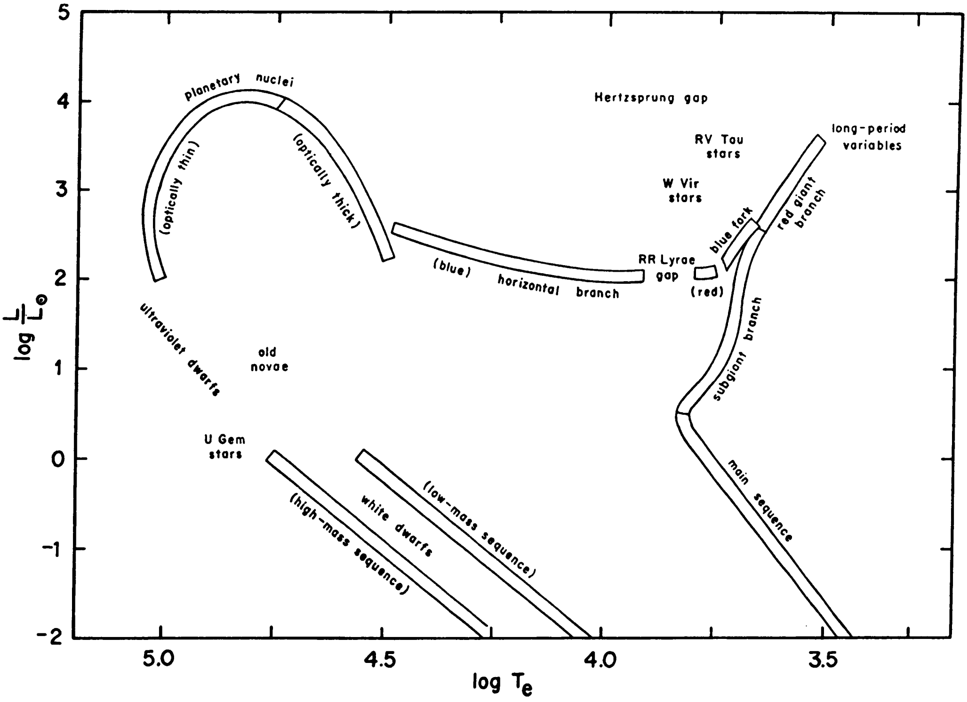

FIG. 2. *Schematic color-magnitude diagram (from Stothers, 1966).*

Central stars of planetary nebulae are fairly highly evolved and some mass loss during or after the red-giant phase (but *before* the mass ejection of the planetary nebula itself) is at least a possibility, but not certain. The amount of this previous mass loss is important (even if the present mass were known) in determining the present composition since carbon, oxygen and neon, as well as helium, are built up in the star's interior whereas the mass ejection is mainly from the hydrogen-rich outer layers. If the total age and original chemical composition of the star were known, its original mass could be determined (since the main sequence age depends on mass and the subsequent evolution is very rapid).

Previous speakers have shown that planetary nebulae belong to stellar population II (including one known member of a globular cluster) but not in its extreme form. This implies some spread in ages and hence a spread in original masses upwards from that of globular cluster red giants M_{gl}. The degree of spread is not known, as previous speakers emphasized, and the 'typical' (or mean) original mass could be only very slightly larger than M_{gl} or it could easily be $1{\cdot}5\ M_{gl}$, say. The numerical value of M_{gl} is itself in doubt at the moment (Faulkner and Iben, 1966; Hartwick *et al.*, 1967): M_{gl} could be as large as $1{\cdot}2\ M_\odot$ if the original composition was helium-poor or as low as 0·7 $M_\odot$ if the original stars were as helium-rich ($\sim$35% by mass) as population I. The present masses of central stars of planetary nebulae are also not known. As Dr. Weidemann will discuss in more detail, there is also a spread in the masses of white dwarfs with masses up to $\sim$0·6 $M_\odot$ 'typical' and (since it is not known whether further mass loss occurs between the planetary nebula and white-dwarf stages) we can only be fairly certain that the central stars have $M \gtrsim 0{\cdot}5\ M_\odot$. The upshot of these uncertainties is that the amount of pre-planetary mass loss could either be negligible or very considerable; original 'typical' masses could be as low as 0·7 $M_\odot$ and as high as 2 $M_\odot$ and the present mass of a 'typical' central star in the range of 0·5 to 1·2 $M_\odot$.

As mentioned above, there is considerable uncertainty in the present chemical composition of the central stars, but a few things are clear: (i) During the previous evolutionary phases considerable nuclear burning must have taken place and the central regions must be rich in C^{12} (and O^{16}, Ne^{20}) and intermediate regions in He^4. (ii) Since only the *extreme* population-II stars are metal-deficient, the original composition of most (but not necessarily all) of the central stars must have involved normal abundances of the metals and oxygen. (iii) The planetary nebulae themselves are not anomalously rich in helium and were ejected from the central stars. Thus, while a large fraction of the outermost hydrogen-rich (original composition) layers may have been ejected, the ejection could *not* have penetrated to the helium-rich layers beneath.

Evolutionary model sequences through helium shell burning are so far only available for a few cases (Iben, 1966; Kippenhahn *et al.*, 1966; Hofmeister, 1967). For a star of mass 5 $M_\odot$, e.g., there is one evolutionary state in which a central core of mass 0·9 $M_\odot$ consists mainly of C^{12} (and O^{16}, Ne^{20}), followed by a *thin* intermediate layer of helium. If mass ejection happened at this stage down to the boundary between the helium layer and the hydrogen envelope, one would have an almost pure carbon star of about 1 $M_\odot$. As we have discussed, typical masses of the original stars of planetary nebulae must be considerably less than 5 $M_\odot$, and it is not yet clear whether almost pure carbon stars are possible candidates for the central stars of planetary nebulae. Nevertheless, I will discuss such homogeneous stellar models next, if only for their simplicity.

3. Homogeneous Models

Before discussing actual models, let us consider Figure 3 which illustrates one general point contrasting models with a concentrated vs. those with an extended energy source. With a concentrated energy source, such as nuclear burning in the core, the heat flux is already close to the full luminosity even in the core and the temperature gradient is appreciable in the core. As a consequence, the temperature in the intermediate regions of the star (for given central temperature T_c and density) is lower and so is the temperature gradient from there on out which determines the value of the luminosity. Conversely, with an extended energy source such as gravitational energy release (or even more so with nuclear shell-source burning), the temperature in the inter-

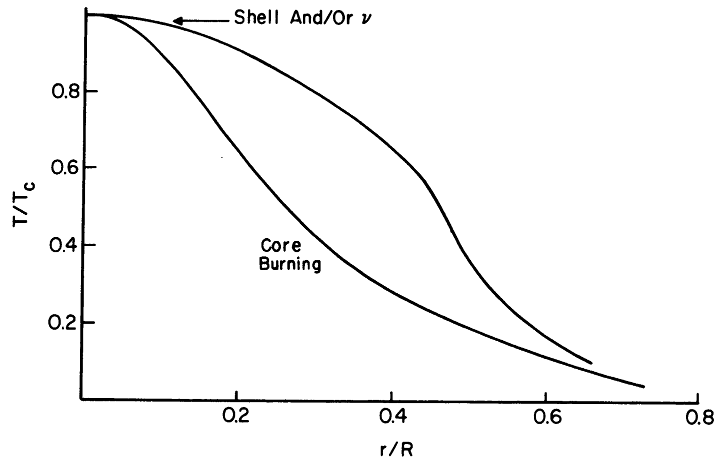

FIG. 3. *Schematic temperature distributions in different types of models.*

mediate regions is close to T_c and the temperature gradient further out, and hence the luminosity is larger. Neutrino energy *loss* (which is fairly concentrated) accentuates the effects of an extended energy source. We shall indeed find that the onset of nuclear core burning in a gravitationally contracting star *lowers* the luminosity L at first (with a slight increase in T_c), whereas the onset of shell burning *increases* L (and models with neutrino energy loss generally have larger optical L).

Explicit evolutionary sequences of models of homogeneous stars consisting mainly

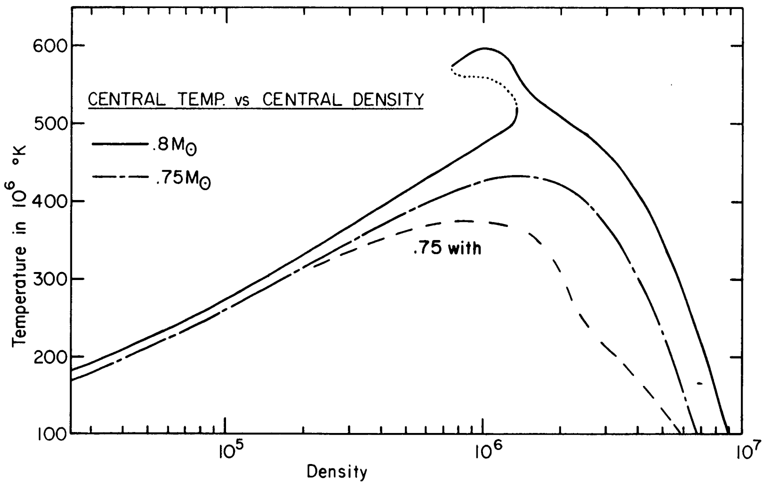

FIG. 4. *Central temperature plotted against central density for pure carbon models ('with' denotes inclusion of neutrino rates).*

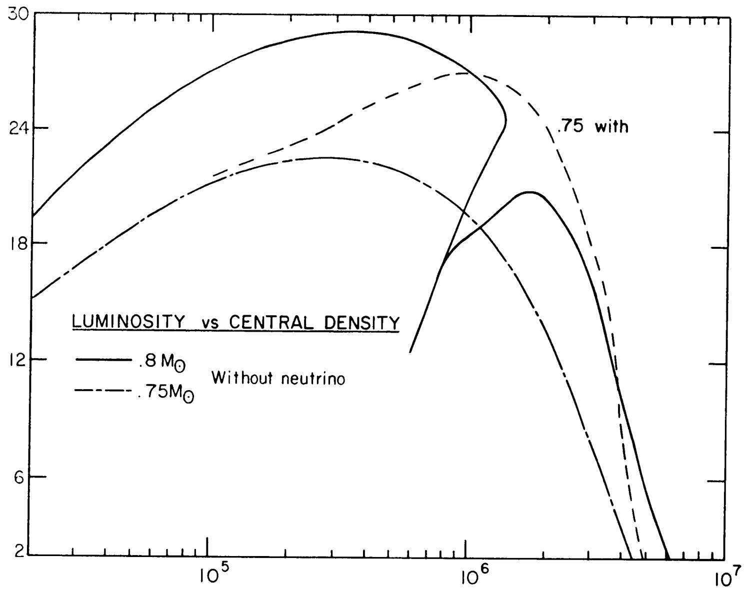

FIG. 5. *Bolometric luminosity plotted against central density for pure-carbon models.*

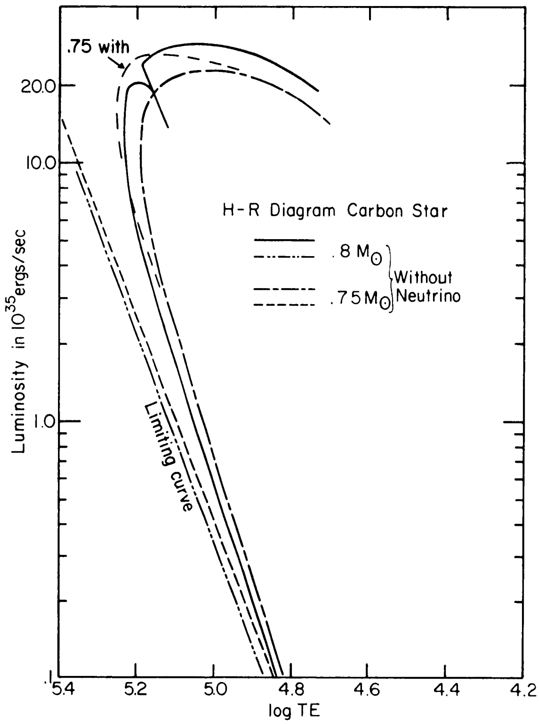

FIG. 6. *Color-magnitude diagram including constant-radius 'limiting curves'.*

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of carbon and oxygen (50% by mass of C^{12}) have now been carried out by a number of authors (L'Ecuyer, 1966; Beaudet, 1967; Beaudet and Salpeter, 1968; Rakavy and Shaviv, 1967). Figures 4–6 are the counterpart of Figure 1, adopted from some actual models for 0·75 $M_\odot$ and 0·8 $M_\odot$ without neutrino reactions and for 0·75 $M_\odot$ with neutrinos. Only the 0·8 $M_\odot$ model burns carbon in its core and its onset is marked by the sudden dip in luminosity. The 'limiting curves' marked on Figure 6 are the color-magnitude tracks at constant radius corresponding to the final equilibrium radius for 0·75 and 0·8 $M_\odot$.

A word about the remaining uncertainties in the models (for a homogeneous star of given mass): The relevant opacity is mainly electron scattering plus a little free-free opacity from the abundant carbon and oxygen atoms. The opacity is therefore fairly well known and errors in the luminosity due to this cause exceeding a factor of 2, are not likely. The various neutrino energy-loss processes have by now been studied fairly carefully. All these processes are predicted on the presence of the Universal Fermi Interaction, which is likely to be correct (but has not yet been rigorously proven from laboratory experiments). If these neutrino reactions *are* present at all, their total combined rate as a function of temperature and density is now known quite accurately ($\pm 20\%$; Beaudet *et al.*, 1967). In all the models published so far, Reeves' (1963) estimate for the energy production rate of the ($C^{12} + C^{12}$)-reaction was used. More recent Cal. Tech. experiments indicate that these rate estimates will have to be lowered, possibly by as much as a factor of about 40.

The minimum mass $M_{\min}$ for which a homogeneous carbon star can initiate appreciable carbon burning in its interior is of interest. As a comparison of the curves for 0·75 and 0·8 $M_\odot$ in Figure 4 shows, the onset of carbon burning is a sudden matter: for the lower mass there is too little burning to affect the model at all. For the slightly larger mass there is just enough carbon-burning just before T_c should reach a maximum for it to raise the value of T_c slightly which in turn makes the carbon-burning rate more appreciable, etc., so that this star burns an appreciable amount of carbon before T_c starts decreasing towards zero again. Without any neutrino processes included $M_{\min} \approx 0{\cdot}8\ M_\odot$ and with neutrinos included $M_{\min} \approx 1{\cdot}00\ M_\odot$, if the old carbon-burning rate is used. If this rate is to be lowered by a factor ~ 40 (as seems likely) these two masses will be raised to $M_{\min} \approx 0{\cdot}95\ M_\odot$ and $1{\cdot}10\ M_\odot$, respectively (Beaudet, 1967).

Figure 7 gives a comparison of the three illustrative models discussed above with the region in the color-magnitude diagram in which the central stars of planetary nebulae are observed. Note that these models have too low a luminosity L and too high a surface temperature. For models with neutrino rates included (which raises L and inhibits carbon burning) slightly higher masses (say 0·9 $M_\odot$ – 1·1 $M_\odot$) will have total bolometric luminosity comparable with the observed ones. However, these homogeneous models with higher mass and luminosity have very much larger surface temperatures than those observed. It behooves us now to look at more realistic inhomogeneous models.

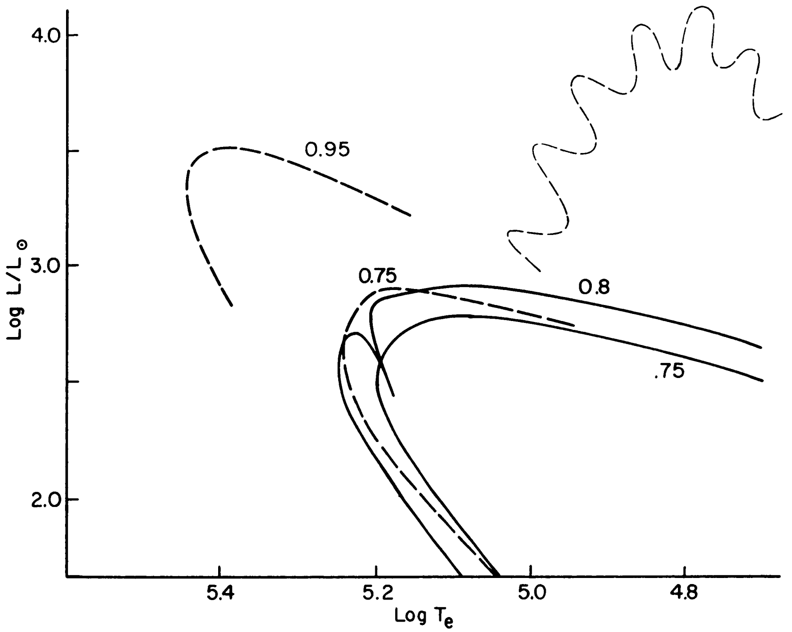

FIG. 7. *Color-magnitude diagram for pure-carbon models (with the region of observed central stars shown schematically in top right-hand corner).*

4. Inhomogeneous Models and Discussion

One kind of chemical inhomogeneity whose effect is easy to estimate is a hydrogen-rich envelope on a star consisting mainly of heavier elements (Cox and Salpeter, 1961). Extremely small amounts of hydrogen are sufficient to appreciably increase the radius and hence decrease the surface temperature T_e, even when the amount of hydrogen is too small for the bottom layer of the envelope to be undergoing any hydrogen burning. In such cases the luminosity is hardly affected at all and only T_e is decreased, but by a smaller and smaller fractional temperature shift as the central density increases. This effect is shown (semi-schematically) in Figure 8 in the curve labelled '10^{-4}H' which represents the 0·75 $M_\odot$ model with neutrinos (dashed curve to the left) but with an envelope of pure hydrogen of mass 10^{-4} $M_\odot$.

As one can see, this evolutionary track in Figure 8 is sufficiently 'reddened' to agree fairly well with the observed positions of central stars of planetary nebulae as regards surface temperature, although not in luminosity L. However, similar arguments apply to more massive carbon stars (0·9–1·2 $M_\odot$ say) which can have reasonable values of L as well as of T_e if they have a small amount of hydrogen in an envelope. Evolutionary time-scales (see also Figure 9) for the stars near the upper end of this mass-range are also at least of the right order of magnitude, $\lesssim 10^5$ years. On purely observational

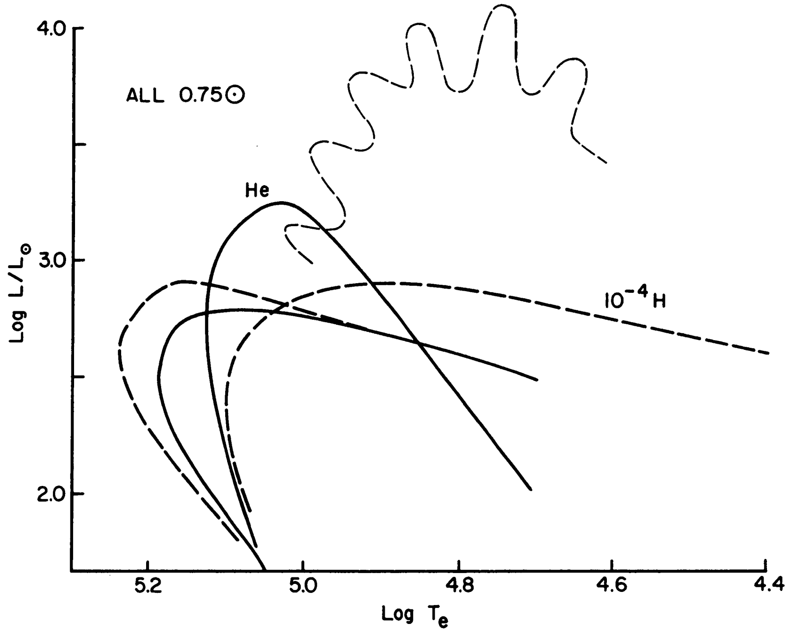

FIG. 8. *Color-magnitude diagram for various models (*He *indicates a helium-burning shell,* 10^{-4}H *a hydrogen envelope).*

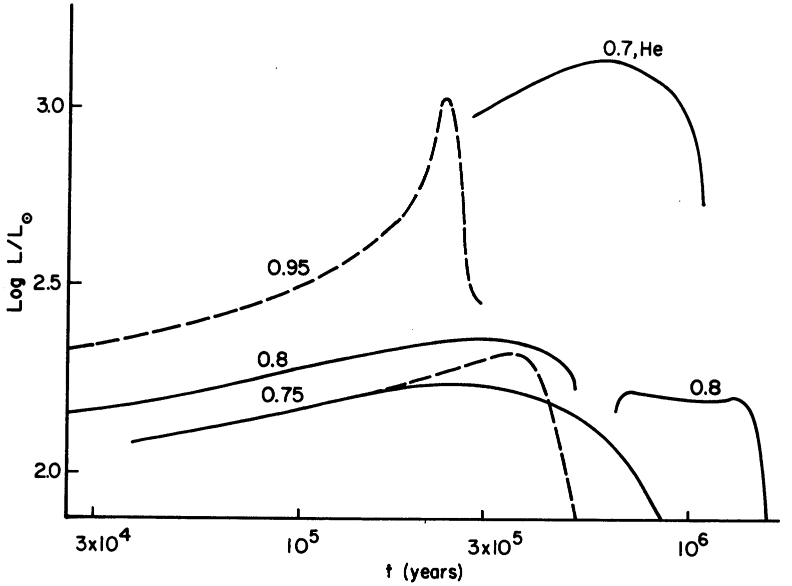

FIG. 9. *Bolometric luminosity plotted against evolutionary time for various models (starting at a stage slightly before maximum luminosity).*

grounds such stars of moderately large mass (but well below the Chandrasekhar limit) consisting mainly of carbon and oxygen (but not burning carbon) with a 'puffed up' envelope consisting of a small amount of hydrogen are possible candidates for central stars of planetary nebulae. On this picture the fairly rapid evolution ($\sim 10^5$ years) implied by the observations would be simply the rather rapid first stages of cooling down from the maximum-luminosity stage toward the white-dwarf stage, which are speeded up by the rather large neutrino luminosities at these high central temperatures.

This picture of carbon stars near 1 $M_\odot$ (a) is predicated on the assumption that the Universal Fermi interaction, and hence our neutrino energy-loss mechanisms, is correct and (b) may or may not be the explanation for the central stars of *some* planetary nebulae, but certainly *not* for all. To have such an appreciable mass consisting largely of carbon (and oxygen) now, the mass of the original main-sequence star must have been considerably larger. As discussed by previous speakers, planetary nebulae may belong to a somewhat mixed stellar population and some fraction of them may be young and have come from original stars of large mass, but presumably a large fraction of them are about as old as 'typical stellar population II' and therefore have come from original stars of moderate mass (0·7–1·4 $M_\odot$ as discussed for globular clusters).

Before turning to stars of smaller mass, a side remark which is likely to apply to all kinds of models: A hydrogen-rich envelope is required to 'redden' highly-evolved models, but the mass in this envelope is likely to be quite small (or else the models would be much redder than the observed central stars). The previous 'blowing-off' of the planetary nebula itself must therefore have removed a large fraction of the originally much more extensive hydrogen-rich envelope of the star. On the other hand we know that the planetary nebula gas is *not* overabundant in helium or oxygen; thus the blowing-off of the nebula could not have removed much of the layers below the envelope which are rich in helium or heavier elements. We therefore have the interesting (but not unexpected) indication that the lower boundary of the ejected material was close to the hydrogen-'heavier' interface where the mean molecular weight (and other quantities) had a discontinuity.

Let us return to stars of smaller mass which still consist mainly of carbon (and oxygen) in their deep interior, but have an additional helium-rich layer (as well as the outermost hydrogen envelope which only affects the radius). If this helium layer contained less than about 10^{-2} $M_\odot$, the helium would not burn at any stage and the layer would be quite unimportant. However, if the helium layer contains a few percent or more of the stellar mass, it does give shell-source burning (at least in the stages near the maximum central temperature) near the bottom of this layer (L'Ecuyer, 1966; Rose, 1966; Sugimoto and Yamamoto, 1966). As mentioned previously, shell-source burning can increase the bolometric luminosity of a star considerably. The curve labelled He in Figure 8 (and in Figure 9) represents (in a semi-schematic way) an

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evolutionary model whose carbon core is gravitationally contracting but with some helium burning in an outer shell.

As Figure 8 shows, shell-source burning increases the luminosity of a low-mass star somewhat, but not sufficiently so, and (as Figure 9 shows) gives much too long an evolutionary time-scale. However, this gloomy picture applies only to the time-averaged model and does not take account of 'thermal instabilities' (Schwarzschild and Härm, 1965). As Rose (1967) showed recently, a star with a helium-burning shell can lead to thermal instabilities which in turn can lead to even more violent events (such as pulsational instability). It will be a while before all the possible ramifications of thermal instabilities have been explored and explicit models involving them are proposed for central stars of planetary nebulae. At the moment I only want to point out optimistically that thermal instabilities can represent extreme forms of relaxation oscillations where peak luminosities are very much higher for a much shorter period (than on steadily evolving models), so that models for low-mass stars ($\lesssim 0{\cdot}7\ M_{\odot}$) may become compatible with the observations.

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DISCUSSION

Reeves: The one numerical value that we have obtained in Dr. Aller's talk is that the ratio of neon to oxygen in planetary nebulae is about ten times smaller than in ordinary stars. This observation does not seem to be compatible with the views presented in your talk.

Salpeter: I consider the abundance ratios in general an embarrassment: We were also told that the ratio of C + O + Ne abundances to H are comparable with or smaller than 'cosmic'. Therefore, these elements could not have been contributed to the nebula by its central star (or else the sum of these abundances would be larger); therefore, one cannot explain anomalous ratios of C to O to Ne in terms of reactions in the central star itself.

Böhm-Vitense: Why must the time-scale be an evolutionary one? Could it be that a small mass-loss determines the time-scale if a small change in a very thin hydrogen shell can move the star appreciably in the HR diagram? Since we estimated yesterday that in this region of the HR diagram a small mass-loss requires a large amount of energy, or continuously decreasing mass-loss might perhaps give an increase of luminosity as observed. Would you think this to be possible?

Savedoff: Not only burning hydrogen, but merely mixing it will change radius and hence color.

Salpeter: We are certainly all agreed that hydrogen envelopes of various kinds can change the color of a star greatly. I personally doubt whether very small amounts of hydrogen can change the luminosity greatly, but it might – at least indirectly, by triggering some instability or dynamic effect.