Star clusters and black holes in galaxies across cosmic time
Proceedings IAU Symposium No. 312, 2014
Y. Meiron, S. Li, F.-K. Liu & R. Spurzem, eds.

doi:10.1017/S1743921315007863

The role of three-body stability in tidally interacting globular clusters

Gareth F. Kennedy[1,2]

[1]National Astronomical Observatories of China, Chinese Academy of Sciences, Beijing 100012, China
email: gareth.f.kennedy@gmail.com

[2]Monash Centre for Astrophysics, Monash University, Clayton, Vic, Australia, 3800

Abstract. The role of stability in the general three-body problem is investigated with regard to the tidal radius of a globular cluster (GC) in a galactic potential. This proceedings is a summary of two papers which outline the stability method (Kennedy 2014a) and compare the predicted stability boundary radius to observations of velocity dispersion profiles in Milky Way GCs (Kennedy 2014b).

Keywords. gravitation, stellar dynamics, methods: analytical, stars: kinematics, globular clusters: general

1. Introduction

The discovery of flattening in the velocity dispersion profile for the galactic globular clusters (GCs) NGC 5139 and NGC 7078 (Scarpa *et al.* 2003), where dark matter is thought not to exist, has generated a lot of excitement with some studies claiming that this is direct evidence for a breakdown of Newton's laws at low accelerations. Explanations for the observed deviation broadly fit into three categories; tidal interactions, dark matter or a modified gravity theory (e.g. MOND). All of these theories can produce a flattening of the velocity dispersion profile for large radii, in the case of MOND this occurs at the critical acceleration of 1.2×10^{-10} m/s^2 Milgrom (1983). The premise of this proceeding is that the velocity dispersion profile will flatten in the outer regions of the cluster where stellar orbits become unstable.

A stability analysis, detailed in Kennedy (2014a), is used to predict the occurrence of unstable stellar orbits in the outermost region of a GC. Stars on unstable orbits around the cluster centre will random walk in orbital binding energy until their eventual escape from the cluster. The timescale for this random walk is currently being investigated using a high resolution N-body simulation of a tidally interacting star cluster on an orbit based on NGC 6341 and will be presented in the third paper in this series. This simulation has run for over 1 Gyr and consists of over 10^5 particles, resolving each star in the cluster. During the random walk stage stars have a different distribution of velocities when compared to stars in an isolated cluster. This manifests as a flattening of the velocity dispersion profile beyond the transition radius between stable and unstable orbits.

The transition radius from stable to unstable orbits was compared to observational data of the velocity dispersion profiles for 15 Milky Way GCs with approximately known orbital parameters in Kennedy (2014b). It was found that the transition radius predicts where the velocity dispersion flattens and that there is no need for any MOND type theories to explain the observations.

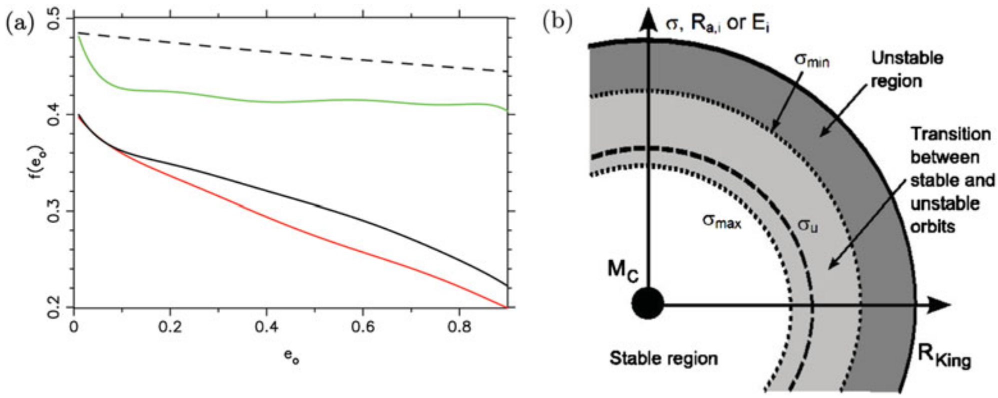

Figure 1. Left panel shows the eccentricity dependence for the transition from stable inner orbits to unstable exterior orbits. The indicative value for the stability boundary (r_c) is shown as a black curve, while the minimum and maximum extents of the partially stable region are the red and green curves respectively. The King radius (Equation 2.2) is shown as the dashed black curve for comparison. Right panel shows a conceptualisation of the stability of stellar orbits in a GC. The distance from the cluster centre associated with the transition from unstable (dark shading) to stable orbits (unshaded inner region) is indicated by the ratio of outer to inner periods σ_u. The region where orbits can be found in either unstable or stable configurations is shown as a light shading between $\sigma_{\min}$ and $\sigma_{\max}$.

2. Stability theory

The method used for determining the stability boundary is described in detail in the first paper in this series Kennedy (2014a) which was based on the stability of the general three-body problem as derived in Mardling (2008) and Mardling (2013).

To describe this stability transition radius the functional form

$$r_t = R_p \left(\frac{M_C}{M_G} \right)^{1/3} f(e) \tag{2.1}$$

is adopted, where M_G is the effective point mass for the Milky Way galaxy, e is the eccentricity of the cluster orbit around the galaxy, and R_p is the distance of closest approach to the galaxy. This form allows direct comparison between the stability boundary and other tidal radii estimates, for example the classical King (1962) tidal radius is given by

$$f(e) = 0.7\,(3 + e)^{-1/3}\,. \tag{2.2}$$

The tidal radius (r_t) will be used to denote the maximum theoretical tidal radius of a GC by using Equations 2.1 and 2.2.

In Kennedy (2014a) the stability boundary was found to be well approximated by three polynomials indicating the minimum and maximum limits of the partial stability region and an indicative value referred to as the chaos radius and denoted by r_c. The three polynomials for $f(e)$ are shown in Figure 1 (a) with the eccentricity function associated with r_c shown as a solid black curve, the reader is referred to Kennedy (2014a) for details. For radii $r < r_{\min}$ all stars are expected to be on stable orbits, whereas for $r > r_{\max}$ they are expected to be on unstable orbits. A schematic diagram of the cluster stability is shown in Figure 1 (b) where the ratio of GC-galaxy orbital period to the orbital period of the star in the GC (σ) is used as a proxy for radius.

https://doi.org/10.1017/S1743921315007863 Published online by Cambridge University Press

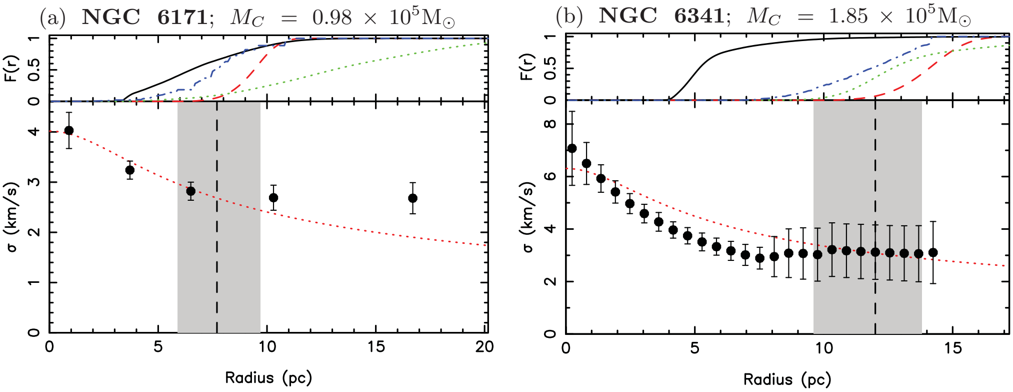

Figure 2. The observed velocity dispersion profile for two GCs. The vertical lines show the best fit to the flattening of the velocity dispersion with errors denoted by the shaded region and the red dotted curve indicates the theoretical velocity dispersion profile for the quoted cluster mass. The cumulative distributions above each velocity dispersion profile show the chaos radius (solid black curve), MOND radius (red dashed curve), tidal radius (green dotted curve) and best fit flattening radius (dot-dashed blue curve).

3. Implications for observations

The bottom panels in Figure 2 show the velocity dispersion as a function of distance from the cluster centre for two of the 15 GCs presented in Kennedy (2014b). References for the velocity dispersion data and basis for the GC-galaxy orbital parameters are given in Kennedy (2014b). The equilibrium velocity dispersion as a function of cluster radius, r, for a Plummer sphere is Dejonghe (1987)

$$\sigma^2(R) = \frac{3\pi G M_C}{64 r_h}\left(1 + \frac{r^2}{r_h^2}\right)^{-1/2} \tag{3.1}$$

where M_C is the cluster mass, r_h is the half-mass radius and G has its usual value. This function is shown as the dotted curves for the quoted cluster masses in Figure 2.

The key requirements for predicting the flattening of GCs is the cluster mass and the GC-galaxy orbital parameters. Both of which have large observational uncertainties. Uncertainties in the orbital elements (R_p and e) and cluster mass (M_C) are treated in Kennedy (2014b) using 10^6 realisations of the GC-galaxy orbit for each GC in a realistic galactic potential taking the observed GC proper motions as initial conditions. For each set of orbital elements and cluster mass the predicted radii for each model are determined and the resulting cumulative distributions for each radii are shown in the top panels of Figure 2 for NGC 6171 and NGC 6341.

The MOND radius r_m as calculated for each cluster from where the acceleration from the cluster potential goes beneath the MOND limit of $a_0 = 1.2 \times 10^{-10}$ m/s^2 is shown in the top panels of Figure 2 as a dashed red line. The transition from stable to unstable orbits r_c (Figure 1 a) is shown as a solid black line, the tidal radius r_t (Equation 2.2) is shown as a dotted green line and the flattening radius that best fits the data is shown as a dot-dashed blue line. The region of best fit for the flattening radius is also shown as the shaded region in the bottom panels.

Both NGC 6171 and NGC 6341 are promising candidates for distinguishing Newtonian and MOND models since these clusters have small relative errors for both the cluster mass and orbital parameters (Kennedy 2014b). For NGC 6341 the existing data is quite good, although further observations of the velocity dispersion at large radii would better

https://doi.org/10.1017/S1743921315007863 Published online by Cambridge University Press

probe the region close to the tidal radius. In the case of NGC 6171 more resolution in radius is required, although the data coverage of the flattening region and the small error bars make this cluster a very good candidate.

Both clusters are rotating strongly enough that velocity dispersion inside the half-mass radius must be modelled in more detail (Drukier *et al.* 2007 and Bellazzini *et al.* 2012). Interestingly increased rotation is predicted by the chaos diffusion model Kennedy (2014a). Specifically, the region between the chaos radius (r_c) and the maximum radius ($f_{\max}$ in Figure 1 a) will have a different rotational profile compared to the rest of the cluster. This is due to preferential removal of stars on prograde orbits relative to the GC-galaxy orbit compared to stars on retrograde orbits. Thus more stars on retrograde orbits will exist in the outer regions on the cluster, leading to a net rotation in this region.

To answer questions regarding the emergence of rotation and the timescale for stars to escape via chaotic diffusion, an N-body simulation based on the mass and orbital parameters of NGC 6341 is currently in progress, the results from which will be presented in an upcoming publication.

4. Conclusions

Flattening of the velocity dispersion of GCs is predicted to occur beyond the stability radius by consideration of Newtonian three-body stability. Kennedy (2014a) presented an easy to use stability radius which has a much stronger dependence on the eccentricity of the cluster-galaxy orbit than the classical tidal radius. As the stability radius depends on the GC-galaxy orbit and not just on the cluster mass, further observations of the motion of GCs will provide a way of distinguishing these predictions from MOND models.

At present the orbital determinations are not quite sufficient to definitively rule out those MOND models that predict flattening for all 15 GCs (see Kennedy 2014b). However NGC 6171 and NGC 6341 have been identified as promising candidates for distinguishing Newtonian and MOND models, requiring only moderate improvement in the observations. In particular, NGC 6171 may already be showing evidence of chaotic diffusion of stars leading to flattening at the chaos radius as seen in the overlap between the black and blue curves in Figure 2 (a). Clarification of this phenomenon will require more resolution in the velocity dispersion and better orbital determination.

References

Bellazzini, M., Bragaglia, A., Carretta, E., Gratton, R. G., Lucatello, S., Catanzaro, G., & Leone, F. 2012, *A&A*, 538, A18

Dejonghe, H. 1987, *MNRAS*, 224, 13

Drukier, G. A., Cohn, H. N., Lugger, P. M., Slavin, S. D., Berrington, R. C., & Murphy, B. W. 2007, *AJ*, 133, 1041

Kennedy, G. F. 2014, *MNRAS*, 444, 3328

Kennedy, G. F. 2014, *MNRAS*, 445, 4443

King, I. 1962, *AJ*, 67, 471

Mardling, R. A. 2008, *Lecture Notes in Physics, Vol. 760: The Cambridge N-body Lectures*, 59

Mardling, R. A. 2013, *MNRAS*, 435, 2187

Milgrom, M. 1983, *ApJ*, 270, 365

Scarpa, R., Marconi, G., & Gilmozzi, R. 2003, *A&AL*, 405, L15