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Propagation-induced Circular Polarisation in Synchrotron Sources

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Abstract: The small degree of circular polarisation observed in some synchrotron sources has a frequency dependence that is not consistent with simple predictions based on the intrinsic circular polarisation of synchrotron emission. The suggestion is explored that the circular polarisation arises as a propagation effect within the source. The physical basis of this alternative mechanism is the fact that the natural wave modes of a synchrotron emitting gas are linearly polarised, allowing partial conversion of linear into circular polarisation as in a quarter-wave plate. A relativistic rotation measure (RRM) is defined to characterise the magnitude of this effect.

Keywords: polarisation — radiative transfer — radio continuum: general.

1 Introduction

It is well known that synchrotron radiation has an intrinsic linear polarisation. Theory implies that there should also be a small component of circular polarisation (e.g. Legg & Westfold 1968). The predicted degree of circular polarisation, r_c, is of order $1/\gamma$, where γ is the Lorentz factor of the emitting particle. The typical frequency radiated by a particle is $\omega \sim \Omega_e \gamma^2$, where $\Omega_e = eB/m$ is the cyclotron frequency. It follows that the degree of circular polarisation should vary according to $r_c \sim \omega^{-1/2} \propto \lambda^{\frac{1}{2}}$, where λ is the wavelength of the radiation. Through the 1970s there were a number of measurements of r_c for extragalactic synchrotron sources (e.g. Biraud 1969; Conway et al. 1971; Berge & Seielstad 1972; Seaquist 1973; Roberts et al. 1975; Weiler & de Pater 1980), and also for the plerionic component of the Crab Nebula (Weiler 1975), although more recent data cast doubt on the significance of the reported value of r_c (Wilson & Weiler 1997). In particular Roberts et al. (1975) detected circular polarisation in eight quasars, all of which showed evidence of synchrotron self-absorption, and some of which showed evidence of time variability in r_c. The data on sources for which r_c was measured at more than one frequency were not consistent with the predicted $r_c \sim \omega^{-1/2}$ law for the intrinsic circular polarisation of synchrotron radiation from a single source. Roberts et al. suggested that these data could be interpreted in terms of a multi-component model for the various sources, with r_c at different frequencies being dominated by different subsources. The suggestion by Roberts et al. includes the possibility that some of the variation of r_c with frequency can be attributed to the change in the sense of polarisation as a source becomes self-absorbed (Melrose 1972, 1980; Jones & O'Dell 1977a, 1997b; Weiler & de Pater 1980). Nevertheless, neither the time variability nor the frequency dependence of r_c is readily compatible with an interpretation in terms of the intrinsic circular polarisation of synchrotron emission. The possibility that the observed circular polarisation is due to a propagation effect needs to considered in detail.

In this paper we discuss an alternative way in which a circularly polarised component could arise as a propagation effect, referred to as circular repolarisation by Pacholczyk (1973). The main emphasis in our discussion is, first, to explain in principle how this alternative mechanism can produce circular polarisation and, second, to make some rough estimates of the conditions under which the effect might be significant in cases such as plerions and compact extragalactic sources where there may be only relativistic particles and no cold plasma. [Pacholczyk (1973) discussed the case where the ambient medium consists of cold plasma with an admixture of highly relativistic electrons.] This investigation is partly motivated by a current program of observations of circular polarisation with the Australia Telescope (R. Norris, D. Rayner, private communication 1997). In principle it is now possible to detect circular polarisation with a much higher sensitivity than was available in the earlier observations reported by Roberts et al. (1975), and this paper is preliminary to a more detailed discussion of the various possible interpretations of circular polarisation of synchrotron sources.

The physical basis for the mechanism considered here is propagation through a medium with elliptically

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(or linearly) polarised wave modes, as discussed in Section 2. In Section 3 we point out that a linearly polarised contribution to the natural modes of the plasma arises from the relativistic electrons (Sazonov 1969; Melrose 1997a). In Section 4 we define a 'relativistic rotation measure' that characterises this effect and consider conditions under which it might be important.

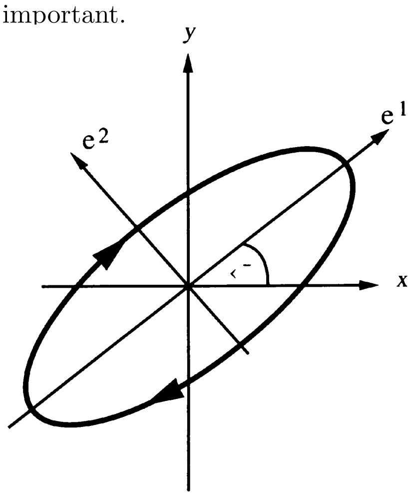

Figure 1—Polarisation ellipse for a wave propagating into the page.

2 Generalised Faraday Rotation

The polarisation of radiation changes as it propagates through any birefringent medium. Birefringence implies that there are two natural wave modes which may be described by their polarisations, which are necessarily orthogonal to each other, and by Δk, the difference in their wavenumbers. In a cold plasma the natural wave modes may be assumed circularly polarised for present purposes. The propagation effect is then Faraday rotation, which causes the plane of any linear polarisation to rotate and which does not alter the degree of circular polarisation. In a medium whose natural modes are linearly or elliptically polarised, the counterpart of Faraday rotation, which we refer to as 'generalised Faraday rotation', can lead to a partial conversion of linear into circular polarisation. Such conversion is the basis for the alternative mechanism for the production of circular polarisation discussed in this paper.

Arbitrarily polarised radiation may be separated into an unpolarised component and a completely polarised component. In general the polarised component is elliptical, as illustrated in Figure 1. The directions $\mathbf{e}^1$ and $\mathbf{e}^2$ in Figure 1 define the major and minor axes of the polarisation ellipse. An arbitrary elliptical polarisation can be represented by a point P on the Poincaré sphere, as illustrated in Figure 2. The north and south pole correspond to opposite circular polarisations, and points on the equator separated by 180° correspond to orthogonal linear polarisations, as indicated in Figure 2a. The cartesian components of the point P are related to the Stokes parameters, I, Q, U, V, through $q = Q/I = \cos(2\chi)\cos(2\Psi)$, $u = U/I = \cos(2\chi)\sin(2\Psi)$, $v = V/I = \sin(2\chi)$, which also define the parameters Ψ and χ, as illustrated in Figure 2b. Faraday rotation corresponds to Ψ changing at constant χ. Partial conversion of linear into circular polarisation is possible through any process that causes χ to change.

The natural wave modes of the medium are orthogonally polarised and hence they correspond to points at the opposite ends of a diagonal through the centre of the Poincaré sphere. This diagonal defines an axis that is characteristic of the natural modes of the medium. Generalised Faraday rotation causes the point P to rotate at constant latitude relative to the axis defined by the natural modes of

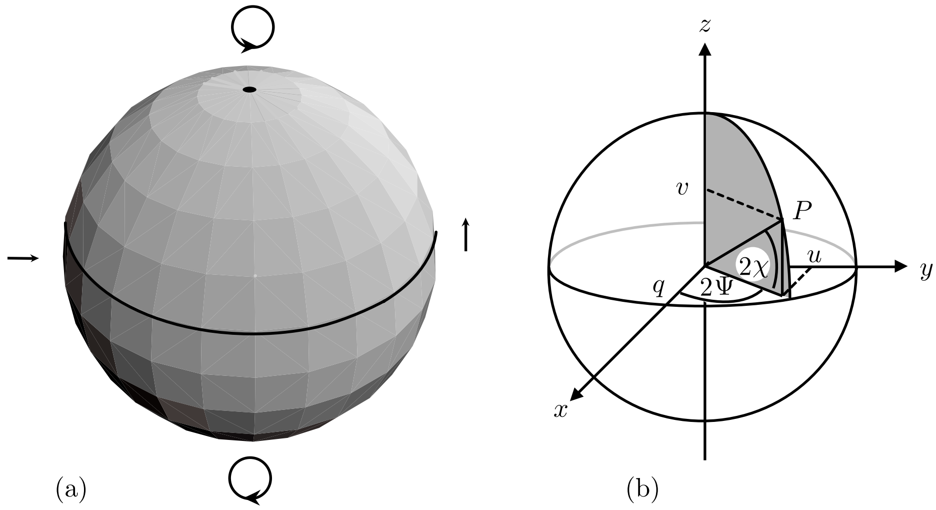

Figure 2—The Poincaré sphere, with the circular polarisations indicated at the poles and the linear polarisations at the equator; (b) the parameters Ψ, χ, q, u, v for an arbitrary point P on the sphere.

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the medium. Faraday rotation, as usually defined, corresponds to circularly polarised natural modes, and then the axis about which this rotation occurs is the vertical axis. This causes the angle Ψ in Figure 1 to rotate at a rate $d\Psi/ds = \frac{1}{2}\Delta k$ per unit distance s along the ray path. Generalised Faraday rotation corresponds to modes with elliptical or linear polarisations, and then (provided the point P is not at one or other end of the axis defined by the two modes) the parameter χ also changes periodically along the ray path, implying a cyclic partial conversion of linear into circular polarisation.

The transfer equation for the Stokes parameters due to generalised Faraday rotation is of the form (e.g. Melrose & McPhedran 1991, p. 188)

$$\frac{d}{ds}\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\rho_V & \rho_U \\ 0 & \rho_V & 0 & -\rho_Q \\ 0 & -\rho_U & \rho_Q & 0 \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}, \quad (1)$$

$$\rho_Q = -\Delta k\,\frac{T^2-1}{T^2+1}\cos(2\psi)\,,$$

$$\rho_U = -\Delta k\,\frac{T^2-1}{T^2+1}\sin(2\psi)\,,$$

$$\rho_V = -\Delta k\,\frac{2T}{T^2+1}\,, \quad (2)$$

where s denotes distance along the ray path, and where T is the axial ratio of the polarisation ellipse of one of the modes. (Interchange of the modes corresponds to $\Delta k \to -\Delta k$, $T \to -1/T$.)

The case where the natural modes are linearly polarised ($T = \infty$ or $T = 0$), as in a uniaxial crystal, is familiar in another context: a quarter-wave plate. For linearly polarised modes the axis defined by the two modes is in the equatorial plane of the Poincaré sphere. If the polarisation point is initially on the equator at a longitude 90° to this axis, then generalised Faraday rotation causes P to rotate about a great circle that passes through both the north and south poles. A quarter-wave plate uses this effect, with the thickness of the plate adjusted such that the rotation is through just 90° of this great circle, so that the initial linear polarisation (at ±45° to the planes of polarisation of the two natural modes) is converted into circular polarisation.

An example of a more general case is illustrated in Figure 3 where the natural modes are highly elliptical ($T \gg 1$) and a sample polarisation point moves around the solid path, which is a circle at constant latitude relative to the axis shown by the solid arrow directed radially from the centre of the sphere. It is apparent from this example that if the wave modes are elliptical or linear, then radiation that is initially linearly polarised develops a circularly polarised component as the polarisation changes in a periodic manner along the ray path (e.g. Pacholczyk & Swihart 1970). The wave modes of a cold plasma are significantly elliptically polarised for a very small range of angles ($\sim \Omega_e/\omega$) about propagation perpendicular to the direction of the magnetic field. However, a more likely cause of significant elliptical polarisation of the natural modes is the relativistic electrons themselves.

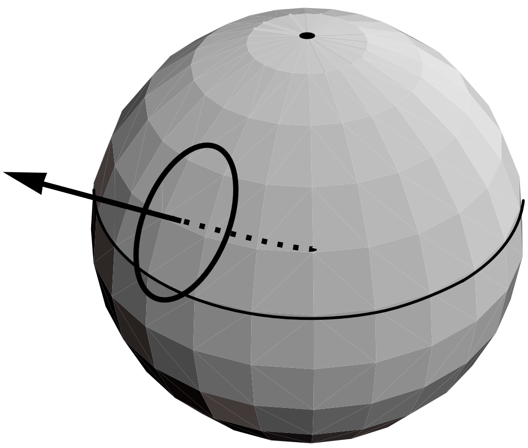

Figure 3—Representative point for radiation in an anisotropic medium rotates about the diagonal joining the points for the two natural modes.

3 Elliptically Polarised Natural Modes

The natural wave modes of a gas of highly relativistic particles are close to linearly polarised for nearly all angles of propagation and for frequencies typical of synchrotron emission by these highly relativistic particles (Sazonov 1969; Melrose 1997a). Hence, as the synchrotron radiation propagates through the source it experiences generalised Faraday rotation due to the contribution of the highly relativistic particles to the polarisation of the natural modes of the ambient plasma. The actual properties of the natural wave modes in a synchrotron source depend on contributions from both the cold plasma and from the highly relativistic particles. In general the modes are elliptically polarised with the ellipticity determined by the ratio of the number densities of cold and highly relativistic particles. The wave modes are nearly circular if the cold plasma dominates and nearly linear if the relativistic particles dominate.

In some sources it may be that the relativistic particles are electron–positron pairs (e.g. Wilson & Weiler 1997). Electrons and positrons contribute with the same sign to the linearly polarised component and with the opposite sign to the circularly polarised component. If the distributions of electrons and positrons are the same the wave modes can have no circular component and, more importantly, the intrinsic circular polarisation from the synchrotron radiation sums to zero. Hence, the alternative mechanism proposed here can produce some circular

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polarisation from an electron–positron pair plasma, despite the fact that the intrinsic circular polarisation is zero in this case.

An important qualitative point in the following discussion is that the orientation of the magnetic field projected onto the plane of the sky must vary along the line of sight through the source for generalised Faraday rotation to occur (cf. Hodge 1982). If this were not the case, the polarisation point for the emitting synchrotron radiation would lie on the axis defined by the two natural modes and would not change along the ray path. Thus, for example, generalised Faraday rotation in the near half of the source could partially convert the linear polarisation of radiation from the far half of the source into circular polarisation provided that the orientation of the magnetic field in the two halves is significantly different.

The relative phase difference between the two natural modes determines the angle through which the polarisation point rotates, cf. Figure 3. This relative phase, $\Delta\phi \sim \Delta k\, L$ say, depends on the difference Δk in wavenumber between the two modes, and the distance L across the source region where the relativistic particles are present. If $\Delta\phi$ is very small, then the difference between the natural modes has no significant effect, and the polarisation of the radiation escaping from the source region is the intrinsic polarisation determined by the synchrotron emission alone. In the opposite limit, when $\Delta\phi$ is very large, the polarisation point rotates many times about the axis. This case corresponds to the two natural modes propagating independently of each other. The net polarisation in this case may be found by separating the emission into the two natural modes before integrating over the source. The intermediate case $\Delta\phi \sim 1$ allows substantial conversion of linear into circular polarisation, but the cyclic nature of the change then implies that the polarisation of the escaping radiation would be a strong function of $\Delta\phi$. In particular, because $\Delta\phi$ is a strong function of frequency, r_c would also be a strong function of frequency. Moreover, any variation in $\Delta\phi$ across the beamwidth of the telescope would reduce the observable circular polarisation.

These properties suggest that significant circular polarisation might arise as a propagation effect under a variety of different conditions in a synchrotron source. Two specific examples are the following:

(1) The relativistic particles dominate, so that the natural modes are linearly polarised; one could then explain a measured value of r_c either due to $\Delta\,\phi \sim 1$ (producing a substantial circular polarisation) over a small fraction ($\sim r_c$) of the source, or by $\Delta\phi \gtrsim r_c$ over a substantial fraction of the source.

(2) The admixture of relativistic particles and cold plasma causes the natural modes to have a small circular component, and one has $\Delta\phi \gg 1$ across the source. This case has already been discussed by Pacholczyk (1973) and is considered only briefly here.

We discuss these two possibilities in the next section.

4 Relativistic Rotation Measure (RRM)

We discuss the specific requirements for significant circular polarisation to arise due to generalised Faraday rotation by referring to the familiar rotation measure (RM) and defining a relativistic rotation measure (RRM) that is its counterpart due to the contribution of the highly relativistic particles.

The parameter RM is defined for the case where only the cold plasma is present and is such that one has $\Delta\phi = \frac{1}{2}\mathrm{RM}\,\lambda^2$, where the dependence on λ follows from $\Delta k \propto \omega^{-2}$ for a cold plasma. (The factor 2 arises from the plane of polarisation rotating through 2π when the relative phase changes by π, that is, from $\Delta\Psi = 2\Delta\phi$ for circularly polarised modes.) The conventional RM is defined by

$$\mathrm{RM} = \frac{e^3}{8\pi^2\varepsilon_0 m^2 c^3}\int ds\, n_e B\cos\theta\,, \tag{3}$$

where the integral is along the ray path, with n_e the number density of the cold electrons and θ the angle between the magnetic field and the direction of propagation. Writing the integral in (3) in the form $L\langle n_e B\cos\theta\rangle$ and expressing L in parsecs, n_e per cubic centimetre and B in gauss, (3) becomes $\mathrm{RM} = 0\cdot 8\times 10^6\, L\langle n_e B\cos\theta\rangle$ rad m^{-2}.

By analogy we define the RRM for the case where only the relativistic gas is considered, so that the natural modes are linearly polarised with $\Delta k \propto \omega^{-3}$ (Sazonov 1969; Melrose 1997a). In this case we write $\Delta\phi = \frac{1}{2}\mathrm{RRM}\,\lambda^3$. The definition of RRM depends on the details of the distribution of relativistic particles. We assume a power-law distribution of particles with energy spectrum

$$N(\gamma) = \begin{cases} N_0\gamma^{-\beta} & \text{for } \gamma_1<\gamma<\gamma_2 \\ 0 & \text{otherwise}\end{cases}\,,$$

$$N_0 = \begin{cases} n_r(\beta-1)/(\gamma_1^{1-\beta}-\gamma_2^{1-\beta})^{-1} & \text{for } \beta\neq 1\,, \\ n_r\ \ln(\gamma_2/\gamma_1) & \text{for } \beta = 1\end{cases}\,, \tag{4}$$

where n_r is the number density of the relativistic particles, β is the power law index and γ_1, γ_2 are cutoff values for the Lorentz factor. Assuming $\beta > 2$ and $\gamma_2 \gg \gamma_1$, the difference between the two modes (Melrose 1997a, 1997b) implies

$$\mathrm{RRM} = \frac{e^4}{4\pi^3\varepsilon_0 m^3 c^4}\,\frac{\beta-1}{\beta-2}\int ds\, n_r B^2\sin^2\theta\,\gamma_1\,. \tag{5}$$

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Writing the integral in equation (5) in the form $L\langle n_r\gamma_1 B^2 \sin^2\theta\rangle$ and expressing L in parsecs, n_r per cubic centimetre and B in gauss, (5) becomes

$$\mathrm{RRM} = 3\times 10^4\, L\langle n_r\gamma_1 B^2\sin^2\theta\rangle\,\mathrm{rad\,m^{-3}}\,. \quad (6)$$

Returning to the two examples in the previous section, example (1) has two specific requirements: (a) the relativistic plasma must dominate over the cold plasma in determining the wave properties, and (b) the relative phase $\Delta\phi$ needs to be either ~ 1 over a small portion of the source or $\gtrsim r_c$ over a substantial portion of the source. Condition (a) requires that Δk be dominated by the relativistic particles rather than the cold plasma, which is satisfied for $n_{\rm cold} \ll (n_{\rm rel}\gamma_1)\ln(\omega/\Omega_e\gamma_1)$, where $mc^2\gamma_1$ is the lower cutoff energy of a power-law distribution of relativistic electrons (Melrose 1997b). This requirement can be expressed in the form $\eta_r \gtrsim 1$, with

$$\eta_r = \mathrm{RRM}\,\lambda/\mathrm{RM_s}\,, \quad (7)$$

where the subscript on $\mathrm{RM_s}$ is to emphasise that the relevant RM is that obtained by integrating along only the portion of the ray path within the source where the cold plasma and relativistic electrons coexist. Condition (b) requires $\mathrm{RRM}\,\lambda^3 \sim 1$ over a small portion of the source or $\mathrm{RRM}\,\lambda^3 \sim r_c$ over a substantial portion of the source. Inserting (3) and (5) into (7), $\mathrm{RRM}\,\lambda^3 \sim 1$ requires $n_r\gamma_1/n_e \gtrsim \omega/\Omega_e$, with $\omega = 2\pi c/\lambda$. This condition is likely to be satisfied only in relativistic pair-dominated plasmas where there are essentially no cold electrons. In particular this condition may be satisfied in pulsar winds and pulsar nebulae (plerions). It might also be satisfied in pair-dominated relativistic plasmas in compact extragalactic sources. Let us assume approximate equipartition, that is, that the energy density in the relativistic electrons, $\sim n_r\gamma_1 mc^2$, is of order the magnetic energy density. Then one has

$$\mathrm{RRM} \sim 10^9\, LB^4\,\mathrm{rad\,m^{-3}}\,, \quad (8)$$

with L in parsec and B in gauss. For example, in a plerion with $L \sim 1$ pc and for $\lambda \sim 20$ cm, (8) requires $B \sim 20$ mG. On the other hand, if one requires $\mathrm{RRM}\lambda^3 \sim 10^{-3}$ over a large part of the source, then $B \sim 1$ mG may be adequate to account for the observed values of $r_c \sim 10^{-3}$. It is interesting to note that this estimate of the field strength required for propagation to cause the circular polarisation is similar to that estimated by Weiler (1975) for the Crab Nebula based on the intrinsic polarisation of synchrotron radiation. For compact extragalactic sources the value of L is larger, requiring a correspondingly smaller value of B; again the values required are of the same order of magnitude as those required for the intrinsic polarisation of synchrotron radiation (e.g. Weiler & de Pater 1980). However, the frequency dependence is more like $r_c \propto \omega^{-3}$ than that ($r_c \propto \omega^{-1/2}$) predicted for a uniform, optically thin synchrotron source. We conclude that for both applications, a more detailed investigation of the two alternatives is clearly warranted.

The other possibility (2) in the previous section is when the relativistic gas causes the natural modes to be only very slightly elliptical. The relative phase difference $\Delta\phi$ is then determined by the cold plasma, and provided one has $\Delta\phi \gg 1$, the resulting circular polarisation should be of order the eccentricity of the modes, which is $\sim \eta_r$. It follows that this possibility implies $r_c \propto \omega^{-1}$, as found by Pacholczyk (1973), compared with $r_c \propto \omega^{-1/2}$ for the intrinsic circular polarisation of synchrotron radiation. The existing data do not appear consistent with $r_c \propto \omega^{-a}$ with either $a = 0{\cdot}5$ or $a = 1$. However, further multi-frequency data are required to clarify whether or not this possibility is a viable one.

5 Discussion and Conclusions

Partial conversion of linear into circular polarisation is possible in synchrotron sources due to the relativistic electrons contributing an intrinsically linearly polarised component to the natural wave modes of the ambient medium. This effect may be characterised by the parameter RRM defined by (5).

The conditions required for the observed circular polarisation of some synchrotron sources to be due to this effect are restrictive but not impossibly so. We discuss two situations in which the mechanism might operate. One situation is when cold plasma is absent, and then one requires either $\mathrm{RRM}\lambda^3 \sim 1$ in a small portion of the source or $\mathrm{RRM}\lambda^3 \sim 10^{-3}$ over a large portion of the source. Estimates based on (6) or (8) suggest that this condition may be satisfied for plerions and for compact extragalactic sources; the magnetic field strengths required are of the same order of magnitude as those required to account for the circular polarisation in terms of the intrinsic circular polarisation of synchrotron radiation. The other situation, which has been discussed previously in this context by Pacholczyk (1973), is when there is a mixture of cold plasma and relativistic particles in the source. Provided that the relative phase difference across the source (which is dominated by the cold electrons in this case) is large, the emerging polarisation should have a circular polarisation similar to that of the natural modes, given roughly by (7).

It is highly desirable to model in detail all the different ways in which circular polarisation might be produced in synchrotron sources in order to compare and contrast them. Although it is a small effect, the circular polarisation of synchrotron sources is potentially significant because, whatever its cause, it depends on parameters different to those that determine the other observable properties of synchrotron radiation (e.g. Weiler & de Pater 1980).

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References

Berge, G. L., and Seielstad, G. A. 1972, ApJ, 77, 810
Biraud, F. 1969, A&A, 1, 156
Conway, R. G., Gilbert, J. A., Raimond, E., and Weiler, K. W., 1971, MNRAS, 152, 1
Hodge, P. E. 1982, ApJ, 263, 595
Jones, T. W., and O'Dell, S. L. 1977a, ApJ, 214, 522
Jones, T. W., and O'Dell, S. L. 1977b, ApJ, 215, 236
Legg, M. P. C., and Westfold, K. C. 1968, ApJ, 154, 499
Melrose, D. B. 1972, PASA, 2, 140
Melrose, D. B. 1980, Plasma Astrophysics, Vol. 1 (New York: Gordon & Breach), p. 217
Melrose, D. B. 1997a, Phys. Rev. E, 56, 3527
Melrose, D. B. 1997b, J. Plasma Phys., 58, 735
Melrose, D. B., and McPhedran, R. C. 1991, Electromagnetic Processes in Dispersive Media (Cambridge University Press), p. 188
Pacholczyk, A. G. 1973, MNRAS, 163, 29
Pacholczyk, A. G., and Swihart, T. L. 1970, ApJ, 161, 415
Roberts, J. A., Roger, R. S., Ribes, J.-C., Cooke, D. J., Murray, J. D., Cooper, B. F. C., and Biraud, F. 1975, Aust. J. Phys., 28, 325
Sazonov, V. N. 1969, Sov. Phys. JETP, 29, 578
Seaquist, E. R. 1973, A&A, 22, 299
Weiler, K. W. 1975, Nature, 253, 24
Weiler, K. W., and de Pater, I. 1980, A&A, 91, 41
Wilson, A. S., and Weiler, K. W. 1997, ApJ, 475, 661