BULL. AUSTRAL. MATH. SOC. 22C05, 43A30
VOL. 18 (1978), 149-151.

Lacunary sets for connected and totally disconnected compact groups

Michael Frank Hutchinson

This thesis takes some steps towards the solution of the problem of determining precisely which compact groups admit infinite Sidon sets.

The property of admitting an infinite Sidon set turns out to be closely related, though not equivalent, to the property of not being tall, where we say that a compact group is tall if it possesses only finitely many pairwise inequivalent continuous irreducible unitary representations of each degree. More specifically, we use Riesz products to show that every non-tall compact group admits an infinite Sidon set. This generalises the well known result that an infinite abelian compact group always admits an infinite Sidon set and strengthens Corollary 3.4 of Parker [6], which asserts that a non-tall compact group admits an infinite central Sidon set. The above result and its proof will appear in [4].

Conversely, we show that tall compact Lie groups and tall totally disconnected compact groups do not admit infinite Sidon sets. The former result is an easy consequence of Theorem 3 of Cecchini [1], while the latter result follows from our theorem that a local Λ set for a totally disconnected compact group has to have the degrees of its members bounded. We also give examples of tall connected compact groups which do admit infinite Sidon sets. These examples simultaneously provide strong evidence for the conjecture that a connected compact group admits an infinite Sidon set if and only if it is not a semi-simple Lie group. If true, this conjecture would strengthen Theorem 9 of Rider [8].

We pursue the study of tall totally disconnected compact groups in

Received 5 September 1977. Thesis submitted to the University of Sydney, April 1977. Degree approved, October 1977. Supervisor: Dr J.R. McMullen.

some depth. In addition to the lacunary characterisation referred to above, we obtain two structural characterisations of such groups and present several illustrative examples. These examples include an example of a totally disconnected compact group which admits an infinite central Sidon set but no infinite Sidon sets. In obtaining our characterisations of tall totally disconnected compact groups, one of the principal tools is a classical theorem due to Jordan and Blichfeldt concerning the existence of certain abelian normal subgroups of finite linear groups (see Theorem 30.4 of [2]).

Our investigations enable us to answer a question of Price [7] by showing that the dual of a compact group is a local Sidon set if and only if the elements of the dual have bounded degree.

We also prove a number of results on tall pro-p-groups. These are of interest in view of Theorem 2 of McMullen [5] which asserts that an infinite compact group contains either an infinite abelian subgroup or an infinite topologically-2-generator pro-p-group for some odd prime p, this last group being tall.

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