CHAPTER 4

ASTEROSEISMOLOGY: RESULTS AND PROSPECTS

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ASTEROSEISMOLOGY: RESULTS AND PROSPECTS

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ABSTRACT: Asteroseismology is interpreted as an extension of the field of variable stars, and not just as the stellar analogue of helioseismology. The main effects of stellar mass and evolution on oscillation frequencies are discussed with the help of simplified wave-propagation diagrams. Frequency separations resulting from asymptotic expressions are compared with the corresponding results from numerical computations. The validity of asymptotic theory can be gauged in this way. The seismological issues of solar-like stars and Ap stars are discussed in some detail, and a progress report on the equation of state for stellar interiors is given. The review ends with a summary of properties and important physical problems for selected classes of variable stars.

1. INTRODUCTION

From the moment that variable stars were recognized as stellar pulsators, the pulsation data became an important diagnostic tool to test stellar models. In 'classical' variable stars often only one and sometimes two modes have been observed. More recently, especially with the observational successes of solar oscillations, there has been an intensified effort to find many modes also in stars. While this new field of multi-mode stellar variability has been called asteroseismology, in analogy to helioseismology, we would like to understand asteroseismology more generally to be a method of testing stellar structure and evolution theory, using <u>all</u> available pulsation data, and not just observed frequencies. Such a broader definition is important, because we will always have to deal with a small number of

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observed modes. There is no hope to obtain the analogue of the highly spatially resolved data of solar oscillations; this will preclude any attempt to solve the inverse problem. Furthermore, the lack of spatially resolved data and the small amount of observed modes has also the following consequences: (i) mode identification (radial order n, angular degree ℓ, azimuthal order m) is difficult and (ii) the basic physical uncertainties (e.g. convection) will probably have to be parametrized. Therefore it is important that all pulsation data (not just frequencies, but also growth rates, phases, the fact that a mode exists, and sometimes ceases to exist, etc.) will have to be taken into account.

Within this broad definition asteroseismology is not a separate science but becomes a natural development of the field of variable stars. Even if there will never be the plethora of solar data, there is already enough material around to embarrass the theorists. This is manifested, for instance, in the two periods of double-mode Cepheids or in the observed frequency differences of Ap stars. And sometimes even one single period combined with other information can be too much, as is the case with Spica and its defunct radial mode (Shobbrook et al. 1972, Odell 1980).

2. CHANGES OF PROPAGATION PROPERTIES OF OSCILLATION MODES WITH MASS AND EVOLUTION

The influence of stellar mass and age on oscillation properties is best discussed with propagation diagrams. We adopt the asymptotic discussion of Deubner and Gough (1984), which itself is based on Lamb (1932) [see also Christensen-Dalsgaard (1986)]. For wavelengths much shorter than the solar radius, normal oscillation modes can be quite accurately discussed using the simplified wave equation

$$\psi'' + K^2(r)\psi = 0. \tag{2.1}$$

Here, $\psi = \sqrt{\rho}\, c^2 \,\mathrm{div}(\delta\mathbf{R})$, ρ and c are density and sound speed of the equilibrium configuration, and $\delta\mathbf{R}$ is the fluid displacement vector. The local wave number is given by

$$K^2(r) = \frac{\omega^2-\omega_c^2}{c^2} + \frac{\ell(\ell+1)}{r^2}\left(N^2/\omega^2 - 1 \right), \tag{2.2}$$

with the acoustic cut-off frequency ω_c defined by

$$\omega_c^2 = \frac{c^2}{4H^2}\left(1 - 2\frac{dH}{dr} \right), \tag{2.3}$$

and the Brunt-Väisälä frequency N by

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$$N^2 = g\ (\ 1/H - g/c^2\ ), \tag{2.4}$$

where H is the density scale height and g the local gravity. From the form of (2.1) (to which upper and lower boundary conditions must be added), one immediately realizes that in propagation zones necessarily $K^2 > 0$.

Our present qualitative discussion of the influence of mass and evolution on oscillation frequencies aims at showing the maximum of effects with a minimum of curves. Here, we restrict ourselves to the role of N^2 and $\omega_c{}^2$ in K. For finer details we again refer the reader to Deubner and Gough (1984), Christensen-Dalsgaard (1984) or Gough (1985). With the convenient definition of the Lamb frequency S_ℓ

$$(S_\ell)^2 = \frac{\ell(\ell+1)}{r^2}\ c^2, \tag{2.5}$$

we obtain the simplified necessary conditions for propagation of an acoustic wave, $\omega > \omega_c$ and $\omega > S_\ell$. Additionally, in order to have a trapped standing wave, it is also necessary that in some surface layer ω_c becomes greater than ω. In propagation zones (if the adiabatic exponent of the gas is 5/3 and constant) a further simplification follows from the fact that $\omega > g/c$ implies $\omega > \omega_c$. And finally, we choose the approximation of identifying (the absolute value of) g/c with N, which certainly gives the correct order of magnitude in radiative zones (but would be entirely wrong in convection zones, where $N \approx 0$). The advantage of this choice is that the same curves will give information about g modes, too. Let us now consider S_1 (we choose the representative case $\ell = 1$) and N in a model of a 1 $M_\odot$ star (Figure 1). Due to the rather deep convection zone, N cannot represent the increase of ω_c close to the surface, and so the diagram does not show the upper turning point that is caused by the large spatial inhomogeneity near the surface (it will do so, however, in the example of a higher mass, see below). In Figure 1, S_1 defines the penetration depth of the $\ell = 1$ modes; for $\ell > 1$ the corresponding S_ℓ curves would be shifted to the right.

The ensemble of Figures 1-4 shows the principal effects of evolution and mass. The first effect of evolution is an increase of the inner 'N-mountain', caused by the growing spatial chemical inhomogeneity in the central regions. The inner mountain is capable eventually to prevent radial modes from penetrating to the center. Modes with $\ell \geq 1$ can acquire 'dual status', i.e. they become gravity modes in the core and remain acoustic modes in the envelope. Thus they penetrate deeper into the interior, while simultaneously their frequency spectrum becomes denser. The second effect of evolution is the decrease of the outer N-mountain (in our figures only visible in the 1.5 $M_\odot$ star). As we have discussed before, this decrease goes parallel with a decrease in ω_c and it therefore causes a smaller total number of modes.The main effect of mass (in the range around and slightly above 1 $M_\odot$) on the frequency spectrum is related to the disappearance of the convective

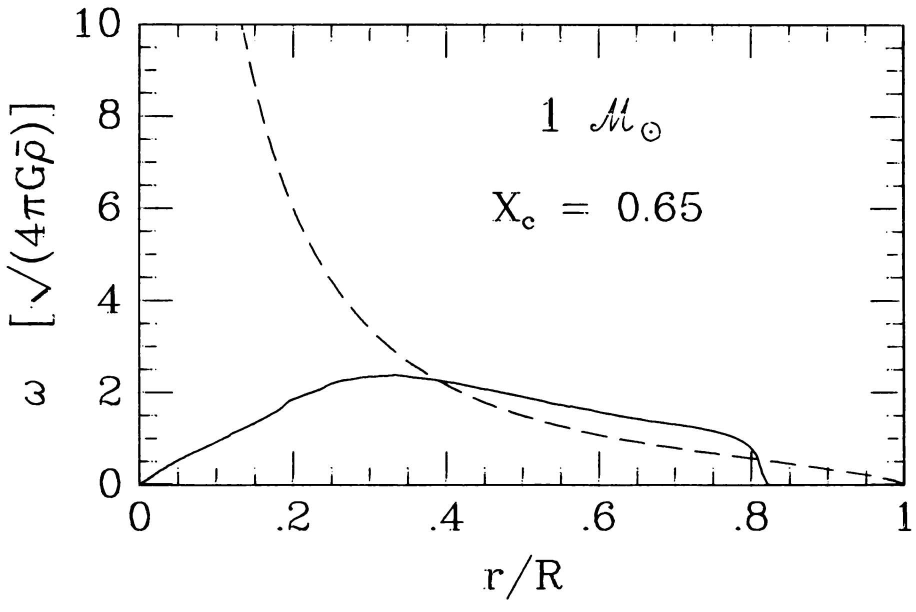

Figure 1. Critical frequencies as functions of the fractional radius r/R. The solid line denotes the Brunt-Väisälä frequency, the dashed line the Lamb frequency S_ℓ for ℓ=1. The model parameters are: hydrogen abundance X = 0.70, heavy-element abundance Z = 0.01, and mixing-length parameter ℓ/H_p = 1.5. Stellar age is indicated by the central hydrogen abundance X_c.

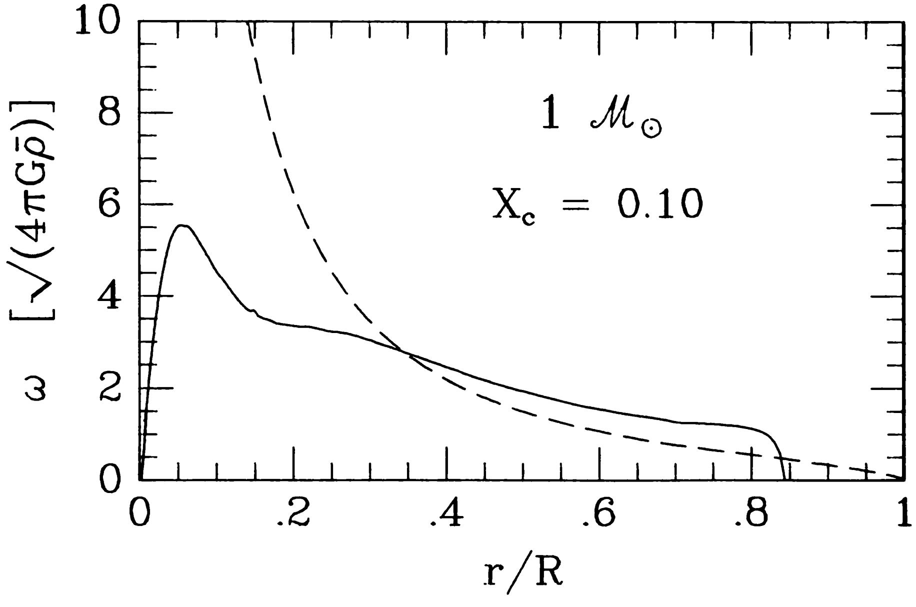

Figure 2. Same as Figure 1, but for a different stellar model.

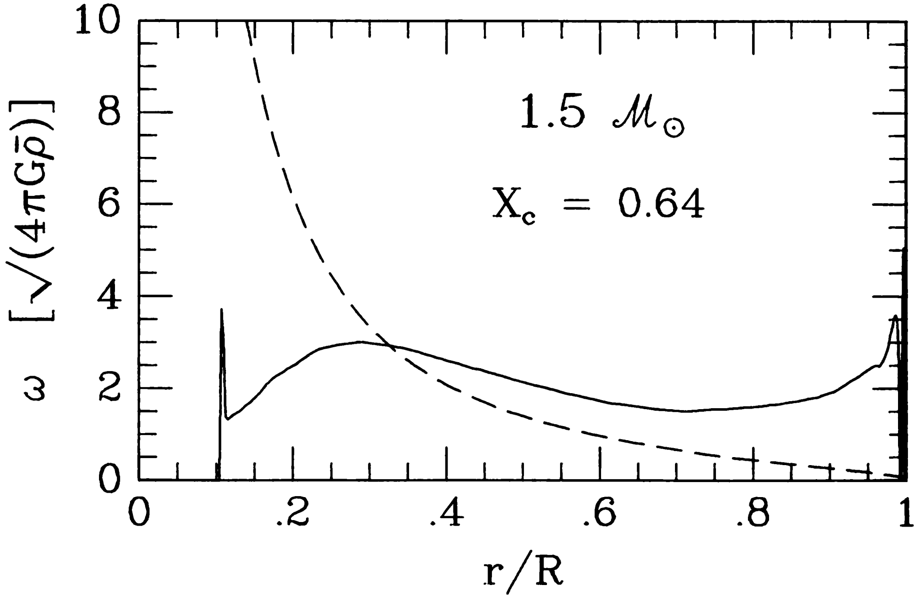

Figure 3. Same as Figure 1, but for a different stellar model.

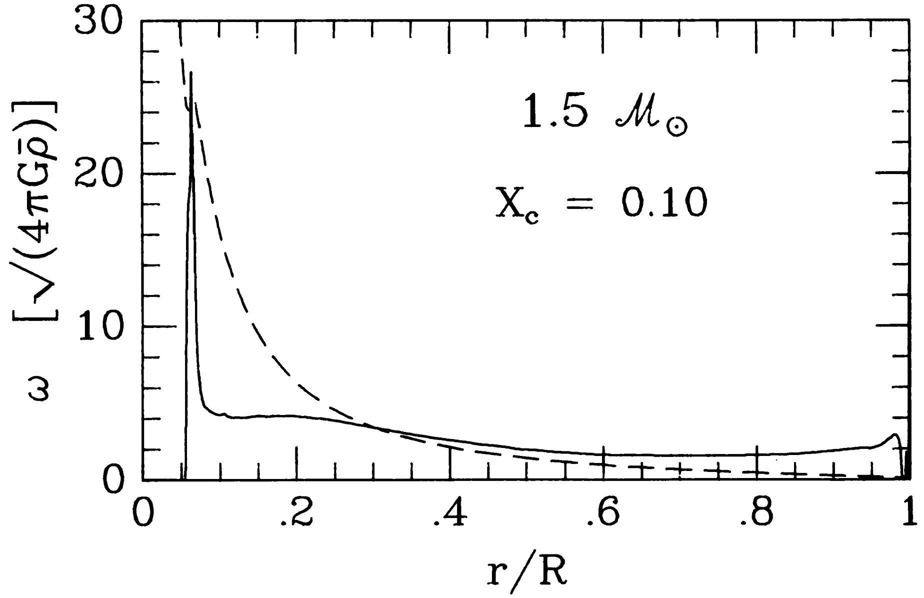

Figure 4. Same as Figure 1, but for a different stellar model.

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envelope and the forming of a convective core. This has a major influence on g-mode propagation.

3. SOLAR-LIKE OSCILLATIONS

Solar high-order acoustic modes and their seismological relevance have been extensively discussed in the literature [recent reviews are Brown et al. (1986), Deubner and Gough (1985) and Gough (1985)]. As mentioned in the previous section, high-order p modes only exist in fairly unevolved stars. Besides the Sun, they are observed in Ap stars [Kurtz (1982), Kurtz (1985)], and there have been reports of observations in Procyon, α Centauri, and ϵ Eridani [Noyes et al. (1984), Gelly et al. (1986)]. Before we address the important seismological issues of these stars, we introduce three definitions pertaining to the structure in the periodogram of high-orderpulsators (given by a set of frequencies $\nu_{n\ell}$).

3.1. Terminology for Periodograms

(i) Large Separation:

$$\Delta_{n\ell} \equiv \nu_{n+1,\ell} - \nu_{n\ell}. \tag{3.1}$$

To first order asymptotic theory it is well known that

$$\Delta_{n\ell}^{-1} \approx 2 \int_0^R (1/c)\, dr. \tag{3.2}$$

(ii) Small Separation:

$$\delta_{n\ell} \equiv (\nu_{n,\ell+1} - \nu_{n\ell}) - \frac{1}{2}(\nu_{n+1,\ell} - \nu_{n\ell}). \tag{3.3}$$

Note that the form of (3.3) differs from the one usually considered in solar applications, which is $\nu_{n\ell} - \nu_{n-1,\ell+2}$ (see, for instance, Gough 1985). The reason is that in stellar periodograms only a pair of two adjacent ℓ-values is know, in contrast to the large number of different observed solar ℓ-values. The ratio between small and large separation is, to second-order asymptotic theory, given by (Tassoul 1980)

$$\frac{\delta_{n\ell}}{\Delta_{n\ell}} \approx \frac{\ell+1}{2\pi^2\nu_{n\ell}} \int_0^R \frac{dc}{dr}\frac{dr}{r} \tag{3.4}$$

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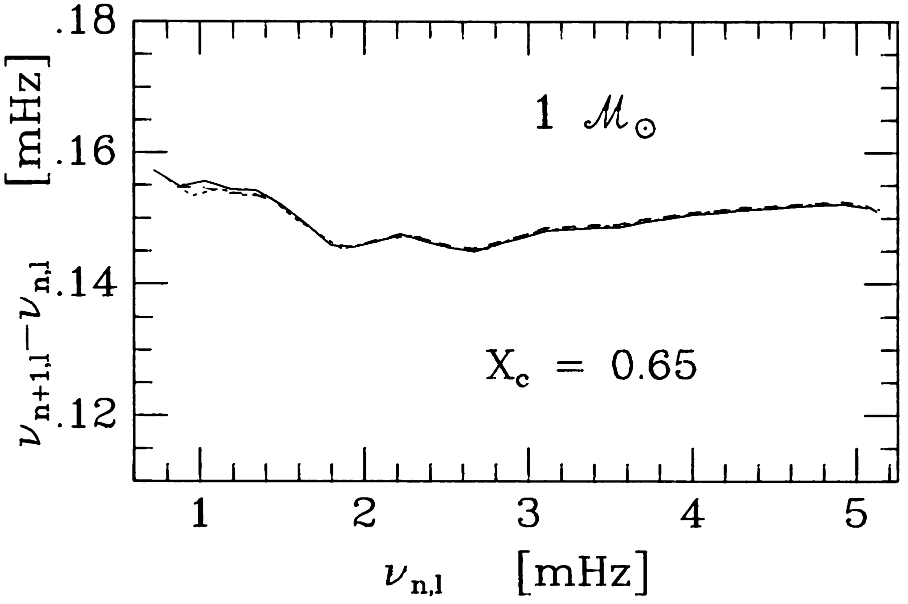

Figure 5. Large frequency separations as functions of oscillation frequency. Solid, dotted, and dashed lines correspond to ℓ = 0,1,2. Stellar model-parameters and indication of age are as in Figure 1. The asymptotic value [equation (3.2)] for this model is 0.148 mHz

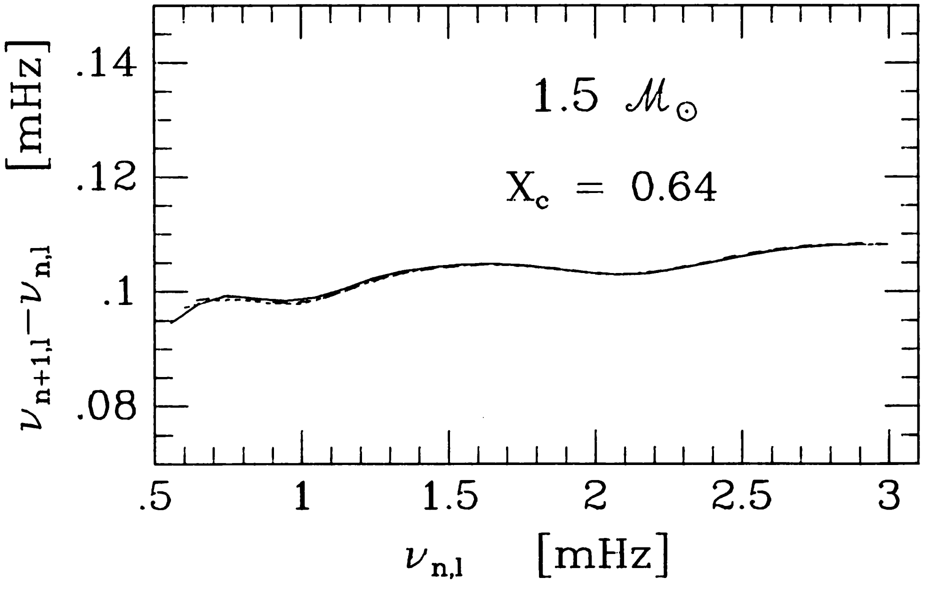

Figure 6. Same as Figure 5, but for a different stellar model. The asymptotic value is 0.107 mHz.

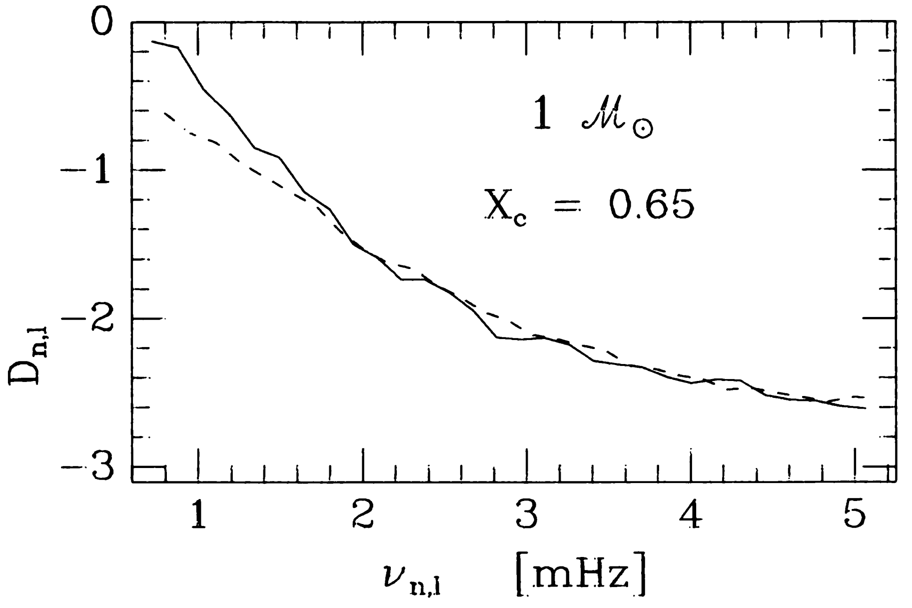

Figure 7. Normalized small frequency separations (in units of μHz) $D_{n,\ell} = 2\pi^2 \nu_{n\ell}((\nu_{n,\ell+1} - \nu_{n\ell})/(\nu_{n+1,\ell} - \nu_{n\ell}) - 1/2)/(\ell + 1)$ as functions of oscillation frequency. The solid line corresponds to $\ell = 0$, the dashed line to $\ell = 1$. The asymptotic value of $D_{n,\ell}$ [see equation (3.4)] for this model is -2.99 μHz.

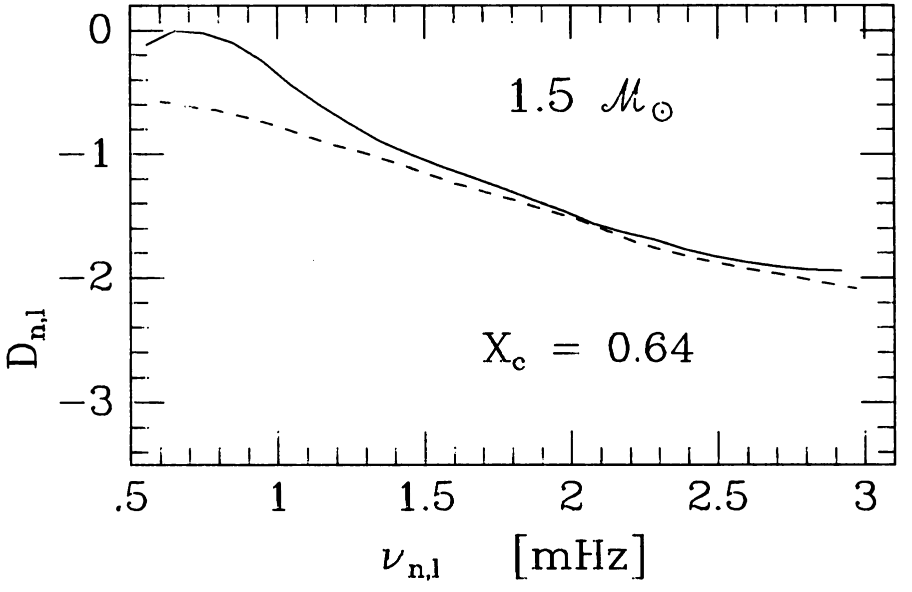

Figure 8. Same as Figure 7, but for a different stellar model. The asymptotic value for this model is -3.14 μHz.

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(iii) Fine Structure: By fine structure we understand the m dependence (m = azimuthal order) of frequencies due to symmetry breaking effects such as rotation or magnetic fields.

Since sound speed increases steeply from the surface to the center of a star, $\Delta_{n\ell}$ probes more the surface regions and $\delta_{n\ell}$ more the central regions. An important diagnostic tool in the form of Δ- δ diagrams was introduced by Christensen-Dalsgaard (1984). For examples, in which Δ and δ are shown as functions of mass and age, see Christensen-Dalsgaard (1986; these proceedings). Such diagrams are of great help in interpreting asteroseismological data. Since in stellar models the asymptotic expressions (3.2) and (3.4) are much simpler to handle than full-fledged numerical frequency calculations, we need to have an idea about their validity. Figures 5 to 8 show examples of small and large separations for two typical stellar models, and the corresponding asymptotic values are given in the captions.

3.2. Procyon, α Centauri, and ε Eridani

Gelly et al. (1986) have reported p modes in Procyon and α Centauri. Only the large separation has been observed. Noyes et al. (1984) have reported three p-mode frequencies and the large separation for ε Eridani. To illustrate the potential of such observations for testing stellar structure and evolution, consider the recent controversial theoretical articles dealing with Noyes et al.'s (1984) observations. While Guenther and Demarque (1986) have concluded that a model of a very old star (≈ 12 Gyr) fits the data best (though they are aware of indications of stellar activity that speak against such a high age), Soderblom and Däppen (1987 and these proceedings) conclude that a model of a very young star (≲ 1Gyr) is equally well suited, provided that one accepts the unusually small value of the mixing-length parameter of $\ell/H_p \approx 0.45$. More and better observed frequencies should help to resolve this issue.

3.3. Ap stars

More extensive data - and therefore even greater puzzles for theorists - are known for the class of Ap stars, for which high order p modes have been observed (Kurtz 1982,1985).

Table 1. The observed frequencies and amplitudes of HD24712 (from Kurtz 1985)

	f(mHz) ±0.000004	A(mmag) ±0.03
f1	2.720907	1.16
f2	2.652927	1.05
f3	2.687404	0.63

Table 1 shows the frequency spectrum of HD24712 (=HR1217) reported in Kurtz and Seeman (1983). They have interpreted the spectrum in terms of a large separation $\Delta_{n\ell}$ for ℓ=0 (f1 - f3 and f3 - f2 are both about 34 μHz). However,this explanation runs into difficulties, first, because the corresponding radial order would be n≈80, i.e. four times higher than in the Sun, and second, because there is no main sequence model yielding the correct integral (3.2). Shibahashi and Saio (1984) constructed models of evolved stars that indeed explain the large separation, but it turns out that, in accordance with what we have said in section 2, for these evolved models at these frequencies no acoustic mode can be trapped. Therefore Shibahashi and Saio (1984) proposed a modified $T(\tau)$-relation that could push the critical acoustic frequency upward and thus allow trapped modes at the observed frequency. Magnetic field and chemical inhomogeneities were conjectured to be the cause of this modification. Gough (private communication) considered that the modes might exist in a nonadiabatic treatment. Imposing outgoing-wave boundary conditions, he found enough driving (kappa mechanism) to compensate amply for mechanical energy losses.

Besides these problems with the large separations, there is also one associated with the small separation. The numerical value of the small separation is best fitted with the interpretation ℓ=0,1. However, since it is observed that the peaks are modulated by the stellar rotation (note the amplitudes in Table 1), one is forced to adopt (at least) a ℓ=1,2 interpretation. Then, however, the small separation is too small to be understood [note the factor (ℓ+1) in equation (3.4)].

The quality of the observational material on Ap stars is excellent: the fine structure of the modes is also visible (Kurtz 1982). To give an idea, we remark that typical periods of modes are between 6 and 12 minutes, and a typical rotation period is about 20 days. The frequency splitting of a (2ℓ+1)-times degenerate multiplet (radial order n, angular degree ℓ) is, according to Ledoux (1951)

$$\omega_{n\ell m} = (1-C_{n\ell})m\Omega + \omega_{n\ell 0}. \qquad (3.5)$$

Here, $C_{n\ell}$ depends on ℓ and n and the stellar structure, but not on the azimuthal order m. Uniform rotation with frequency Ω is assumed in (3.5). Kurtz's (1982) data suggest that the frequencies in the multiplets are precisely equidistant with separation Ω and thus not compatible with (3.5). For this reason he proposed the so-called 'oblique-pulsator-model', which assumes that we observe instead of (2ℓ+1) modes [with frequencies given by equation (3.1)] just one with a single azimuthal quantum number m, that refers to the magnetic field axis and not to the rotation axis. The frequency splitting in the nonrotating observer's frame is then explained as a pure kinematic effect of the rotating pulsator. [To understand this note that the signal on earth is modulated by a periodic function with frequency Ω; thus in Fourier space a multiplet with separation Ω appears; the multiplet has the finite number of (2ℓ+1) terms due to the invariance properties of the vector space of spherical harmonic functions of degree ℓ ; see Kurtz (1982) for a discussion of explicit examples for the cases ℓ=1,2.] Despite an undeniable beauty of this explanation there are problems with it. In the presence of rotation, modes having the quantum number m with respect to the magnetic field axis are not pure eigenstates and there would be inevitably mixing with the other modes in the multiplets (all that of course under the assumption that the deviations from sphericity are small enough so that the quantum number ℓ still makes sense). The destruction rate of a mode would be of order ΩC. Without further justification Kurtz (1982) thus assumes $C_{n\ell}$ =0. Beyond this ad-hoc postulate two solutions have been proposed. Dolez and Gough (1982) argued that nonadiabatic effects might cause excitation and de-excitation of a given mode with the rate $|\tau|$ >> ΩC, thus not leaving enough time for mixing. Dziembowski and Goode (1985) carried out a perturbative analysis of modes in the presence of magnetic fields and rotation, thus allowing $C_{n\ell} \neq 0$. They showed that this 'generalized oblique-pulsator model' essentially has the properties of Kurtz's (1982) model, if the effect of the magnetic field on splitting dominates that of rotation. Kurtz and Shibahashi (1986) have applied this model to the star HR3831. The main problem with this approach is that the field is too strong to be considered as small perturbation. Thus spherical harmonic functions can become inappropriate, and the large and small frequency separations (3.1) and (3.3) will have to be defined differently (Goode and Dziembowski, work in progress).

3.4. The Problem of the Equation of State

An important physical issue to be addressed by solar and stellar oscillations is the equation of state. The principal open problem is the number of excited states of hydrogen and helium in the zones of partial ionization. While for many astrophysical applications simple equation-of-state recipes can be sufficient, it has been shown (Däppen 1987) that for finer helioseismological applications (e.g. helium abundance determination) such simple formalisms are not adequate. In contrast to various other improvements over the simple Saha equation, about which no disagreement exist, there are widely divergent opinions on the internal partition function of bound systems. There has been a recent controversy about the so-called Planck-Larkin partition function (Rouse 1983, Ebeling et al. 1985). The Planck-Larkin partition function essentially limits the number of excited states to those having a binding energy $\gtrsim$ kT. Optical hydrogen spectra, however, show more lines than predicted by the Planck-Larkin partition function (Däppen et al. 1987). Rogers (1986) explains this fact by allowing resonances that are not counted in the partition function but could be seen in optical spectra. Thus the Planck-Larkin controversy cannot be resolved with optical experiments, but thermodynamical properties will have to be known. Stellar models with and without Planck-Larkin partition function will have to be compared. While thermodynamical quantities based on the Planck-Larkin partition function will soon become available (Rogers, private communication), an advanced and very smooth version of a more conventional equation of state has been developed in the framework of an ongoing opacity recomputation (Hummer and Mihalas 1987, Mihalas et al. 1987, Däppen et al. 1987). If observational constraints on the partition functions can be obtained, helio- and asteroseismology could answer this question from microphysics.

4. 'CLASSICAL' VARIABLE STARS

This section is by no means thought to be a thorough or systematic introduction to the field of variable stars. Its only purpose is to list very briefly some classes of variable stars with their most important properties. In the spirit of our unifying view, we merely hope to bridge the gap between the (already) 'conventional' asteroseismology and the time-honored subject of variable stars.

4.1. β Cepheids and δ Scuti stars

<u>β Cepheids</u>:

Masses:	10 - 20 $M_\odot$
Luminosity classes:	IV - V
Log T_{eff}	4.3 - 4.4
Periods:	0.1 - 0.3 days
Evolutionary Status:	Main Sequence or Early Post-Main Sequence
Modes:	radial and nonradial: p_0 or p_1 or p_2
Driving:	unknown

<u>δ Scuti stars</u>:

Masses:	1.4 - 2.0 $M_\odot$
Luminosity classes:	V - III
Log T_{eff}	3.85 - 3.95
Periods:	0.02 - 0.3 days
Evolutionary Status:	Main Sequence or Early Post-Main Sequence
Modes:	radial and nonradial: p_0 - p_4 (often $p_0 + p_1 + p_2$ or $p_0 + p_2$)
Driving:	kappa mechanism

There have been no serious attempts to do seismology for δ Scutis or β Cepheids despite the fact that for some of these objects ℓ values have been assigned by means of numerical simulation of line profile changes (see Smith 1985 and references therein). The problem of theoretical diagnostics is not easy, because of the complexity of the frequency spectrum of evolved stars (see section 2). If, however, in the future more frequencies will be observed, they will give the potential to determine the N profile in the interior and thus perhaps answer the question of mixing.

4.2. Population I Cepheids and RR Lyrae stars

Cepheids:

Masses:	4 - 20 $M_\odot$
Luminosity class:	I
Log T_{eff}	3.68 - 3.82
Periods:	1 - 50 days
Evolutionary Status:	central helium burning
Modes:	only radial: p_0 or p_1 or, perhaps, p_2 (13 stars with p_0 and p_1)
Driving:	kappa mechanism

RR Lyrae stars:

Masses:	0.55 - 0.65 $M_\odot$
Luminosity class:	III
Log T_{eff}	3.72 - 3.85
Periods:	0.4 - 0.8 days
Evolutionary Status:	central helium burning
Modes:	only radial: p_0 or p_1 or, perhaps, p_2 (13 stars with p_0 and p_1)

Cepheids and RR Lyrae stars are highly evolved stars, and therefore they present an opportunity to test the theory of advanced stellar evolution (with the mass-loss problem). Since Cepheids are used for distance calibrations, it is crucial to have independent ways to determine their absolute luminosity. RR Lyrae stars are among the oldest stars in the universe, and it is of great importance to know their age and chemical composition precisely.

Although even one pulsation period can give interesting information, far more constraints on stellar structure and evolution are obtained from multiperiodic objects [see Cox (1980, 1982) for comprehensive reviews; Dziembowski (1984) for a brief introduction]. The ratio of two observed periods has a high diagnostic value, as was first recognized by Peterson (1973). Multiperiodic Cepheids and RR Lyrae stars also allow to study nonlinear phenomena, which in turn can give additional diagnostic insight. Dziembowski and Kovácz (1984) suggest that double-mode Cepheids and RR Lyrae stars could involve 2:1 resonance phenomena between one of the observed modes and an unobserved third mode. Thus three periods would be known, if the interpretation could be confirmed. However, a systematic search for such resonances in RR-Lyrae-star models showed that difficulties remain (Kovácz 1985). Moreover, Buchler and Kovácz (1986) showed that it is possible to explain double-mode pulsation without a three-mode resonance. Another interesting diagnostic potential could reside in the observed periodic amplitude modulation of RR Lyrae stars (Moskalik 1985).

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