

# RADIAL PULSATIIONS OF $\delta$ SCTI STARS: THEORY IN COMPARISON WITH OBSERVATIONS

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**Abstract.** In this paper the pulsations of  $\delta$  Sct star models are compared with observational data. The locations of the blue edges of the instability strips were computed as a function of helium abundance. Linear non-adiabatic pulsations were computed following Dziembowski's programme.

## 1. Introduction, Method of Computation and Models

The purpose of our work was to study the pulsations of  $\delta$  Sct star models and to compare the results with available observational data. It concerns, firstly, the location of the blue edge of the instability strip which is a function of helium abundance in stellar envelopes. Secondly, comparison of theoretical and observational periods permits us to determine modes of pulsations if we assume that  $\delta$  Sct stars have 'normal' masses (from 1.5 to 3.0  $M_{\odot}$ ). Finally, we were interested in investigating the pulsational instability of a model with effective diffusion of chemical elements which can represent a 'metallic-line' Am star.

We have chosen some models along the evolutionary tracks of 1.5 and 2.25 solar mass stars (Iben, 1967) as well as some models of 1.8  $M_{\odot}$  stars with constant luminosity ( $M_{\text{bol}} = 1^m 5$ ). The chemical composition was taken as  $X, Y, Z = 0.70, 0.28, 0.02$  and 0.60, 0.38, 0.02. The opacity coefficient was calculated by means of Christy's formula (Christy, 1966) or its slight modification (Pamjatnykh, 1973). Convection was taken into account only in static models, and mixing length was taken equal to pressure scale height. We have considered only chemically uniform envelope models (except in the case of the effective diffusion model) containing 70% of the stellar mass.

The linear nonadiabatic analysis of pulsations was carried out by means of a programme which was kindly proposed by W. Dziembowski (Warsaw, Poland).

## 2. Results of Computations

The basic results of our computations are the following:

(1) The periods of pulsations and the location of the blue edges in the four lowest modes were determined. We shall mention here some characteristics:

- (a) The blue edge is hotter for harmonics than for the fundamental mode.
- (b) In the instability region, the amplitude growth rate is larger for higher modes, hence the lowest modes (the fundamental mode at least) are not probably dominant in pulsations. However, this problem may be finally solved only by means of nonlinear theory.
- (c) The amplitude growth rate increases along the instability strip as the luminosity

increases; for example, the amplitude growth time in models of  $1.5 M_{\odot}$  near the main sequence is of the order of hundreds and thousands of years while, for models of  $2.25 M_{\odot}$  along a corresponding evolutionary track, this time is less than ten years.

(d) The location of the instability strip blue edge for the fundamental mode fits excellently the results of Cox *et al.* (1973). For harmonic modes, we find that the blue edges have lower temperatures by 0.01–0.015 in  $\lg T_e$ . Disagreement is probably caused by differences in the opacity law.

(2) The model of a  $1.5 M_{\odot}$  main sequence star is pulsationally stable if there is effective diffusion of chemical elements. A chemically uniform model at this point on the HR diagram is unstable. The region of second helium ionization which is most responsible for the maintenance of pulsations is practically absent in the model with effective diffusion, and the hydrogen ionization region is unable to maintain the pulsations. This is a possible model for ‘metallic-line’ Am stars. Diffusion of helium in this model was studied by Vauclair *et al.* (1974).

### 3. Comparison with Observations

For the comparison with observations we have plotted 60  $\delta$  Sct-type stars with known periods and *uvby $\beta$* -photometry on the HR diagram (see Figure 1). The observational data were taken from the review by Baglin *et al.* (1973) and from the catalogue of Lindemann and Hauck (1973). The transformation from observational quantities to theoretical ones was made using the calibrations of  $T_e((b-y)_0, c_1^0)$  by Breger (1973) and of  $M_v(\beta, c_1)$  by Crawford (1970). Interstellar reddening was taken into account according to Crawford (1970). Bolometric corrections are small for A stars, and we have taken  $M_{bol} \equiv M_v$ .

The location of the blue edge of the instability strip is shown also in the figure. Eighty per cent of the stars lie in the instability region if the helium abundance is  $Y=0.28$ , and almost all stars if  $Y=0.38$ . Exceptions in the latter case are  $\gamma$  CrB and o Ser; these two stars are components of spectroscopic binary systems. Three of the four other  $\delta$  Sct stars which are components of spectroscopic binary systems also lie to the left of the blue edge for  $Y=0.28$ . Perhaps, these stars have an overabundance of helium in their outer layers or new calibrations of *uvby $\beta$* -photometry are necessary for these stars. It is possible that  $\gamma$  CrB and o Ser have pulsations of an unusual nature.

Thus, the theoretical blue edges coincide rather well with the observational one for a helium abundance between  $Y=0.28$  and  $Y=0.38$ . The upper limit may be somewhat high if some differences between our results and the results of Cox *et al.* (1973) are taken into account.

The figure also shows the initial main sequence for the models of Iben (1967) and Breger (1973); the latter ZAMS is characterised by  $\lg g=4.33$ . The blue edge crosses the initial main sequence at  $\lg T_e=3.915-920$  for  $Y=0.28$  and at  $\lg T_e=3.930-3.935$  for  $Y=0.38$ .

The modes of pulsation of individual stars are also shown in the figure. These modes

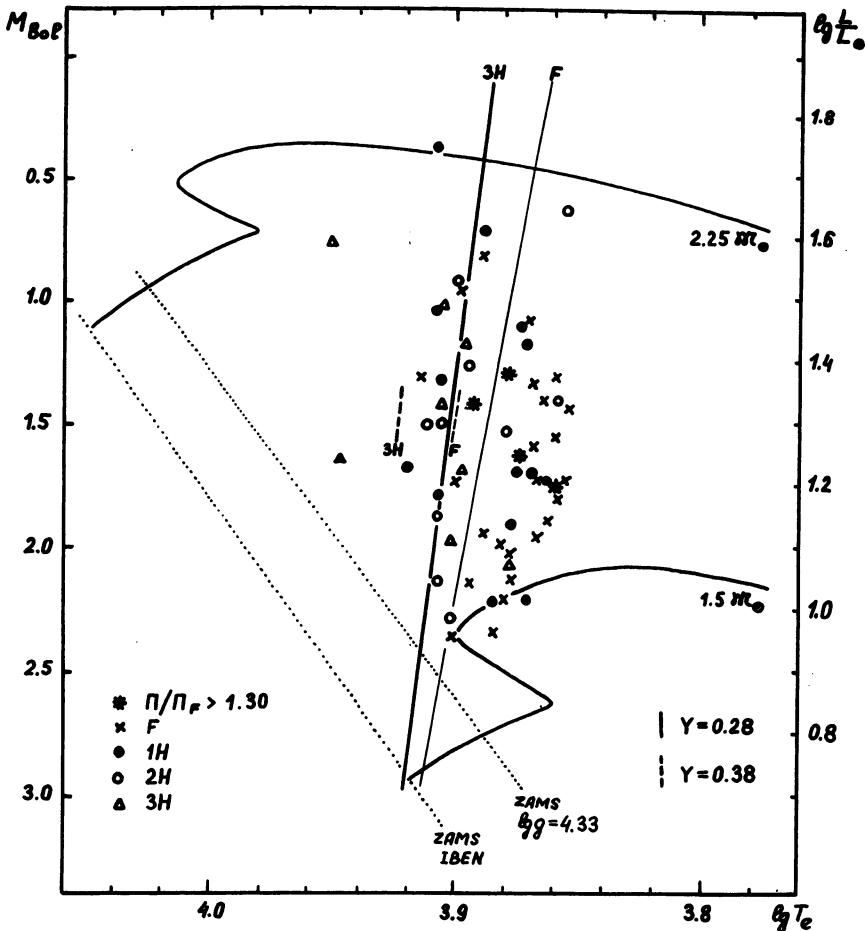


Fig. 1. The location of the  $\delta$  Sct-type stars with known periods and  $uvby\beta$ -photometry in the theoretical HR diagram, and the modes of pulsation of these stars. The theoretical blue edges of the instability strip are shown for chemical compositions  $X, Y, Z = 0.70, 0.28, 0.02$  and  $0.60, 0.38, 0.02$ .

were determined by a comparison of the observational period with the theoretical one which corresponds to ‘normal’ mass ( $1.5\text{--}2.25 M_{\odot}$ ) and to the given location of the star in HR diagram. The distribution of stars according to modes is as follows: 4 stars have periods more than 30% longer than the theoretical period of the fundamental mode; 24 stars pulsate in the fundamental mode; 14, 10 and 8 stars pulsate in the first, second and third harmonics respectively. We see that more stars pulsate in the fundamental mode than in any harmonic, and a few stars have periods even longer than the fundamental period. However, from linear computations it follows that the fundamental mode cannot possibly be dominant in the pulsations. Thus, it appears that some  $\delta$  Sct stars have periods which are too long for their location on the HR diagram. Unfortunately, the linear theory is not able to give the final answer to

the question on the dominant mode, so that the reality of the disagreement is as yet doubtful. We note only that this 'disagreement' may be overcome by a change in the calibrations of the *uvby* $\beta$ -photometry or by a significant decrease in the mass of some stars in comparison with the 'normal' mass. The first candidate as such an 'unusual' star is perhaps  $\delta$  Sct itself which has the longest period and largest amplitude of pulsation among  $\delta$  Sct-type stars.

In conclusion we note that most of the stars pulsating in low modes have lower temperatures than stars pulsating in high modes. For example, to the left of the line  $\lg T_e = 3.895$  lie 14 (78%) of the 18 stars pulsating in the second and third harmonics, and only 9 (21%) of the 42 stars pulsating in the lowest modes. This picture is qualitatively consistent with the fact that the theoretical blue edges are displaced towards higher temperatures on the passage from fundamental to higher modes.

I am grateful to Dr W. Dziembowski for his programme of computation of linear nonadiabatic pulsations and to Dr M. S. Frolow for fruitful discussions.

All computations were carried out by the M-222 computer of the Astronomical Council of the U.S.S.R. Academy of Sciences.

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### DISCUSSION

*M. Breger:* Many of the stars in the diagram we have seen are cluster variables for which the luminosities are known from cluster moduli. Some others have well-measured trigonometric parallaxes. This shows that most of the stars have normal masses and that the luminosity calibrations of the *uvby* system apply to most stars in the previous diagram. The star whose  $Q$  value deviates most from calculations is  $\delta$  Scuti itself. For this star  $\beta$  and  $(b-y)$  give very different temperatures, implying blanketing problems. Yet neither temperature gives a reasonable radial  $Q$  value.

*D. H. P. Jones:* Surely  $(\beta, c_1)$  gives you a gravity rather than a luminosity: you have to know the radius also to find the luminosity. Moreover blanketing affects  $c_1$ .

$q$  Puppis is a bad case.