ROTATIONAL DYNAMICS OF JANUS AND EPIMETHEUS

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Abstract. We investigate simplified models of flat rotational motion of the coorbital satellites of Saturn, Janus and Epimetheus. We try to verify the hypothesis of chaotic rotation of the moons, caused by gravitational interaction between them. The possibility of parametric resonance in the librations of Janus is also investigated.

1. Introduction

The orbital dynamics of coorbital satellites of Saturn, Janus (1980 S1) and Epimetheus (1980 S3), caused interest in past years. Their revolution periods of approximately $16^h 40^m$ differ by only 28s and the sum of their radii is more than three times the difference between their mean orbital radii. The papers by Dermott and Murray (1981a,b), Yoder *et al.* (1983), Harrington and Seidelmann (1981) explain the miracle of the coorbitals by their gravitational interaction. The satellites do not collide by pursuing horseshoe shaped orbits in which the satellites librate around their mean positions in a rotating system (Yoder *et al.*, 1983). The ratio of the amplitudes of the librations is proportional to the inverse ratio of the masses of the satellites and strongly depends on it. Numerical integrations made by Harrington and Seidelmann(1981) showed that the period of the horseshoe type orbits is of order 3000 days and the satellites never get closer than about 6°. They found also that mutual perturbations between coorbitals and the perturbations from Saturn's oblateness and eight major satellites do not cause any instability in the motion over the period of 100 years.

On the other hand, the satellites are assumed to be in synchronous rotation with negligible librations (Stooke, 1993a,b). To our knowledge, no one studied the effects of the gravitational interactions between the

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coorbital satellites on their rotational motion. The aim of this paper is to perform a simplified analysis of the problem. It concerns two models of planar rotation of the satellites.

The inertial characteristics of the moons were derived by the method described in Goździewski and Maciejewski(1995), on the basis of the models of topography by Stooke (1993a,b). The main moments of inertia (A, B, C), in the units of $M \times r_o^2$ (M – mass, r_o – mean radius), are estimated to be (0.38, 0.40, 0.48) for Janus and (0.35, 0.48, 0.51) for Epimetheus. Estimation of their formal errors is of the order of few percent. For further analysis the value $n^2 = 3(B - A)/C$, $A \leq B \leq C$, is essential; it is 0.14 for Janus and 0.76 for Epimetheus.

2. Model I: Flat Dynamics of "Chaotic (?) Potato"

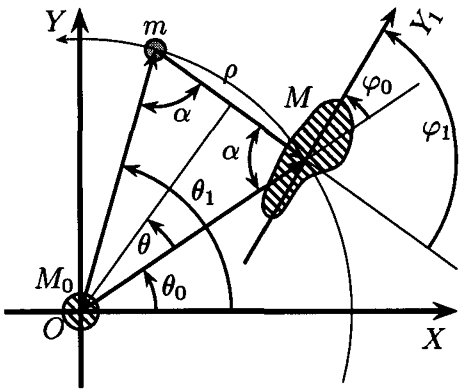

Figure 1. Geometry of the flat rotation of a coorbital moon.

We start from the model of the motion of Janus and Epimetheus, illustrated in Figure 1. We assume flat rotation of the moons, and additionally

- bodies M and m move in *circular* orbits around the central body M_0,
- the angular separation of M and m varies periodically with time according to the law $2\theta = 2\theta(t) = \pi - (\pi - \theta_{min}) \sin(\Omega t)$; this is an effect of mutual and external perturbations,
- both bodies *do not* have constant angular velocity, more specifically: $\theta_0 = \omega t - \theta(t), \quad \theta_1 = \omega t + \theta(t)$.

The equation describing planar oscillations of the body M has the form

$$\frac{d^2\varphi_0}{dt^2} + \frac{3}{2}\frac{\mu_0}{R^3}\frac{B-A}{C}\sin(2\varphi_0) + \frac{3}{2}\frac{\mu_1}{\rho^3}\frac{B-A}{C}\sin(2\varphi_1) = -\dot{\omega}_0,$$

where $\rho = 2R\sin\theta$ and $\omega_0 = \dot{\theta}_0 = \omega - \dot{\theta}$, $\mu_0 = GM_0$, $\mu_1 = Gm$, $2\varphi_1 = \pi - 2\theta + 2\varphi_0$. We assume that $\omega^2 = \mu_0/R^3$ and we put $n^2 = 3(B - A)/C$,

$\epsilon = m/M_0$. The equation of the planar oscillations can be rewritten in the form

$$\frac{d^2\varphi_0}{dt^2} + \frac{1}{2}\omega^2 n^2 \sin(2\varphi_0) + \frac{1}{2}\epsilon\omega^2 n^2 \frac{1}{8\sin^3\theta} \sin(2(\theta - \varphi_0)) = -\dot{\omega}_0.$$

Let us put $q = 2\phi_0$, $p = \dot{q} + \omega_0$. This allows us to rewrite equations of the planar oscillations in the Hamiltonian form:

$$H = \frac{1}{2}(p - \omega_0)^2 - \omega^2 n^2 \cos(q) + \epsilon\omega^2 n^2 \frac{1}{8\sin^3\theta} \cos(2\theta - q)$$

Analysis of the system depends on two parameters ϵ and Ω that scale the value of perturbations.

The Hamiltonian represents a dynamical system with one and half degree of freedom. It can be investigated by the map after the period of perturbation, i.e. the period $T = 2\pi/\Omega$.

Figure 2 shows the relative orbital motion of the coorbitals as derived from numerical integration (points, the orbits were initially circular), compared to our simplified model of relative motion (line). As was mentioned, the minimal separation θ_{min} of the moons depends strongly on their mass ratio and can be as small as a few degrees (Dermott and Murray, 1981a,b). Unfortunately, the densities and masses of the satellites are not well known. In our example, the masses were $M_J = 6.0 \times 10^{-9} M_S$, $M_E = 1.0 \times 10^{-9} M_S$. This corresponds to $\theta_{min} \simeq 6°$ and the separation varies with the period of about 3000 days (case B). Another probable case (A) is $M_J = 2.8 \times 10^{-8} M_S$, $M_E = 8.0 \times 10^{-9} M_S$ then the period would be about 2700 days with $\theta_{min} = 15°$.

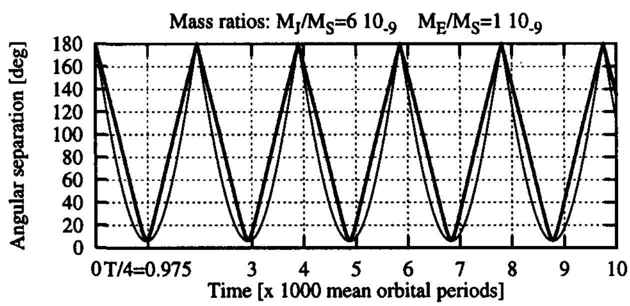

Figure 2. Relative motion of the coorbital satellites. Points represent numerical integration of the three body problem with gravitational interaction of the third mass, line — the simplified harmonic model.

Typical derived results are shown on Figure 3. Case B corresponds to the situation when the mutual perturbations are relatively big (the maximal

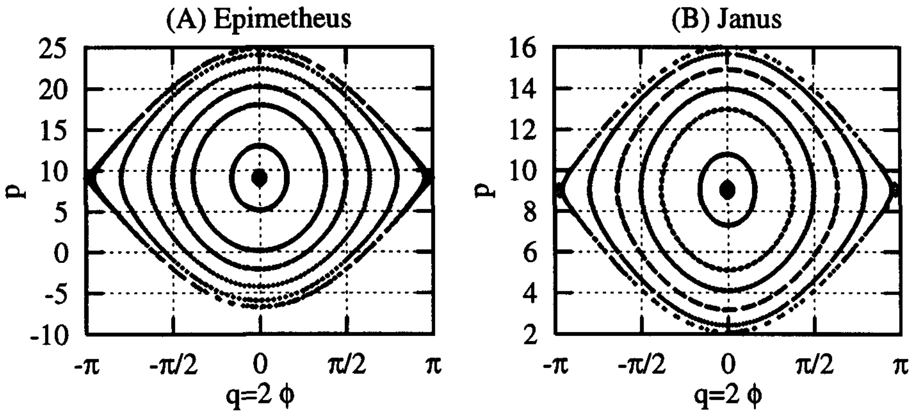

Figure 3. Poincaré maps after the period of orbital libration for Epimetheus and Janus. Cases (A) and (B) of mass ratios (see the text for explanation).

torque is of the order of 10^{-5} of the torque exerted by Saturn, compared to the next substantial value of 10^{-6} caused by Mimas). The motion is regular. We did not find a case whith the motion being irregular when varying the parameters in the acceptable range. This agrees with the conclusion of Stooke (1994a), that the existing images of Janus support the assumption of synchronous rotation of the moon.

3. Model II: Consequences of Beletskii's Equation

Let us now assume that the moons move on elliptic, Keplerian orbits, in the field of the central body, Saturn (in fact, eccentricities are $e_J = 0.007 \pm 0.002$, $e_E = 0.007 \pm 0.002$ (Yoder *et al.*, 1983). Perturbations from other bodies are neglected. The previous sections justify it. In this case, the planar librations are described by the Hamiltonian function of the form:

$$H = \frac{1}{2}(\frac{p}{f(\nu)} - 2f(\nu))^2 - f(\nu)n^2\cos(q), \quad f(\nu) = 1 + e\cos(\nu).$$

As before, one of the principal axes of the satellite is normal to the orbital plane, $n^2 = 3(B - A)/C$. The system may be mapped after the orbital period (here $T = 2\pi$). At the circular orbit there exists the equilibrium solution $q = 0$, $p = 2$ corresponding to the synchronous rotation of the satellite. For $e > 0$ this solution yields a periodic solution (Beletskii, 1965). Analysis of its stability shows, that it may be unstable for some specific values of parameters — the oscillations may admit parametric resonance.

It is well known, that the Beletskii's system is not integrable and can admit chaotic behavior (Maciejewski, 1995). Investigating it numerically

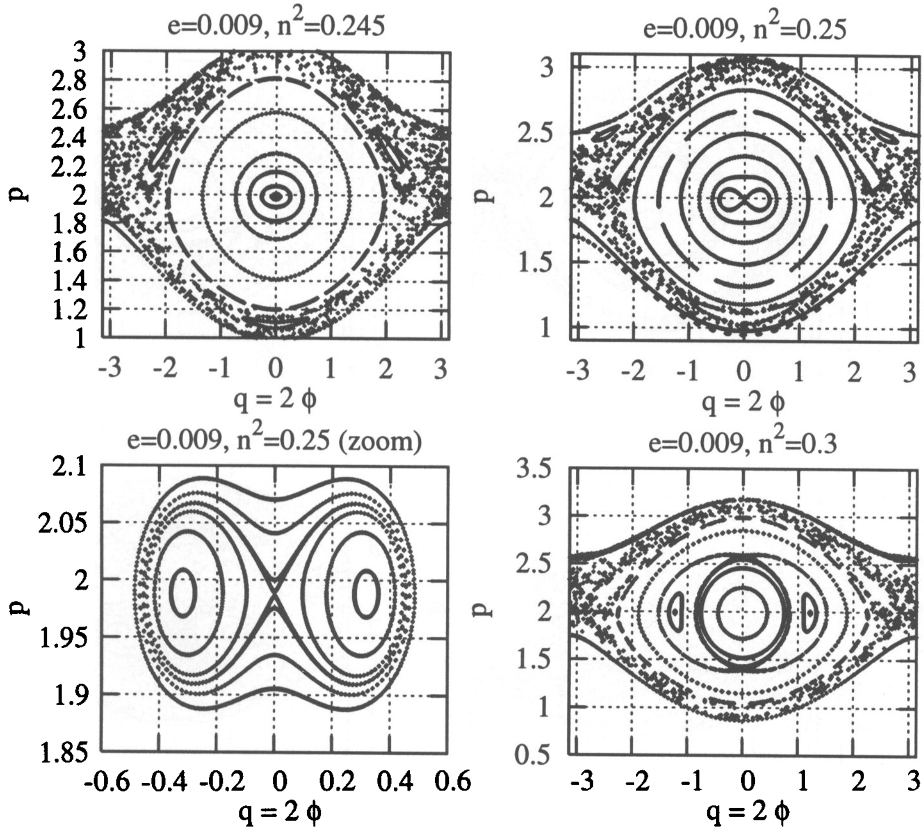

Figure 4. Transition through the parametric resonance $n^2 = 0.25$.

we found, that the parametric resonance at $n^2 = 1/4$ has dramatic consequences, illustrated on Figures 4 and 5. We conclude that at the zone of parametric resonance the synchronous rotation becomes highly unstable. The moon would not rotate synchronously, but oscillate with big (of the order of tens degrees) amplitude (Figure 4). The system is almost completely chaotic even for **very small** eccentricities (Figure 5). The parameters corresponding to Janus are very close to the critical area of parametric resonance, although it is probably not possible to fit them exactly into the critical area of the parameters.

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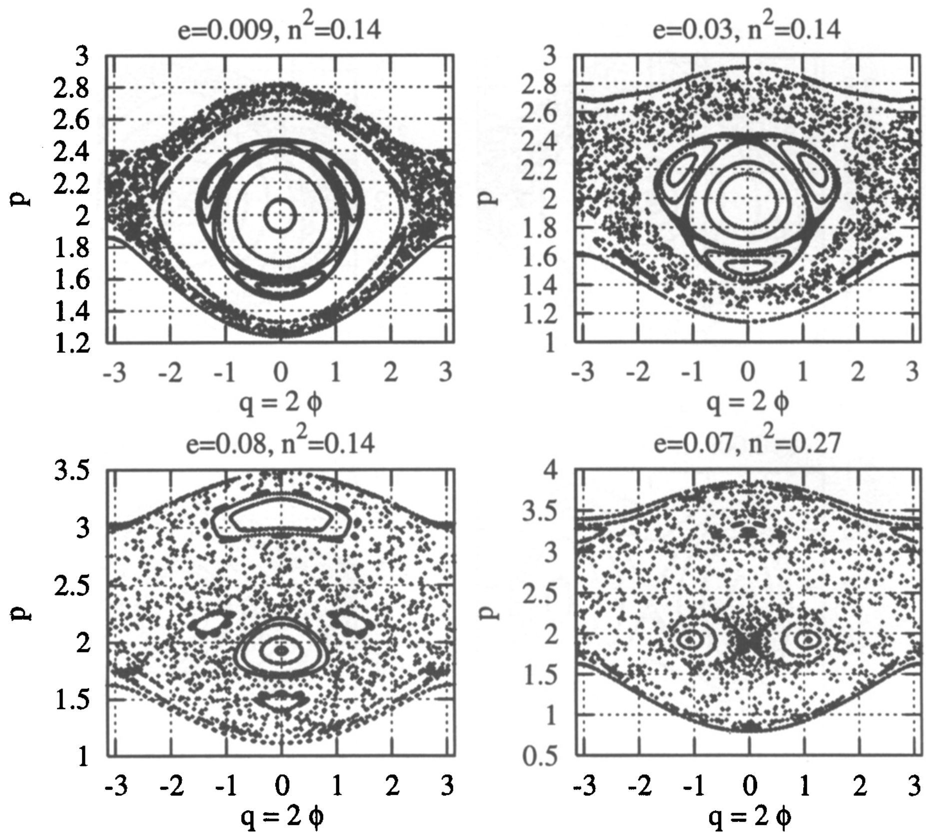

Figure 5. Poincaré maps of the Beletskii equation.

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