THE $(\log g, \log T_{\text{eff}})$-DIAGRAM, A FUNDAMENTAL HR-DIAGRAM

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Abstract. Spectroscopic and photometric parallaxes are based on the assumption that to each point in the relevant region of $(g, T_{\text{eff}}, \boldsymbol{a})$-space there corresponds a unique mass. The implications of a breakdown in this assumption are discussed briefly and an alternative approach to problems that traditionally have required spectroscopic or photometric parallaxes is presented. This alternative approach involves the use of the $(\log g, \log T_{\text{eff}})$-diagram as an HR-diagram.

I would like to draw your attention to a source of errors in spectroscopic and photometric parallaxes that we have not yet discussed at this meeting. That is, the possible failure of the basic assumption that gravity-sensitive spectral features are uniquely related to the absolute visual magnitude M_v.

For the purposes of this discussion let us define *spectroscopic observations*, to be all those in which the quantities measured, S, are selected attributes of a star's emergent flux distribution F_ν. Note that this definition of the term 'spectroscopic' is broader than usual; it covers the full range of available spectral resolution, from high-dispersion spectrophotometry to broad-band photometry, and includes visual spectral classification.

It is well known that, to the extent that the plane-parallel assumption is valid, the structure of a star's atmosphere, and hence its emergent flux distribution F_ν, can be parameterized in terms of g, T_{eff}, and surface chemical composition $\boldsymbol{a}$ (for the purpose of this discussion we can ignore the possible influence on F_ν of non-thermal velocity fields, magnetic fields, etc., although these effects can in principle be included [see, for example Peat 1969]). Thus a given spectral feature S can be written as a function of the above three variables:

$$S = fn(F_\nu) = f(g, T_{\text{eff}}, \boldsymbol{a}). \tag{1}$$

It follows that g, T_{eff} and $\boldsymbol{a}$ are the principal arguments of *all* such observations.

From the preceeding argument it is obvious that the basic physical parameters of a star $(\mathfrak{M}, L, R)$ cannot be determined directly from spectroscopic data. At most, one can derive a mass-luminosity ratio $\mathfrak{M}/L$ (see, for example, Keenan, 1963). This implies, in particular, that the determination of spectroscopic parallaxes is a process of limited validity; i.e. the observational parameters employed in the derivation of a spectroscopic parallax are not explicit functions of the quantity being determined (the ordinate of the conventional HR-diagram, M_v). Typically, a *surface gravity sensitive* parameter, such as a line-strength ratio or the equivalent-width of Hγ, is calibrated against M_v. This spectroscopic parameter is then called, incorrectly, a 'luminosity criterion'.

If a heterogeneous stellar sample is being investigated, spectroscopic parallaxes

B. Hauck and B. E. Westerlund (eds.), Problems of Calibration of Absolute Magnitudes and Temperature of Stars, 86–89.

are liable to be misleading. This can be seen from Equation (3) (below); two stars can have identical (g, $T_{\rm eff}$)-values, and therefore display identical 'luminosity criteria', but have different masses and, hence, different luminosities. Clearly, the basic assumption involved in the method of spectroscopic parallaxes is that there is a unique, single-valued relation between g and M_v. This is equivalent to the assumption that to each (g, $T_{\rm eff}$, $\boldsymbol{a}$, $B.C.$)-value there corresponds a unique value of the total mass $\mathfrak{M}$ (here $B.C.$ denotes the bolometric correction). This must not be regarded as a second order effect; may I remind you that the variation in M_v corresponding to a change $\Delta \log\mathfrak{M}$ in the mass is

$$\Delta M_v = 2.50 \cdot \Delta \log \mathfrak{M} \,(\text{mag.}) . \qquad (2)$$

Of course, the above assumptions are justified in many cases. Nevertheless, we must always exercise care; even in the best case (i.e., the study of open clusters) there are evolutionary phases during which the tracks of stars of different masses can cross. It is imperative, as we push our studies into the field, and, in particular, when we study faint samples that may contain objects that belong to the old disk or halo populations or when we study stars in external galaxies, that we keep the central assumption of the method of spectroscopic parallaxes firmly in mind. Let us consider two cases where this assumption breaks down: (i) in the study of the high-latitude faint blue stars one can easily encounter variations of mass by a factor of two at a given (g, $T_{\rm eff}$, $\boldsymbol{a}$)-value. Thus, ΔM_v can be of the order of $0\overset{\rm m}{.}75$, and (ii) in the study of the distribution of A-stars perpendicular to the galactic plane one will begin to include appreciable numbers of blue horizontal-branch stars in the sample for $V \gtrsim 10\overset{\rm m}{.}0$ – such stars will have their M_v-values overestimated by $\sim 1\overset{\rm m}{.}5$ if a Population I calibration is adopted.

There is, nevertheless, an alternative approach that can, in some important contexts, eliminate the need for spectroscopic parallaxes. That is, to use g directly as an ordinate of the HR-diagram.

Let us define a *physical HR-diagram* to be one that (i) preserves the morphology of the original (M_v, spectral type)-diagram, and (ii) has arguments that are explicit functions of the fundamental *physical* characteristics of a star ($\mathfrak{M}$, L, R). The most widely used physical HR-diagram is the [$\log L$, $\log T_{\rm eff}$]-diagram. Nevertheless, surface gravity is an equally valid ordinate; from the definitions of g and $T_{\rm eff}$ we can derive the following relation between g and L:

$$g = [\text{constant} \cdot T_{\rm eff}^4 \cdot \mathfrak{M}]\, L^{-1} . \qquad (3)$$

From this equation it is obvious that, for a given value of $\mathfrak{M}$, g is uniquely related to L at each $T_{\rm eff}$. Thus, under the condition of constant mass, a curve in the ($\log L$, $\log T_{\rm eff}$)-plane maps one-to-one into the ($\log g$, $\log T_{\rm eff}$)-plane. In most cases of practical interest, variations in $\mathfrak{M}$ do not destroy the correspondence.

We can readily appreciate, from a historical point-of-view, why the ($\log L$, $\log T_{\rm eff}$)-diagram is at present the most widely used physical HR-diagram. Nevertheless, the ($\log g$, $\log T_{\rm eff}$)-diagram is the fundamental diagram when one is dealing with spectroscopic observations. This fundamental nature of the $(\log g, \log T_{\text{eff}})$-diagram was first emphasized by Morgan (1937). Morgan noted that the features observed in stellar spectra are explicit functions of the two principal *physical* parameters of a stellar atmosphere, viz., the surface gravity g and the effective temperature T_{eff}. Thus, the spectral changes that form the basis of a visual classification scheme correspond, primarily, to changes in g and T_{eff}. As Morgan pointed out, a diagram that displays the variation of these two parameters along the principal stellar sequence is of central importance in attempts to understand stellar classification problems.

But the $(\log g, \log T_{\text{eff}})$-diagram can play a much wider role than that outlined above. For example, one of the most important applications of the $(\log g, \log T_{\text{eff}})$-diagram lies in the study of stellar evolution. Theoretical stellar evolution tracks can be presented just as readily in the $(\log g, \log T_{\text{eff}})$-diagram as in the conventional $(\log L, \log T_{\text{eff}})$-diagram. Provided that care is taken to ensure that the derived g and T_{eff} values are correctly related to the global quantities $\mathfrak{M}/R^2$ and L/R^2 (Newell *et al.*, 1969) then the predictions of stellar structure computations can be compared directly with observations, without reference to luminosities; i.e. *the comparison is independent of distances and bolometric corrections*. This technique is obviously one of considerable power in observational tests of stellar interior theory and in attempts to understand observed stellar sequences. In particular, it allows both cluster and *field* stars to be compared with each other and with theory in a manner that is not possible in the conventional HR-diagram. Such comparisons can lead to luminosity-independent determinations of stellar ages. Examples of comparisons of this type are to be found in the work of Hyland (1967), Norris (1971), Osborn (1971), Gross (1972), and Newell (1973).

In summary, those involved in the calibration of spectral features against absolute magnitude are urged to keep the central assumption of their approach firmly in mind and to ensure, for each group of stars studied, that they are justified in assuming a unique, single-valued relationship between g and M_v. The $(\log g, \log T_{\text{eff}})$-diagram provides an approach that is independent of spectroscopic parallaxes and, in my opinion, is valuable enough to justify considerable effort being spent on the problem of calibrating our spectroscopic data against g.

Acknowledgements

I am indebted to the faculty and graduate students of the Yale Astronomy Department for many useful discussions and to Dr W. W. Morgan for his enthusiastic support of this study.

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DISCUSSION

Pecker: The problem with the s-shaped tracks at the hydrogen-exhaustion phase may not be as serious as you have indicated. A recent study by Lesh and Aizenmann places the ultra-short period variables in this part of the diagram and thus we may be able to distinguish between stars undergoing this rapid evolution and stars that are not.

Newell: Such a discriminant can alert us to the fact that we are working in a region where the mass of a star may be uncertain, but it cannot remove the problem with spectroscopic parallaxes.

Demarque: On the question of the identification of β Cephei variables with the hydrogen exhaustion phase, it has, some years ago, been looked into by Percy. On the basis of the frequency of β Cephei stars, Percy concluded that they are too numerous to be associated with such a rapid phase of evolution, and must rather be near the end of their core burning phase.