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ABSTRACT

The evolution of an $0.6 M_{\odot}$ stellar model during core helium burning is presented. Following the off-center ignition of helium in the "core" flash, the star remains on the red giant branch for $> 10^6$ years, undergoing twelve additional flashes. After leaving the giant branch, the star evolves on the horizontal branch for 8.15×10^7 years before returning to the giant branch and undergoing strong helium-shell flashes. The implications for horizontal branch and RR Lyrae stars are discussed.

1. INTRODUCTION

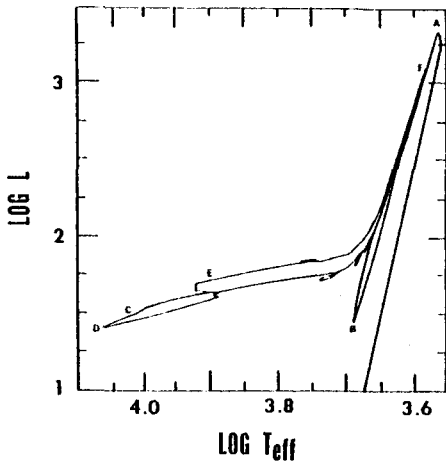


Figure 1. The evolutionary H-R diagram for an $0.6 M_{\odot}$ model. See text.

This paper discusses the evolution of a low-mass star from the ignition of helium to just after its exhaustion in the core. In the H-R diagram this covers the phases of evolution from the tip of the Red Giant Branch (RGB) through the Horizontal Branch (HB) to the Asymptotic Giant Branch (AGB). Thus, it is the purpose of this paper to consider some matters relating to the structure and evolution of HB stars, including RR Lyrae and BL Herculis variables. In Figure 1 is plotted the evolutionary track on the H-R diagram for a stellar model of $0.6 M_{\odot}$ with a composition of $X = 0.9$ and $Z = 0.001$. Several of the main points of evolution are marked on the figure. The various phases of evolution are discussed in the following sections.

2. HELIUM IGNITION (POINTS A-B)

In low-mass stars, the ignition of helium occurs in a degenerate core at the tip of the RGB. The mass of the core (M_C) at ignition can be approximated by

$$M_C \approx 0.474 - 0.038(M-0.8) - 0.24(Y-0.3) - 0.007(3+\log Z) + 0.017(F_V-1) - 0.10(F_{3\alpha}-1), \quad (1)$$

where $F_V = \epsilon_V/(\epsilon_V)_{SG}$ and $F_{3\alpha} = \epsilon_{3\alpha}/(\epsilon_{3\alpha})_{SG}$ are the ratios of the actual neutrino loss and energy generation rates, respectively, compared to those used by Sweigart and Gross (1978; hereafter SG II). The first four terms have been taken from SG II, while the last two have been derived from their results and those of Tarbell and Rood (1975, 1976). It has long been recognized that M_C is a crucial parameter in determining HB structure and evolution (SG II and references therein).

As a result of neutrino "cooling," ignition occurs off-center. The location of the igniting shell, $M_r(T_{max})$, is even more sensitive to variations in the mass, composition, etc. From SG II we can approximate

$$M_r(T_{max}) \approx 0.156 - 0.185(M-0.7) - 0.655(Y-0.3) + 0.137(F_V-1) \quad (2)$$

Around $Z = 0.001$ there is very little Z dependence, while the dependence on $F_{3\alpha}$, presumably large, has not yet been determined.

From Despain and Starrfield (1981) we can approximate the linear growth rate of a temperature perturbation at $M_r(T_{max})$ by,

$$\lambda \approx [v\epsilon - F|T''|(3+v-\kappa_T)]/C_p T, \quad (3)$$

where $v = (\partial \ln \epsilon / \partial \ln T)_\rho$, $F = 64\pi^2 a c r^4 T^3 / 3\kappa$, $T'' = d^2 T / dM_r^2$, and $\kappa_T = (\partial \ln \kappa / \partial \ln T)_\rho$. Generally $v+3 > \kappa_T$ so that the core must fulfill three conditions (in addition to being degenerate) to be unstable:

- i.) It must be hot enough at the temperature maximum for the nascent helium burning to overcome neutrino losses ($\epsilon > 0$).
- ii.) It must be sufficiently isothermal
- and iii.) sufficiently massive for T'' to be small. This explains why the flash does not occur immediately after the onset of degeneracy in the core.

Figure 2 shows the run of temperature, energy generation, and linear growth rate for the $0.6 M_\odot$ model at the tip of the RGB. The temperature maximum, and the point of maximum instability, lies at $M_r = .256 M_\odot$. The thermal runaway occurs at this point.

The description of the flash that follows assumes that the star remains in hydrostatic equilibrium. Deupree and Cole (1980) report

recent results that indicate a breakdown in hydrostatic equilibrium and a much more energetic flash. The outcome is significantly different.

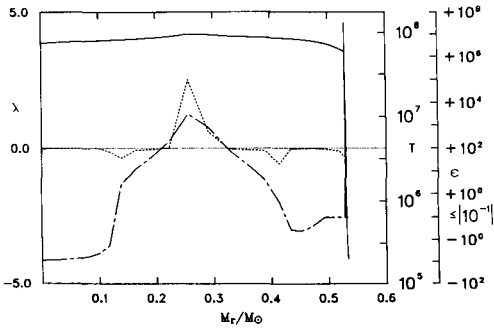


Figure 2. The temperature \rightarrow , energy generation \dashrightarrow , and linear growth rate \dashrightarrow versus M_r for a model at the tip of the RGB.

imum extent 7 years after the peak in energy generation (Despain, 1981) and persisting for 3000 years thereafter.

Initially the unstable shell is quite degenerate; the temperature rises rapidly with very little change in density. However, as the degeneracy weakens the density also begins to decrease. The time of maximum energy generation is reached while the shell is still mildly degenerate ($\log T = 8.38$ and $\log \rho = 5.19$); maximum temperature ($2.5 \times 10^8 \text{K}$) is achieved five hours later, after which the shell expands rapidly, "quenching" the flash.

As is typical in flashes, a convective shell develops just above $M_r(T_{\max})$; the shell grows in mass during the flash, reaching its max-

imum extent 7 years after the peak in energy generation (Despain, 1981) and persisting for 3000 years thereafter. At the peak of the flash the surface of the star is still virtually unaffected by the events occurring in the core. Since reaching the tip of the RGB the outer envelope has been slowly contracting. Following the peak, as the hydrogen shell is expanded and extinguished; the envelope undergoes a more precipitous contraction and a decline in surface luminosity. The stellar radius decreases by about a factor of 16 and the luminosity drops nearly two orders of magnitude. The surface temperature, however, changes only slightly; the star "slides" down the RGB arriving at point B 8000 years after the peak in energy generation.

It should be pointed out that the energy from the flash has not lifted the degeneracy of the core below $M_r = 0.248 M_{\odot}$. Whether or not the degeneracy of the entire core is lifted during the flash is a function of $M_r(T_{\max})$. Thomas (1967), Mengel and Gross (1976), and Paczynski and Tremaine (1977) also describe flashes in which the degeneracy is not lifted. The location of the flash in their cases ranges from 0.25 to 0.40 M_{\odot} . In contrast, Demarque and Mengel (1971) report a flash occurring at 0.092 M_{\odot} in which the degeneracy of the entire core is lifted.

3. EVOLUTION TO THE HB (B-C)

A prerequisite for existence on the HB would appear to be a convective, helium-burning core; thus if the flash has not lifted the

degeneracy, the star must make further adjustments before evolving to the HB. At point B (Fig. 1), both the hydrogen- and helium-burning shells have been extinguished by the post-flash expansion. The core begins re-contraction as the outer envelope expands; the star again ascends the RGB. The core contraction leads to re-ignition of the helium-burning shell. However, the shell is unstable and the star undergoes several additional flashes. In this case the shell is no longer strongly degenerate. The flashes are essentially "thin-shell" flashes much like those that occur on the AGB, with one notable exception--they occur at every smaller core masses, as $M_r(T_{\max})$ moves inward.

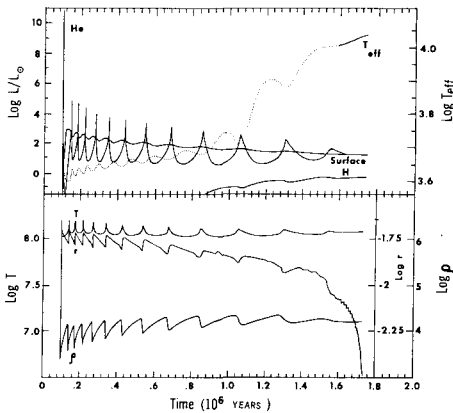


Figure 3. (Upper) L_H , L_{He} , L_{surface} , and T_{eff} vs time, following the flash. (Lower) T , r , and ρ at $M_r(T_{\max})$ vs. time.

T_{eff} (point D), followed by a "typical" HB evolution, as described by Sweigart and Gross (1976, SG I) for a model of this mass and composition (points D-E). There are, however, two major differences between this evolution and that of SG I:

- i.) the lifetime is approximately 15% shorter (7.23×10^7 years when $Y_c = 0.05$ and 8.15×10^7 years total);

and ii.) the evolutionary track is cooler by about 0.03 in $\log T_{\text{eff}}$.

The decrease in HB lifetime may be explained by the following factors:

- i.) At ZAHB there is 3-4% less helium in the core ($Y_c = 0.927$), because of the delayed evolution to the HB.
- ii.) The luminosity of the track is about 6% higher. This may be caused by the difference in rates used for the $3\alpha \rightarrow {}^{12}\text{C}$

Figure 3 (upper) shows the variation of the various luminosities and of the effective temperature during the evolution from the flash to arrival on the HB. The lower panel illustrates the behavior of T , r , and ρ at $M_r(T_{\max})$ for the same time. Note that $M_r(T_{\max})$ is decreasing to zero during the time shown. As can be seen the model takes approximately 1.6×10^6 years to evolve to the HB.

4. HB EVOLUTION (C-E)

For purposes of definition the Zero Age HB has been defined as the time when $M_r(T_{\max}) = 0$ for the first time (point C). The subsequent evolution is marked by a rapid (3.2×10^5 yr) evolution to maximum

reaction; the rate used in this study (Fowler, Caughlan, and Zimmerman, 1973) is 40 to 45% larger than that of SG I. It should be noted that the most recent rate (Fowler, Caughlan, and Zimmerman, 1975, FCZ II) is 60 to 70% higher than that of SG I.

- iii.) The amount of ^{12}C burned to ^{16}O during the HB is less. This is caused by a decrease in the rate of $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ (see FCZ II). Since there is 3 times as much energy per α liberated in $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ as in $3\alpha \rightarrow ^{12}\text{C}$ this decreases the HB lifetime by about 4% (Iben and Faulkner, 1968; Iben and Rood, 1970).

The decrease in theoretical lifetime can be translated into an increase in the primordial helium abundance of about 3% predicted by the "R-test" on globular clusters (Renzini, 1977), bringing this value closer to abundances determined by pulsational tests (Deupree, *et al.*, 1979).

The change in T_{eff} may be accounted for as follows. For this composition and mass the hydrogen-shell is virtually extinct during the HB, so it retains its RGB profile. The evolutionary shell thickness (Despain and Starrfield, 1981) is significantly thinner than that assumed by SG I, which alters the ZAHB location (Gross, 1973).

If the hydrogen shell is active on the HB it quickly adjusts its profile to the prevailing conditions, eliminating the change in T_{eff} . A perusal of the models of SG I suggests that the temperature change may only affect the stars on the Blue HB. Those in the RR Lyrae instability strip all have active hydrogen-burning shells. This is another factor that should be taken into account when considering Blue HB morphology.

5. EVOLUTION TO THE AGB (E-F)

As helium is exhausted in the core, the core begins contraction and the hydrogen-burning shell re-ignites. As is characteristic of a stellar structure with an inert, contracting core and a developing shell source, the star evolves toward the RGB, arriving at its base 3×10^6 years after core He exhaustion. It crosses the instability strip at a higher luminosity than the HB in about 10^5 years, during which time it would presumably be a BL Herculis variable. The core contraction also leads to ignition of helium in a shell, which is unstable to subsequent flashes. The star has become a thermal-pulsing AGB star.

6. SUMMARY

There are several points that can be drawn from the evolutionary results discussed above:

- i.) When the core flash occurs far from the stellar center it does not lift the degeneracy of the entire core.
 - ii.) Until the degeneracy is lifted by subsequent helium burning the star remains a red giant, undergoing repeated shell flashes.
 - iii.) The horizontal branch lifetimes may be shorter than previously indicated (SG I) because of RG helium burning and differing reaction rates;
- and iv.) the morphology of the blue end of the HB may be significantly affected by the (assumed) width of the extinct hydrogen shell. The RR Lyrae stars are not affected because their hydrogen-burning shells are active.

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DISCUSSION

STOBLE: Did your calculations take into account the effect of mass spread along the horizontal branch to explain its width?

DESPAIN: I have not looked into that in detail. However, as you increase the mass in the hydrogen burning shell, the zero age horizontal branch location goes to the red. A spread in hydrogen shell widths might exist which tends to unbunch the stars at the blue end of the horizontal branch.

J. COX: If you had allowed mass loss during the helium flash, would that have affected the core flashes?

DESPAIN: I don't think so. The core and the envelope can be very decoupled. The assumption has been made that there is always hydrostatic equilibrium, but Deupree shows that that is not always good. I assume that there is no mass loss at the red giant branch before the flashes. If the core structure represents some typical horizontal branch model, which is what is intended, then depending upon the location of the temperature maximum you still get these secondary pulses. It is the same as shell flashes on the asymptotic branch.

KING: I guess your models don't show this, but I'm curious to know if you have any feeling whether any mixing might occur between the shell and the outer envelope.

DESPAIN: The convection always falls short. The convective shell moves out until the thermal time scales for radiation diffusion from the top of the shell equals the long e folding time of the shell width. The maximum convective phase is reached 7 years after the peak helium burning luminosity. I've increased the energy generation rates artificially to do some numerical experiments to see if I could push the convective region out. With a luminosity up by a factor of 10 convection moves out, but not far enough, at least, for hydrostatic equilibrium.