# MODEL ATMOSPHERES WITH PERIODIC SHOCKS

G. H. Bowen
Astronomy Program, Physics Department
Iowa State University, Ames, Iowa 50011

Abstract. The pulsation of a long-period variable star generates shock waves which dramatically affect the structure of the star's atmosphere and produce conditions that lead to rapid mass loss. Numerical modeling of atmospheres with periodic shocks is being pursued to increase our understanding of the processes involved and of the evolutionary consequences for the stars. It is characteristic of these complex dynamical systems that most effects result from the interaction of various time-dependent processes. For example, rapid mass loss in the models is a joint consequence of the enormous extension of the atmosphere caused by shocks, and of radiation pressure on grains formed in the cool outer region; it is also affected by thermal relaxation processes, which determine the temperature distribution. The progress and significance of these modeling calculations will be reviewed.

## 1 INTRODUCTION

It has become clear in recent years that stars on the asymptotic giant branch (AGB) normally undergo mass loss so prodigious that it changes the entire course of their subsequent evolution. These stars end their lives as white dwarfs, most of relatively low mass (Weidemann 1987). The most rapid mass loss appears to be associated with evolutionary phases in which there is large-amplitude, long-period pulsation, notably in the Mira-class variables and in OH/IR sources. What changes occur in the outer structure and in the behavior of such stars because of their pulsation? How does mass loss result?

This paper is an attempt to summarize the results of modeling calculations which bear on those questions. The emphasis is necessarily on my own work, although a brief description of some other work will be given. A very great deal remains to be done. In order to make progress it has been necessary to make numerous approximations and to use estimated values for various input parameters that are uncertainly known at best. Nevertheless a picture has developed which seems to make sense, which increases our understanding of long period variable (LPV) stars, and which points to the areas most urgently needing further work.

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One very important lesson learned from this modeling is an appreciation of the complexity of these stars' behavior. Simple pictures and simple solutions are appealing, but these are not simple systems. Many of the phenomena they exhibit can not be analyzed properly by studying one isolated process at a time. It would lead to very wrong conclusions, for example, to assume that the atmosphere of a Mira variable is normally in hydrostatic equilibrium and that when periodically disturbed by a shock wave it quickly returns to the original equilibrium conditions, which then prevail until the next shock passes through. Atmospheres in the dynamical models, and no doubt in the stars, are very different from that indeed. Their structure and behavior are determined by interactions, throughout the cycle, of hydrodynamic processes, including but not limited to shock waves; of thermodynamic processes, including but not limited to radiative transfer; of ionization and other chemical processes; of grain formation, growth, optics, and dynamics -- by a wide variety of time-dependent processes, in fact, many of which may be far from equilibrium in much of the extended atmosphere for much of the time. LPV atmospheres thus present a challenging problem -- difficult, rather messy, but very intriguing.

## 2 MODELING METHOD

The approach used in LPV atmosphere modeling calculations has been to consider the atmosphere separately from the interior, as follows. One constructs a model atmosphere whose inner boundary is placed at or somewhat inside the photosphere and studies the response of the model to periodic driving at the inner boundary which simulates the effects of a pulsating interior. Possible effects of the atmosphere on the interior have been neglected, which surely is a reasonable approximation for most purposes; the atmosphere may play a significant role in limiting the pulsation amplitude, however, as will be discussed later. In any case this approach has made possible much progress toward understanding the behavior of the atmosphere and the mechanism of mass loss.

Table 1. Assumed typical ranges for parameters of Mira variables.

| | |
|---|---|
| Mass | 1-2 $M_\odot$ |
| Period | 200-500 days |
| Radius | 150-350 $R_\odot$ |
| Effective temperature | 2800-3000 K |
| Shock amplitude | 25-35 km $s^{-1}$ |
| Wind velocity | 10 km $s^{-1}$ |
| Mass loss rate | $10^{-7}$ to $2\times10^{-6}$ $M_\odot$ $yr^{-1}$ |

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The objects most studied in modeling calculations of this kind are the Mira variables. These are cool giant LPVs of high luminosity, which commonly have considerable circumstellar dust and fairly large mass loss rates. Stellar properties assumed to be typical of Miras are shown in Table 1 (Willson 1982, 1988). Most LPV models have had parameters in or near these ranges.

No attempt will be made to give here a complete review of the relevant literature. For discussions of possible mass loss mechanisms, the excellent reviews by Castor (1981) and by Holzer & MacGregor (1985) are recommended. Modeling calculations which seem particularly significant with respect to the structure of LPV atmospheres have been carried out by Willson, by Hill, by Wood, and by Bowen; these will be described briefly.

Willson used analytic methods and a ballistic approximation for the motion of gas in the atmosphere to gain insight into the behavior resulting from shocks and to elucidate the conditions under which mass loss can occur (Willson & Hill 1979, Hill & Willson 1979). This work was elaborated and extended in later publications (e.g. Willson & Bowen 1985, 1986a, 1986b). Although it omits essential physical effects, it does provide a useful limiting case.

Numerical hydrodynamic calculations have been carried out by Hill (Willson & Hill 1979, Hill & Willson 1979), by Wood (1979), and by Bowen (1988a, 1988b). All assumed spherical symmetry but made no other assumptions about the form of the solution. Each then wrote the basic hydrodynamic equations, including artificial viscosity, and integrated these to obtain a description of the dynamical model. Significant differences in their work include the following. Hill used mostly a 5-$M_{\odot}$ model with a radius close to that now believed to correspond to the first overtone mode for the period employed; Wood used only a 1-$M_{\odot}$ overtone model; Bowen explored a grid of models that extended over a sizable range of masses and periods, including both fundamental and overtone modes, and was supplemented by systematic variation of other model parameters. Hill assumed that all thermodynamic processes, including shocks, were isothermal (except for one interesting case, not pursued further, in which an abrupt change to adiabatic behavior was made at large radii, where low density would be expected to give slow recombination of ionized hydrogen); Wood tried both isothermal and adiabatic conditions; Bowen used both completely isothermal and completely adiabatic conditions, but introduced the use in most models of density- and temperature-dependent thermal relaxation rates. Hill assumed a uniform, constant temperature throughout the model atmosphere at all times; Wood calculated the temperature as a function of radius (in his isothermal models) using a fictitious optical depth chosen to fit his interior calculations; Bowen used the Eddington approximation for a gray spherical atmosphere to calculate the radiative equilibrium temperature at each radius, assuming a uniform opacity estimated from the results of Alexander et al. (1983) to be suitable for the inner atmosphere. Both Wood and Bowen, but not Hill, made calculations for models that included radiation pressure on dust, with the assumed amount

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of dust (more precisely, its cross-section for radiation pressure) calculated in a simple, ad hoc way from the local radiative equilibrium temperature.

The results of all three of these investigators are similar, to the extent that their work overlaps. Shocks are formed in all cases, and the atmosphere becomes greatly extended. Completely isothermal models with no dust have very low mass loss rates, and completely adiabatic models have extremely high mass loss rates. Wood showed that addition of a suitably adjusted amount of dust to his isothermal model gave a realistic mass loss rate; Bowen confirmed this and explored the dust effects throughout his extended grid of models. In broad outline, at least, the picture seems solid and secure.

## 3 MODELING RESULTS

In order to illustrate typical results in some detail I shall use a specific model of my own. It is the 1.0-$M_\odot$, 320-day, fundamental mode model whose characteristics are listed in Table 2. Figures 1-4 show the behavior of this model, which will be discussed.

Table 2. Characteristics of Illustrative Model

| Input parameters: | Calculated values: |
|---|---|
| $M = 1.0\ M_\odot$ | Initial static model: |
| $P = 320$ days | $L = 4000\ L_\odot$ |
| Fundamental mode | $g = 0.48$ cm $s^{-2}$ |
| $R = 240\ R_\odot$ | $v_{esc} = 40$ km $s^{-1}$ |
| $T_{eff} = 2960$ K | $H = 0.025\ R_\star$ |
| $T_{eff}$ amplitude = 200 K | (scale height) |
| Piston vel. ampl. = 3 km $s^{-1}$ | Dynamical model: |
| Dust: $T_{cond} = 1400$ K | $\dot{m} = 2.5 \times 10^{-7}\ M_\odot\ yr^{-1}$ |
| Radiation pressure | Max. shock ampl. = 33.5 km $s^{-1}$ |
| cross section calc. | Wind speed = 11-12 km $s^{-1}$ |
| to give $a_{rad} = 0.95g$ | $M_{bol}$ variation = 1.07 |

The input parameters and the calculations for this model were very similar to those described in Bowen (1988a). They differ in the use of more and finer zones in the model and of shorter time steps, to give better modeling; of a deeper inner model boundary (about 0.9 $R_\star$) to permit better study of conditions inside the photosphere; of slightly adjusted values of the dust condensation temperature (to 1400 K), of the assumed opacity (to $4 \times 10^{-4}$ cm$^2$ gm$^{-1}$), and of the fraction of Lyman alpha radiation assumed to escape (to 0.1%); and of an imposed variation in

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the effective temperature (amplitude = 200 K), as part of the driving. Also included were the collisional transfer of energy from grains to gas atoms (Bowen 1988b).

A stable, reproducible dynamical model is generated by starting with a model in hydrostatic equilibrium, then increasing the velocity amplitude of the inner boundary (the "piston") slowly and smoothly from a tiny initial value (say 1 cm $s^{-1}$) to the desired final value. If piston oscillations are begun abruptly at full amplitude the first wave, moving outward in the steep density gradient of the static atmosphere, grows rapidly into an extremely strong shock which accelerates most of the model to speeds much greater than the escape velocity and destroys the model. When the piston amplitude is increased slowly, however, the model changes without disastrous transients to a very different density distribution which permits steady state behavior with periodic shocks. The adjustment of the model to a change in the driving amplitude is quite fast, in fact, requiring rather few pulsation cycles. Presumably stars can similarly adjust rather quickly and easily to changes in oscillation amplitude that might occur.

Figure 1. Density as a function of radius at phases 0.00, 0.25, 0.50, and 0.75. (Phase 0.00 shown bold.) The density distribution in the static model is shown for comparison.

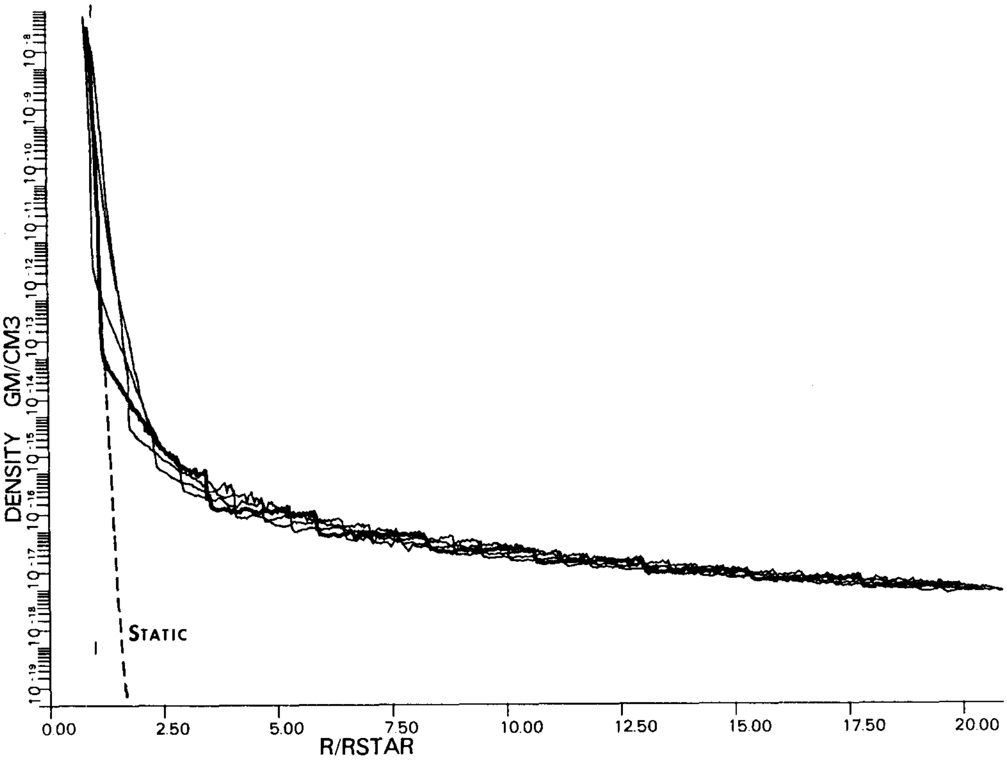

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Figure 2. Radius of selected shells as a function of phase. The innermost line is the piston. The bold line is the photosphere.

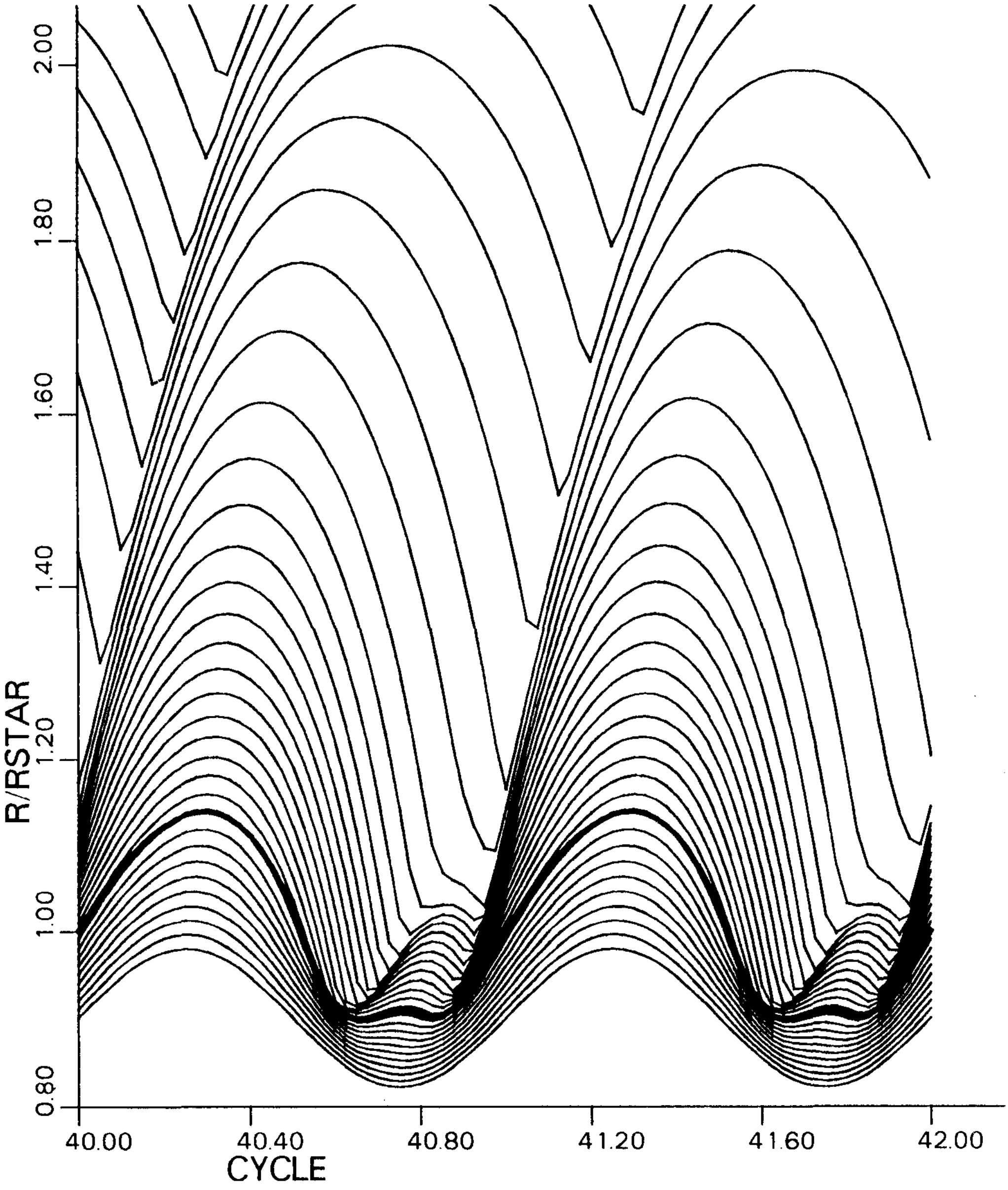

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Figure 1 shows the density distribution for the illustrative model at four phases. Apart from phase-dependent differences resulting from variations in the photosphere radius and from the formation and propagation of shocks, the curve can be described as consisting of two parts: inside the first shock, a steep exponential section like that of the static model; outside the first shock, a very gentle slope -- i.e. a very large scale height, which in fact increases with radius. The effect is to give a truly enormous extension of the atmosphere. The total mass in the outer region is not large, but it is this mass (and the low density gradient there) which makes possible the stable, steady state response of the model to periodic shocks, the formation of substantial amounts of circumstellar dust, and the development of an outflowing wind that gives rapid mass loss from the star.

If the piston amplitude is increased, the innermost shock is formed at slightly smaller radius, where the density and power dissipation are greater; beyond that radius the density curve runs parallel to the previous one, keeping a higher value at all radii. (The mass in the extended outer region is derived from the dense inner zones, of course, with negligible effect on them.) The effect is to increase the mass loss rate considerably -- at the cost of greater power input, almost all of which is dissipated at the inner shock and lost from there via radiation.

Figure 2 shows the radius as a function of phase for selected shells in the inner part of the model, where the density is relatively large and the mean outward velocity is small. Note the formation of a strong shock just outside the photosphere at about phase 0.0 of each cycle; these propagate smoothly outward, weakening as they go. Between encounters with these shocks, individual shells execute almost periodic motion along roughly ballistic trajectories. Note also the formation of a second shock near the photosphere at about phase 0.6; this does not propagate outward and appears rather limited and weak in the figure. In fact these second shocks dissipate a great deal of power because they occur in regions of relatively high density. To understand their formation, observe that motion inside the photosphere is essentially that of a standing wave whose amplitude increases with radius. At phase 0.0 shells just outside the photosphere start outward fast enough to follow semiballistic paths, but not fast enough to remain "levitated" for a full period, in the sense discussed by Willson (Hill & Willson 1979). They fall back into the material below and produce a shock. The propagating shock beginning at phase 1.0 is formed by the interaction of material whose postshock velocity at phase 0.0 was great enough to keep it levitated for one full period.

Figure 3 shows the same information as Figure 2 for a region that extends far enough outward to include dust formation (roughly 2-3 $R_\star$). Note the rapid outward acceleration that occurs there and the transition beyond that to almost steady outflow. Because the mean outward velocity of the material in this lower density region is fairly large, a given shell encounters the (now weak) propagating shocks at time intervals much longer than one period.

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Figure 3. Radius of selected shells as a function of phase. (Same as Figure 2 except for the scale. Shells between the piston and the photosphere are omitted here for clarity.)

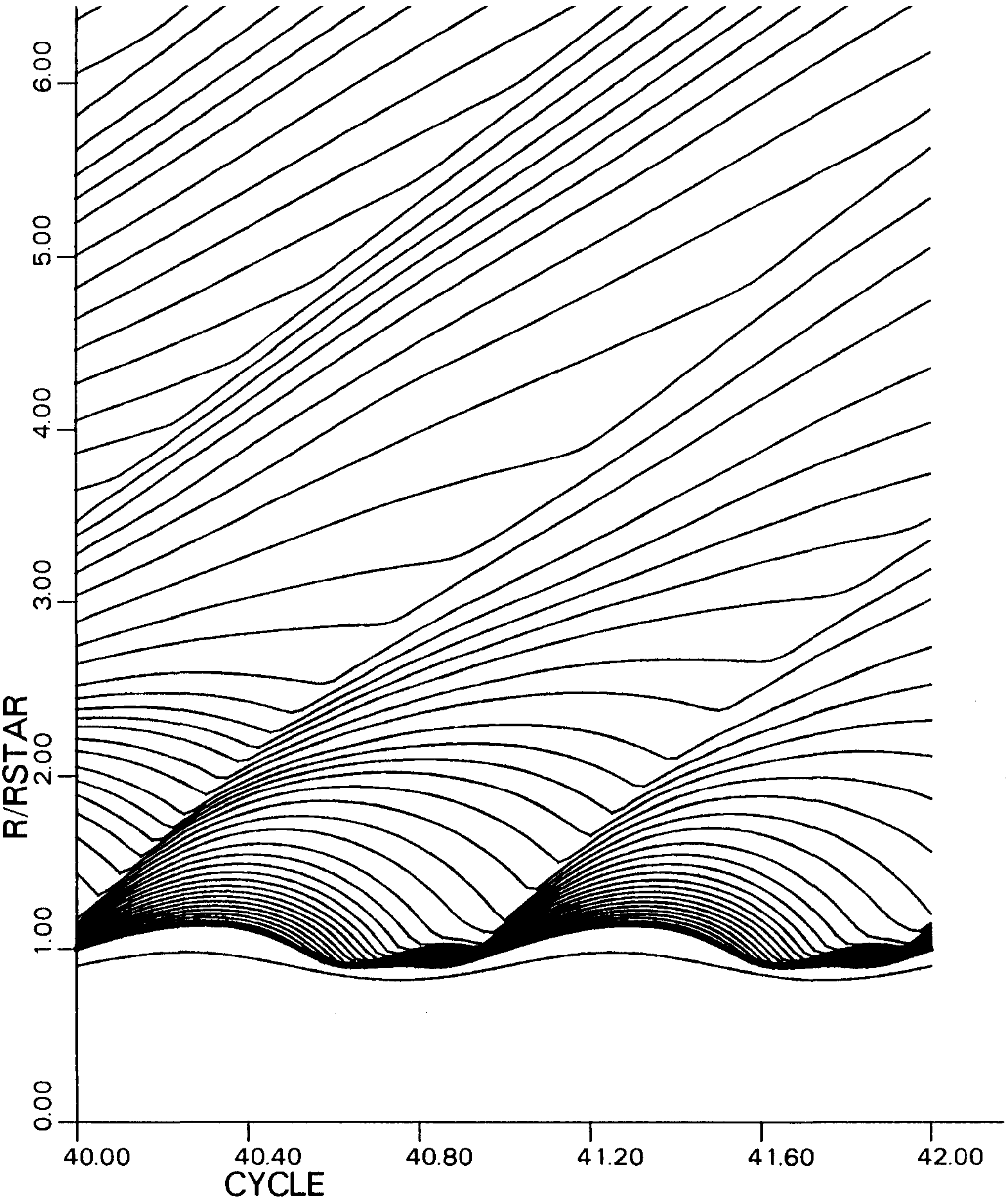

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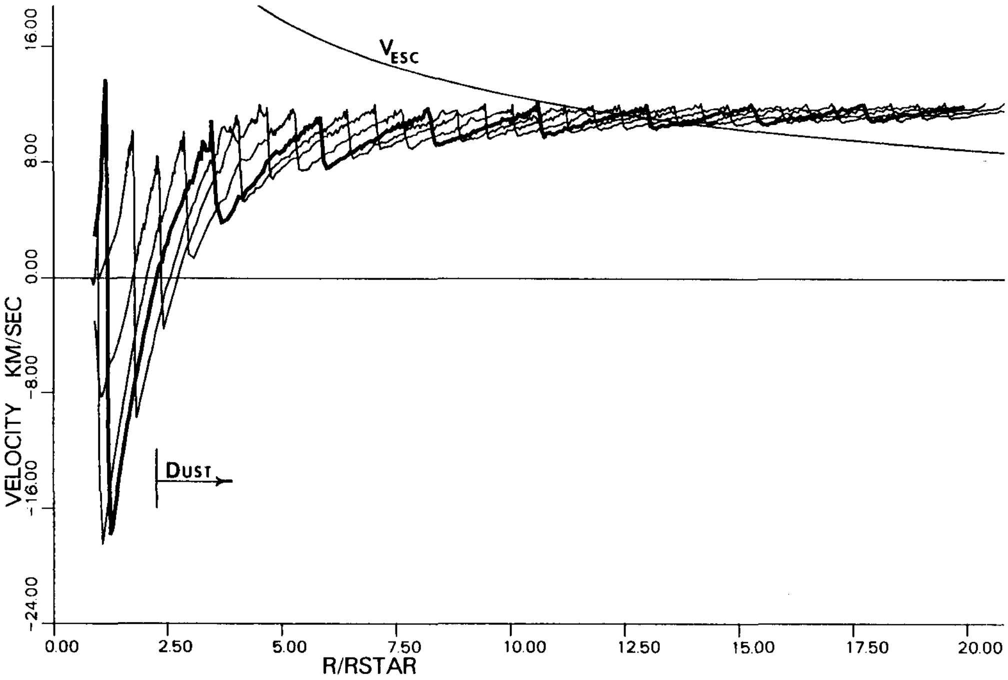
Figure 4. Radial velocity and gas kinetic temperature as functions of radius at phases 0.00, 0.25, 0.50, and 0.75. (Phase 0.0 shown bold.)



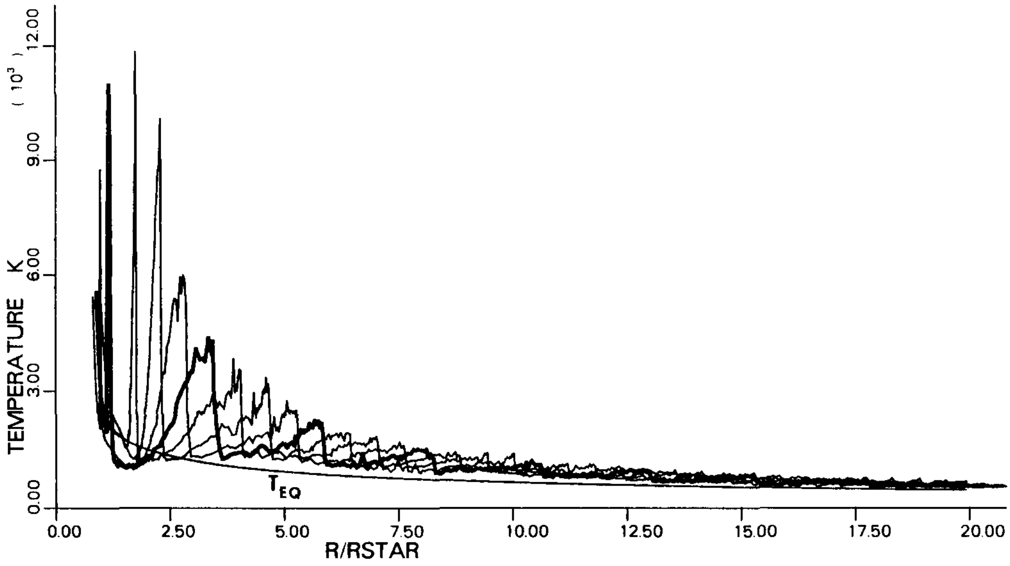


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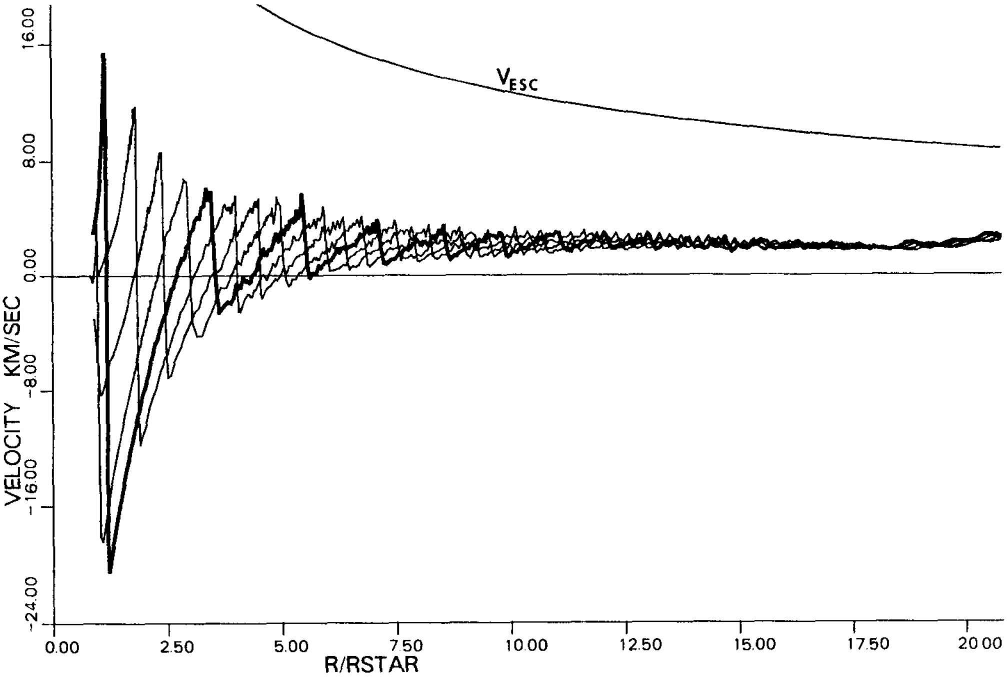
Figure 5. Same as Figure 4 except that dust was omitted from the model.



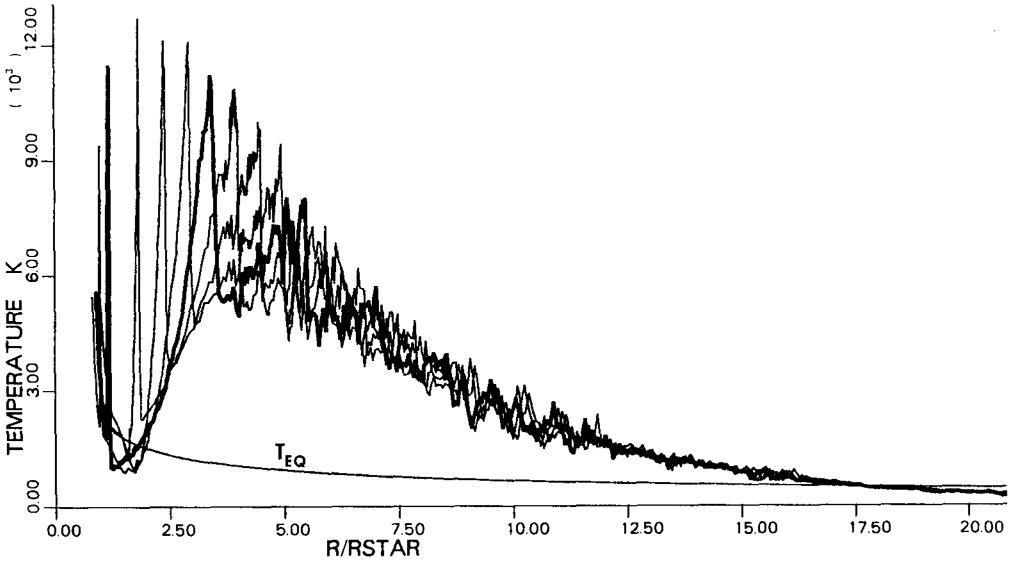


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Figure 4 shows the radial velocity and the gas kinetic temperature as functions of radius for four phases. The velocity plot clearly shows the formation, propagation, and weakening of the shock waves; the rapid acceleration that occurs in the region where dust has formed; and the development of an almost steady wind with a speed exceeding the escape velocity. The temperature plot shows a sharp spike at the position of a strong shock in the relatively dense inner atmosphere, where the collision rates are high and collisional excitation is fast enough to give very rapid radiative cooling; processes there are effectively isothermal. At somewhat greater radii, where the density and temperature are much lower, radiative heating and cooling become so slow that processes there are effectively adiabatic. The dominant cooling mechanism is then adiabatic expansion, which occurs both in the regions between shocks and also at very large radii, because of the general outflow of material. That is sufficient to lower the temperature well below the radiative equilibrium temperature ($T_{eq}$) in some preshock regions and at all large radii; the wind is then quite cool (Bowen 1988a). The inclusion in the calculations of collisional transfer of energy as well as momentum to the gas by grains (Bowen 1988b) gives temperatures in the wind which are somewhat above $T_{eq}$, as seen here.

Figure 5 shows the quite different results obtained when dust is omitted from the calculations. The rapid acceleration seen in Figure 4 in the dust region does not occur. The gas kinetic temperature drops below $T_{eq}$ in the region of very rapid, effectively adiabatic postshock expansion at roughly 1.5-2 $R_\star$. Beyond that, however, expansion is too slow to give much cooling, and the temperature remains far above $T_{eq}$ for a large distance. The temperature gradient does drive a very slow outflow at large r, and there is a small but nonzero mass loss rate. It should be added that off the scale at large r there is a sonic point, where acceleration of the gas to locally supersonic velocities occurs; but the density there is so low that the resulting mass loss rate is very small.

Table 3 summarizes in very schematic fashion the structure of the model of Table 2, with an indication of significant phenomena occurring in each region. A brief acounting of the power budget is also included. The results with other models would be similar. Note that the energy per unit time required to drive the mass loss (0.079 $L_\odot$) is an extremely small fraction of the star's luminosity, whereas the momentum required per unit time (140 $L_\odot/c$) is not small on that scale. Radiation pressure on grains appears to perform the essential function of coupling momentum from the star's radiation field to the gas, so as to make possible the observed outflow.

Note also in Table 3 that the maximum power delivered to the model by the piston is almost exactly equal to the star's mean luminosity. This is more or less fortuitous. (The piston velocity amplitude, 3 km $s^{-1}$, was chosen somewhat arbitrarily to be large enough to give well developed shocks which dissipate substantial power, but small enough to avoid supersonic piston velocities, odd resulting waveforms, and excessive power requirements. A range of driving amplitudes has been

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explored for this and many other models. There is nothing special about this particular choice beyond the considerations mentioned above.) It does suggest a conjecture, however. Interior models for stars of this type show very large oscillation growth rates (Ostlie & Cox 1986). It has never been clear, even with the best nonlinear models of the interior, what determines the limiting amplitude of pulsation. The large power requirement for mechanically driving the inner atmosphere, together with the power dissipation by shocks there, may play an important role in limiting the pulsation amplitude.

Table 3. Summary of the structure and power budget for the model of Figures 1-4. (Schematic)

| Radius | Region | Phenomena | Power |
|---|---|---|---|
| $>20\ R_\star$ | Wind | Wind speed approx. constant | Wind: (Net rate of energy change, from photosphere to large radius) |
| | | | Gravitational $0.069\ L_\odot$ |
| | | | Kinetic $0.0025$ |
| | | Cool wind | Thermal $-0.0012$ |
| | | | Net: $0.070\ L_\odot$ |
| $15\ R_\star$ | Escape velocity exceeded | | |
| | | Heating/cooling by grains is small but significant | |
| | | Heating/cooling by radiation is very slow | |
| | | Shocks very weak | Shock power negligible |
| | | Expansion --> cooling | |
| | | Rapid outward acceleration | |
| $2\text{-}3\ R_\star$ | Dust formation | | |
| | | Shocks propagate and weaken | Postshock radiation:<br>Mean = $55\ L_\odot$<br>Max. = $420\ L_\odot$ |
| | | (approx. ballistic behavior between shocks) | |
| $R_\star$ | Photosphere | Shocks form | Continuum radiation:<br>Mean = $4300\ L_\odot$ |
| | | Weak traveling wave plus Strong standing wave | Wave transport:<br>Mean = $30\ L_\odot$<br>Max. = $4300\ L_\odot$ |
| $0.9\ R_\star$ | Driving region | (Piston) | |

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Table 4 presents data selected from the grid of fundamental mode models described by Bowen (1988a). These show the mass loss rates for a range of masses and for a range of periods extending to values characteristic of OH/IR sources; all other model parameters were held constant to focus attention on the effects of mass and period. There are striking trends toward higher mass loss rates with either a decrease of model mass or an increase of pulsation period. The lower mass models with periods greater than about 500 days have very high mass loss rates and optically thick circumstellar dust -- properties associated with OH/IR sources. Such behavior would have important consequences for evolutionary tracks in mass-luminosity diagrams. An aging, mass losing AGB star, with increasing luminosity and core mass, would also increase in radius and pulsation period, increase its mass loss rate, and proceed more and more rapidly toward loss of all its envelope.

Table 4. Mass loss rates ($M_\odot$ yr$^{-1}$) for fundamental mode models including dust.

| $M/M_\odot$ | 175 d | 250 d | 350 d | 500 d | 700 d | 1000 d |
|---|---|---|---|---|---|---|
| 2.0 | .... | 9. x10$^{-9}$ | 5.8x10$^{-8}$ | 3.1x10$^{-7}$ | 1.2x10$^{-6}$ | 6.5x10$^{-6}$ |
| 1.6 | .... | 3.2x10$^{-8}$ | 1.7x10$^{-7}$ | 6. x10$^{-7}$ | 2.3x10$^{-6}$ | 1.4x10$^{-5}$ |
| 1.2 | 2.2x10$^{-8}$ | 9. x10$^{-8}$ | 2.7x10$^{-7}$ | 9. x10$^{-7}$ | 7. x10$^{-6}$ | 4. x10$^{-5}$ |
| 1.0 | 3.4x10$^{-8}$ | 1. x10$^{-7}$ | 2.5x10$^{-7}$ | 2.1x10$^{-6}$ | 1.6x10$^{-5}$ | 1. x10$^{-4}$ |
| 0.8 | 5. x10$^{-8}$ | 3. x10$^{-7}$ | 7. x10$^{-7}$ | 8. x10$^{-6}$ | 4. x10$^{-4}$ | .... |

## 4 FUTURE DIRECTIONS

Modeling results gained thus far have demonstrated that it is possible to construct reproducible, stable, steady state, spherically symmetric models which show apparently reasonable physical beahavior and have mass loss rates that are at least of the right order of magnitude. What now needs to be done? What are the most fruitful directions for further work?

There are several areas within the theoretical modeling which clearly need much further work. My own nominees for the most urgent of these, because each has potentially major effects on the large-scale structure and behavior of the models are the following.

1- Studies of the time-dependent, nonequilibrium chemistry of the atmosphere -- especially that of hydrogen, of course, since it is present in such large amounts, but extending also to other constituents which are known to be important radiative cooling agents, such as MgII. (By *"chemistry"* *I mean not only conventional molecular reactions but*

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also ionization/recombination.) The chemistry needs to be coupled directly to the hydrodynamic equations in order to treat properly the interactions between the various processes involved.

2- Improved treatment of the region inside and immediately outside the photosphere, and of the coupling between the atmosphere and interior. Energy transfer is very rapid there, and processes take place which probably shape the observable behavior of the star (e.g. light curves) and some aspects of the star's evolution (e.g. the limiting pulsation amplitude, which is a major determinant of the mass loss rate).

3- Studies of dust -- nucleation and growth of grains, their changing optical properties, their dynamics, their coupling to both the gas and to the star's radiation field. Dust plays a key role in these stars, but it has been treated only rather superficially so far.

Comparison of modeling results with observational results is urgently needed, on one hand to help interpret the observational data, and on the other to check on the validity of the models. This should also help to establish more accurately some of the rather uncertainly known parameters that enter the modeling calculations. The modeling has at last reached a level at which it should be rewarding to pursue such studies vigorously. Some work of this kind has been done (e.g. Beach et al. 1988, Brugel et al. 1988). Much more is needed.

And ultimately, of course, the most important goal of all, to me, is to use the understanding thus gained to learn more about the place in stellar evolution of these remarkable stars.

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