Certainly I think the authors have tried to cram too much into the space, and the development of matrix algebra in chapters 9 and 12 would be heavy going for beginners of less than the highest calibre. The banishing of a vital step in § 12.7 to an appendix is surely a sign of overloading, as is "An appeal to a book on mechanics" on p. 21. Layout is pleasant although occasionally cramped. Misprints are few—"$x+y$" for "x and y" on p. 18 and "vector non-zero" for "non-zero vector" on p. 114 are confusing. One answer is undoubtedly wrong (12(*a*)(2)) and some questions appear to have been poorly set [e.g. 6(*b*)(2) where the wording is misleading, 10(*b*)(1) where the officer must surely not know his men and 12(*b*)(5) where a stochastic matrix is given an incomplete definition] but the general standard is good. The book will certainly be stimulating for the brighter sixth-form pupil.

M. PETERSON

KARLIN, S., *Total Positivity*, Vol. I (Stanford University Press; London: Oxford University Press, 1968), xi + 576 pp., 166s. 6d.

Let X and Y be sets of real numbers (or any totally ordered sets) and let $x_1, \ldots, x_m$ and $y_1, \ldots, y_m$ be m-tuples, arranged in increasing order, in X and Y respectively. For a real-valued function $K(x, y)$ denote by $K_{[m]}$ the determinant of $[K(x_i, y_j)]$, $1 \leqq i, j \leqq m$. Call $K(x, y)$ sign-consistent of order m (SC_m) if $\varepsilon_m K_{[m]} \geqq 0$ for all x_i, y_j ($\varepsilon_m = \pm 1$), sign-regular of order r (SR_r) if it is SC_m for $1 \leqq m \leqq r$, totally positive of order r (TP_r) if it is SR_r with $\varepsilon_m = 1$ for $1 \leqq m \leqq r$, and a Pólya frequency function of order r (PF_r) if $K(x, y) = K(x-y)$ and is TP_r. These concepts and their ramifications (such as strictly SC if $\geqq$ is replaced by $>$, confluent forms, etc.) are the central theme of this work.

Total positivity (TP) and related properties play (sometimes indirectly) an important role in surprisingly numerous areas of mathematics including convexity, inequalities, moment spaces, eigenvalues of integral operators, oscillation properties of solutions of differential equations, approximation theory, statistical decision processes, inventory and production problems, reliability theory, stochastic processes of diffusion type, and the study of coupled mechanical systems. The present volume I concentrates primarily on developing the analytical properties of TP_r functions, although some applications (among them applications to approximation theory and to differential equations) are included. A projected second volume will contain further applications, notably to integral operators, statistics, and stochastic processes. The author estimates that more than half of the material presented in the volume under review appears here for the first time; and much of the remaining part is presented for the first time in a unified and sometimes novel form.

After a brief summary of some formulae involving matrices and determinants, chapter 1 contains in 35 pages an overall view of TP and its applications and may be regarded as a prospectus of the entire 2-volume work. The subject of chapter 2 is a hierarchy of notions of sign structure and their interrelations, of chapter 3, operations preserving TP, of chapter 4, smoothness properties of TP_r functions, of chapters 5 and 6, the variation-diminishing property of SR_r functions and its applications, of chapters 7 and 8, PF_r functions in the continuous and discrete case respectively, of chapter 9, periodic PF_r functions, and of chapter 10, the role of TP_r functions in connection with differential equations.

This volume contains an enormous amount of material, the organization of which presents a formidable problem, and it would be idle to pretend that the book is easy to read. The author has performed a very valuable service by relating all this material to a central theme and showing how the concept of sign regularity irradiates and illuminates a great variety of subjects.

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