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Magnetic Helicity, Dynamo Action, Reconnection, and Particle Acceleration

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Abstract. Blackman & Field have shown that a working α-Ω dynamo requires finite but opposite flows of small- and large-scale magnetic helicity through a body's surface. The helicity is accompanied by magnetic energy available for dissipation. The observed solar coronal nonthermal power is consistent with the derived lower limit required. This link between dynamo field generation and nonthermal emission should generally apply to stars, spiral galaxies, and accretion disks.

1. Escape of Magnetic Helicity

The time derivative of the magnetic helicity, defined (Elsässer 1956) by $H^M = \int_V \mathbf{A} \cdot \mathbf{B}\, d\mathbf{x}$, where $\mathbf{A}$ is the vector potential of $\mathbf{B} = \nabla \times \mathbf{A}$, can be written (cf. Field 1986) using only the homogeneous Maxwell equations as $\partial_t(\mathbf{A} \cdot \mathbf{B}) + c\nabla \cdot (\mathbf{E} \times \mathbf{A} + A_0\mathbf{B}) = -2c\mathbf{E} \cdot \mathbf{B}$, where $\mathbf{E} = -\frac{\mathbf{V}}{c} \times \mathbf{B}$. Thus, H^M is conserved for appropriate boundary conditions. Therefore, in the α-Ω dynamo, in which kinetic helicity of the turbulence creates a large-scale field carrying magnetic helicity, there has to be a compensating creation of small-scale field carrying helicity of the opposite sign. From the numerical solution of approximate equations describing the spectra of energy and helicity in MHD turbulence, Pouquet, Frisch, & Leorat (1976) showed that the α effect conserves magnetic helicity by pumping a positive (negative) amount to scales $> L$ (the outer scale of the turbulence) while pumping a negative (positive) amount to scales $\ll L$, where it is subject to Ohmic dissipation. They identified magnetic energy at the large scale with the $\bar{\mathbf{B}}$ of Steenbeck, Krause, & Rädler (1966). Thus, dynamo action leading to an even larger $\bar{B}$ and ever more large-scale helicity can proceed as long as small-scale helicity of opposite sign can be dissipated by Ohmic diffusion.

However, these calculations were based on a magnetic Reynolds number $R_M = 30$, well below that of astrophysical systems like the Sun. For larger R_M, some numerical and analytic calculations which ignore boundary/divergence terms (e.g. Seehafer 1994; Gruzinov & Diamond 1994; Cattaneo & Hughes 1996) argue that large-scale dynamo action is severely curtailed.

Blackman & Field (2000) have shown that dynamo action can occur in large R_M systems only if small-scale helicity can escape to a body's surface, something impossible for the boundary conditions of Seehafer (1994) and Cattaneo & Hughes (1996). If correct, a net amount of small-scale magnetic helicity should be escaping through the solar photosphere. This leads to an observational test for an internal α-Ω dynamo: solar activity is predicted from the emergence and dissipation of helical magnetic flux tubes through the solar surface—this is observed (cf. papers in Pevtsov, Canfield, & Brown 1999).

2. Relation to Dynamo Coefficient α and Energy Dissipation

The time derivatives of the small- and large-scale magnetic helicities must be equal and opposite. Blackman & Field (2000) find the magnitude of this quantity in each hemisphere to be $\dot{H}^M = \frac{4\pi}{3} R_\odot^3 \langle \alpha \bar{B}^2 - \beta \bar{\mathbf{B}} \cdot \nabla \times \bar{\mathbf{B}} \rangle$, where the angle brackets refer to a hemispheric volume average. According to Blackman & Field (2000), both terms in the brackets are of the order $v_0 L \bar{B}^2 / R_\odot$, where $L = 3 \times 10^8$ cm is the outer scale of the turbulence, $v_0 = 10^4\,\mathrm{cm\,s^{-1}}$ is the turbulent velocity at that scale, and $\bar{B} = 100\,\mathrm{G}$ is the mean magnetic field (Parker 1979). We therefore take the first term as representative. Let $H^M = \int H_k^M \,\mathrm{d}k$; keep in mind that both H_k^M and H^M are signed quantities. According to Frisch et al. (1975), for a helical magnetic field to be realizable, its energy spectrum E_k^M must satisfy $E_k^M \geq \frac{1}{8\pi} k |H_k^M|$. Hence, there is an associated minimum power:

$$\dot{E}^M \geq \frac{1}{8\pi} \int k |\dot{H}_k^M| \,\mathrm{d}k \geq \frac{k_{\min}}{8\pi} \int |\dot{H}_k^M| \,\mathrm{d}k \geq \frac{k_{\min}}{8\pi} |\dot{H}^M| = \frac{k_{\min}}{6} \left|\langle \alpha \bar{B}^2 \rangle\right| R_\odot^3.$$

A mode's presence in the lower corona requires $k > k_{\min} = 2\pi / R_\odot$, so $\dot{E}^M \geq \frac{\pi}{3} \left|\langle \alpha \bar{B}^2 \rangle\right| R_\odot^2$. Summing over both polar hemispheres, the total power in ejected magnetic fields satisfies $P \geq \frac{2\pi}{3} \left|\langle \alpha \bar{B}^2 \rangle\right| R_\odot^2 = 4 \times 10^{27}\,\mathrm{erg\,s^{-1}}$, using $\alpha \sim 40\,\mathrm{cm\,s^{-1}}$ (Parker 1979). This energy is available for particle and wind acceleration. This lower limit is in fact consistent with solar observations, as it is $\sim 1/6$ the total measured nonthermal energy loss (Withbroe & Noyes 1977).

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