

where 
$$S = \sum_1^{\infty} \frac{(-)^{n-1}}{2^n} \cos 2^n A, \dots\dots\dots(12)$$

and the series converges as  $n \rightarrow \infty$ .

This formula can be immediately checked in such simple cases as  $A = \frac{1}{2}\pi, \frac{1}{4}\pi, \frac{3}{8}\pi$ .

6. While there seems to be no available rule for summing the series in (12), either to  $n$  terms or to infinity, for a general value of  $A$ , all cases in which  $A$  is a *rational* fraction of  $\pi$  can be dealt with.

Even when the angles of the original triangle are all different, provided they are rational fractions of  $\pi$ , a stage is bound to come when, say, the  $n$ th pedal is equiangular with a previous one. If  $A, B$  are of the form  $\pi p/q, \pi p'/q$ , there will clearly be a repetition of the angles after not more than  $q^2$  steps. In the case of the isosceles triangle, since  $B$  is then not independent of  $A$ , the repetition will come after not more than  $q$  steps; and the summation to be effected will be that of a convergent geometrical progression.

Tuckey (*l.c.*) shows that if the original angles are each a whole number of degrees, the order of the period is twelve, or a submultiple of twelve, and that the starting-point of the period comes not later than the third triangle.

7. A similar, and apparently even less tractable, problem arises when  $A_1, B_1, C_1$  are the points in which the internal bisectors of the angles  $A, B, C$  meet the opposite sides.

D. G. T.

CORRESPONDENCE.

MARK-FREQUENCIES.

To the Editor of the *Mathematical Gazette*.

SIR,—In a recent *Gazette* I found the suggestion—somewhat frivolous, if I may say so, for the pages of your austere periodical—that when marks are being analysed the way in which the various marks compete for the leading position might be regarded as a race and made the subject of a sweepstake.

This led me to study the way in which some marks for an algebra paper approached their final frequencies, with the following result.

The first eight marks to attain, and their order in attaining,

|                   |                                 |
|-------------------|---------------------------------|
| frequency 5 were  | 57, 78, 55, 56, 40, 42, 27, 35; |
| frequency 10 were | 55, 75, 80, 78, 42, 81, 27, 56; |
| frequency 15 were | 35, 42, 45, 55, 80, 48, 58, 59; |

while the final order was

45 (fr. 23), 42 (fr. 20), 55 (fr. 18), 68 and 80 (fr. 17), 35, 39 and 58 (fr. 16).

The complete fading away of 57 after a good start, and the sudden spurt of 35 for the lead, followed by its collapse, are only less remarkable than that the ultimate winner did not appear among the leaders till more than half-way through the race.

All of which goes to show how unequal one school is to another and how misleading are our reports if they begin: "The standard in Algebra was in general very satisfactory."

Yours, etc., C. O. T.