

SKETCHES IN HIGHER CATEGORY THEORY

RÉMY TUYÉRAS

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A sketch, in the sense of Charles Ehresmann, provides the data needed to specify a type of mathematical structure. The category of structures for a given sketch has good properties that are ensured by the existence of reflection functors from presheaf categories. For example, a Grothendieck site is a sketch and the reflection functor assigns the sheaf associated to a presheaf.

The present thesis proposes a generalisation of a sketch for higher categories. The motivation for this comes from homotopy theory rather than universal algebra. For the requisite homotopy structure on a category, we introduce *vertebral categories* and *spinal categories*, rather than starting with a Quillen model category in which the weak equivalences are part of the data. For us, the weak equivalences are defined from the vertebrae in much the same way as they are constructed from discs and spheres in topology. The categories of structures for our generalised sketches are categories of fibrant objects in the usual cases. Categories of stacks and spectra are examples. Moreover, our construct of the reflection functor is an extension of the small object argument of Quillen.

Finally, using our algorithm for constructing weak equivalences, we show that Grothendieck's ∞ -groupoids form a spinal category. By combining all the results developed in the present thesis, future work will aim at proving that the category of ∞ -groupoids admits a Quillen model structure and satisfies the *homotopy hypothesis* (which was conjectured by Grothendieck in 1983 and is still unproven).

RÉMY TUYÉRAS,
Department of Mathematics, Macquarie University,
New South Wales 2109, Australia
e-mail: remy.tuyeras@mq.edu.au

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