SOLAR DIAMETER(S)*

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Abstract. Because of the renewed attention now paid to the solar diameter, its variations from equator to pole, or its secular or long-period changes, the question: *what is a solar diameter?* is not meaningless. Two kinds of definitions may be given: either astrophysical, each one relating to a specific physical parameter, or observational, relating to a given quantity to be measured. Only the second kind is directly accessible, and astrophysical definitions should be linked to these quantities, once they are determined with the highest possible accuracy. In practice, all the programs under way refer to the point of the limb where the brightness gradient is maximum, or to a higher order approximation of the shape of the profile. Two of them are compared: the Pic-du-Midi experiment, using fast scans of the limb to define the inflection point after a correction for the blurring effect of the atmosphere, and the SCLERA experiment, using the algorithm called FFTD to eliminate this correction. The advantage of a fast scan is emphasized, and the remark is formulated that, once the signal is digitized and stored, FFTD or any processing of it can be performed. In collecting day-long one-limb scans to calibrate the blurring correction, the authors have found fluctuations of the maximum brightness gradient which provide a new entry to the field of solar oscillations.

1. Introduction

The idea was born already ten years ago, while the controversy around the solar oblateness announced by Dicke and Goldenberg (1967) was very active, on both observational and theoretical sides, to undertake solar diameter measurements at Pic-du-Midi. The good observing conditions already proven, together with a straightforward method, could be expected, indeed, to afford some valuable data in the matter. For a number of cumulative reasons, although the principles had been laid down from the beginning, the program progressed very slowly, and it is not before 1978 that it really began to develop. As a counterpart, several facts occured in the meantime which did not make it obsolete, but rather enhanced its interest, and even caused a branching towards two different goals, as will be seen.

These facts are:

– the publication in 1975 by the SCLERA Group (Hill *et al.*, 1975) of results contradicting Dicke's ones but indicating pulsations of the solar diameter, later on interpreted as photometric fluctuations in the limb profile;

– the extensive development of theoretical and observational work on solar oscillations;

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– the attention recently paid to possible secular variations of the solar diameter (Eddy and Boornazian, 1979), which could represent long period oscillations, and lead to the need for absolute measurements of the angular diameter of the Sun (Parkinson *et al.*, 1980);

– and finally, from our own side, the observation, on daily measurements of the maximum brightness gradient of the solar limb, of oscillations ressembling those found in Doppler or brightness measurements.

This last point caused the splitting of our program into diameter measurements on one hand, and limb oscillation studies on the other one. The present paper will deal mostly with the first part, starting with a preliminary question.

2. What Does Solar Diameter Mean?

Perhaps some thinking on this point could help in clarifying the controversies of the last decade.

The Sun is not a stainless steel ball like those in the bearings. Nobody argues about the significance of the diameter of such a ball, because it is physically defined with an accuracy of the order of the dimension of iron atoms, and we are far from being able to measure it, in practice, with such an accuracy.

The situation has been the same for the Sun as long as everybody believed it to appear with a 'very sharp' edge (something like the surface of the ocean as seen at the horizon), and as long as the observational uncertainties were largely predominant; various authors found various values of the 'diameter', but, admitedly, all of them were measuring the same thing.

Now the scene is completely different. It has been theoretically and observationally proved that nowhere, from the center to the outer corona, does a zero exist in the density and emissivity of the solar atmosphere, so that no infinite brightness gradient is to be observed on the limb. On another hand, sensitivity and accuracy of the observational techniques have gained orders of magnitude, and the effects of the terrestrial atmosphere, which were, if not ignored, at least unexplored as late as fifty years ago, are now seriously taken into account.

Clearly, as soon as the non-existence of a 'vertical' edge is established, a definition of what is to be called 'solar diameter' is required.

3. Two Kinds of Definitions of the Solar Diameter

It would be nice to define the radius of the Sun by the level in its atmosphere where a given *physical* parameter has a specified value or a particular property. It could be, say, the level of the temperature minimum. That would probably lead to make use of different definitions, according to the astrophysical problem under consideration. To change numerically from one kind of radius to another would need a model of the atmosphere. For instance, what is the relationship between the locus of temperature minimum and an equipotential surface of the gravitational field of the Sun, the oblateness of which was the primary aim of Dicke's experiment? But much more severe is the fact that such physical quantities are not directly observable, and that, consequently, a model, with its uncertainties, will be needed to relate them to measurable ones.

Therefore, it appears logical and simpler to take the opposite way: choose a definition by an *observable* parameter, establish it as firmly as possible upon careful measurements, and deliver the concluded numerical value of the diameter as a boundary condition to any model to be computed later on for whatever astrophysical problem. In fact, only definitions of this second type are practicable. But then, the drawback is that, as *observable* parameters result from a number of local physical parameters which may vary in time (oscillations) or from one place to another (e.g. pole/equator), one should compute – unavoidably through a model again – differential coefficients giving the variation of the observable as a function of the variation of the physical parameters, provide they are known from elsewhere. However, if the definition is properly chosen, the variations should be small, and consequently not very sensitive to the uncertainties in the model.

4. The Limb-Profile Observations

Dicke's experiment, based upon what can be called 'integrated photometry', was essentially designed for pole-equator differential measurements. Not surprisingly, all those being developed at the moment, or at least the five of them presented at the Sacramento Peak Meeting in 1980 (Dunn, 1981) make use, in some way, of the limb profile, in modern improvements of the old visual observations. Indeed, the 'steepest part of the profile' still appears as the most evident feature to be observed, provided that more insight is looked for as regards its physical meaning and its dependence from various parameters.

First of all these parameters is the effect produced by the spreading function of the instrument *plus* the terrestrial atmosphere. It is a classical result that the convolution of the edge of a uniformly bright half-plane by a spreading function having a center of symetry gives a smooth symetrical profile having its inflection point exactly on the edge of the object, whereas if the object is limb-darkened, this inflection point is shifted inside increasingly with the width of the transfer function.

Among the five papers mentioned above, two only (SCLERA and Pic-du-Midi) refer to this effect, and consequently the discussion will be limited to these two.

Both experiments operate by radial photoelectric scans of the limb, but they differ in the procedure of these scans, and in the way in which the effect of the atmospheric blurring is treated.

For the clarity of the discussion, it may be useful to recall the parameters which characterize the atmospheric effects on the image of a point-source.

These parameters clearly appear on Figure 1, recovered from old records (Rösch, 1962) of the integrated flux during the occultation of a stellar image by a moving knife-edge (time scale about 2 s for a complete tracing): *scintillation* makes the fluctuations out of occultation; *blurring* (or *spreading*) governs the ratio of the maximum

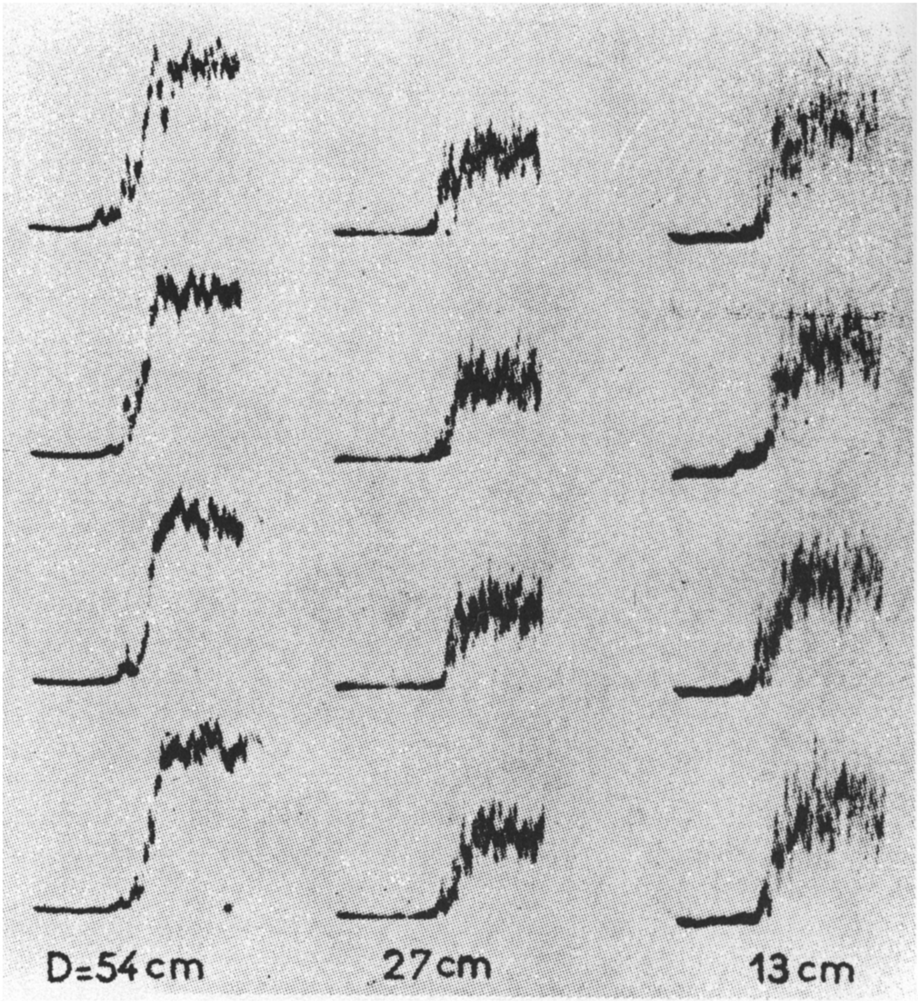

Fig. 1. Occultation of a stellar image by a kinfe-edge, showing the effects of *scintillation, blurring* and *image motion*. Duration of a scan ~ 2 s. D: diameter of the objective.

slope of the tracing to the maximum in the case of a diffraction-limited image (this ratio is equivalent to the reciprocal of the factor D/r_0 defined by Fried, 1966); random *image-motion*, algebraïcally added to the motion of the knife-edge, widens the outer enveloppe of the tracing in the direction of the motion.

How do these parameters influence a scan of the solar limb?

– *Absorption*, if uniform over the field observed, would not affect the brightness ratio between two given points;

– *scintillation* is not detrimental within the extent of the aplanetic patch (i.e. ~ 10″) or somewhat more; the tracings show how it decreases for larger and larger objectives;

– image *blurring* causes the 'shift' of the inflection point, as mentioned above; of course, if the outer conditions are such that a larger objective gains towards its theoretical resolution, as seen from the slopes of the tracings, the shift will be smaller;

– *image motion* will have a dramatic effect, again larger for a small objective than for a large one; first, of course, it will cause a random fluctuation of the measured diameter, but at least this fluctuation is purely additive, and can be treated directly by an arithmetic average; but, second, as it is clear on these pictures and conspicuous too on limb profiles, if the scan is not fast enough as compared to the image motion, the tracing is completely hashed and does not resemble anymore something like a solar limb profile.

Any device intended for solar diameter measurements should take these remarks into account.

5. The Pic-du-Midi Experiment

We deliberately adopted the definition of the solar diameter as the distance between inflection points on opposite limbs, as being the most directly observable parameter; we ought therefore to explore, on one hand, the calibration of the shift *versus* the spreading function which should be known at any time, and on the other hand the stability of the inflection point towards the physical parameters in the solar atmosphere. Figure 2a shows the principle of the observation. The focal image of the Sun (60 mm) is projected onto a CERVIT rod cut with sharp edges, somewhat shorter than a solar diameter. The beams from the opposite limbs are transported by two rhombohedra so as to project both limbs (after a magnification) onto one and the same slit and detector. In front of the slit, a rotating cube, as used by Brandt (1970) for the JOSO Site Survey, produces a scan of the limbs in succession, as seen on Figure 2b; in between the limbs is the scan of a reference patch sampled from the center of the solar disk, which, measured by the same detector, gives at any time the darkening at any point of the limb. Forty scans per second are produced, resulting in a rate of 3000″ per time second; therefore, the 'hashing effect' described above is practically eliminated and the profile is frozen.

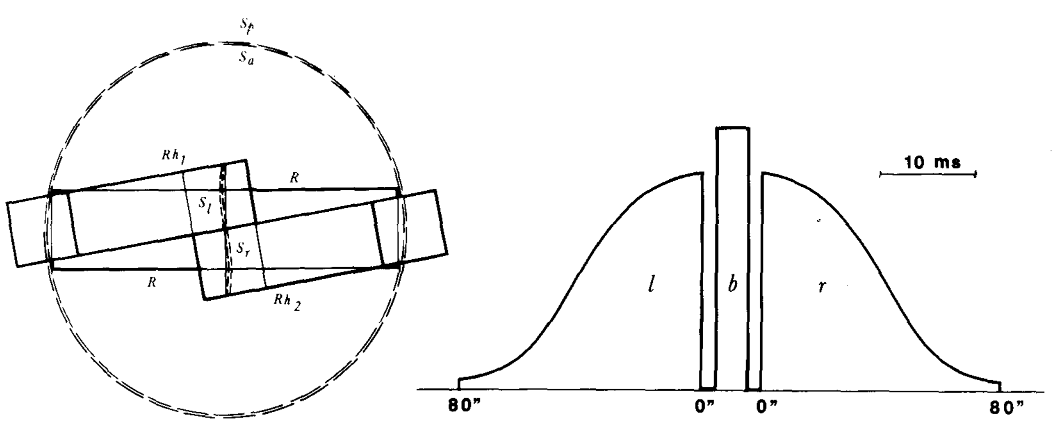

Fig. 2. *Left*: the solar image, as seen looking to the sky. S_p and S_a are the solar images at perihelion and aphelion; R is the CERVIT rod; Rh_1 and Rh_2 are the rhombohedra; S_l and S_r are the portions of the limb as seen through the rhombohedra. *Right*: the profile of a complete scan, on left limb l, calibration beam b, and right limb r, with scales in arc and time seconds.

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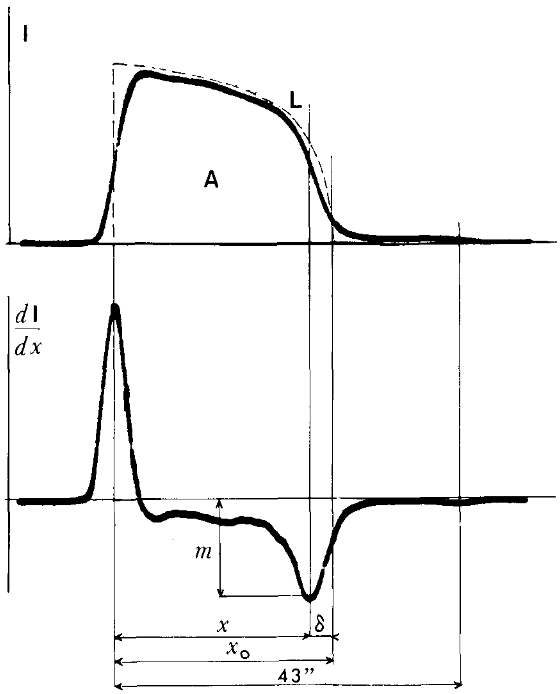

Fig. 3. Oscilloscope tracings of the signal (top) and of its first derivative (bottom), showing the definitions of the abscissae of points of maximum gradient on the true limb L (x_0) and on the blurred limb (x) (δ is the shift), of the maximum gradient itself m, and of the integral flux $\mathscr{A}$.

The upper part of Figure 3 shows a real CRO tracing on one limb. As the length of the slit, parallel to the limb, does not exceed 1″, local brightness fluctuations at the granulation scale can be detected; some are visible on the tracing presented, and consecutive tracings do confirm their solar nature.

Of course, we thought about using diode arrays instead of scanning. But we gave up the idea, at least for the moment, for two reasons: first, we considered that a classical photometric device where a Fabry lens forms a fixed pupil onto an extended cathode whatever the point treated on the image was more reliable than a detector containing a large number of discrete elements which should be carefully calibrated individually; and second, to take profit of the optimal angular resolution, given the actual size of the diodes, calls for a very long equivalent focal length, which in turn raises some problems in optical design. Incidentally, it should be noted that Stebbins (1980) used a diode array the individual element of which covered 1 arc-second.

The lower part of Figure 3 refers to the problem of the shift of the inflection point by the image blurring. It shows, added to the CRO display of the signal, the simultaneous tracing of its first derivative, produced by a differentiating circuit (for the effective work, the signal is sampled and the derivative computed). The dashed line superimposed represents what the *true* profile would be; x and x_0 are the abscissae of the true and apparent inflection point (origin at the limit of the scan) and $\delta = x_0 - x$ is the *shift*. For

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each scan are computed m, the maximum of $\mathrm{d}I/I\,\mathrm{d}x$, the abscissa x of this maximum, and the flux $\mathscr{A}$ integrated over the scan. The basic idea is that if the blurring is sufficiently defined by one parameter, both m and δ are functions of this unique parameter, and therefore, through a calibration to be established, the knowledge of m for each scan would give the correction δ for the same one. By a procedure (using $\mathscr{A}$ as an auxiliary quantity) which has already been described (Rösch and Yerle, 1980), we could make daily plots of δ (with arbitrary origin) versus $1/m$, so as to extrapolate towards a non-blurred profile. Figure 4 is an example of such a plot; the abscissae ε are calibrated in FWHM of the Gaussian spreading function which would give the apparent slope m to a classical limb profile. The increase (in absolute value) of δ with increasing ε is evident. The correction computed by the SCLERA group would show, at the same scale, a very small slope, and leave most of the points below it for the large values of ε; this is not surprising, since the wings of a real spreading function are known to decrease much more slowly than a Gaussian, and they should contribute to increase δ when the atmospheric blurring is important.

We know several reasons which contribute to the dispersion of the points around an average curve. The first one is that the sampling interval of the signal is much too coarse; operation with adequate value is just to start now; next to this improvement an iteration process should be applied for a more accurate plotting; and finally, real oscillations of

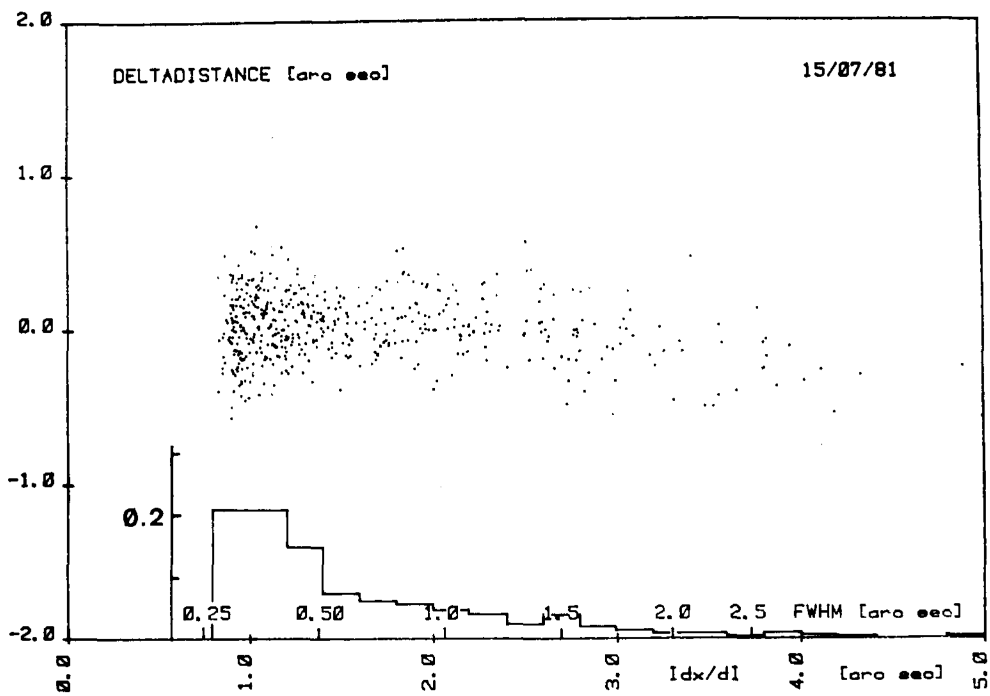

Fig. 4. Shift of the inflection point (arc seconds, arbitrary origin) versus the reciprotical of the maximum brightness gradient. The histogram shows the frequency (in decimal fraction of the total) of the equivalent FWHM in arc-seconds.

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the gradient would influence the diagram, but there are solutions to take that into account. Also some incidental causes of dispersion do exist, as seen on Figure 4, which is a diagram for what we consider as a 'very good day'. The crowding of the points towards low values of ε is conspicuous: as the maximum gradient on the solar limb is not infinite, no points can be observed for $\varepsilon < 0\overset{''}{.}30$. We have other cases where 60% of the points fall below $\varepsilon = 0\overset{''}{.}50$. Anyhow, some features need to be explained, like the increased and asymetrical dispersion for small ε's, or the apparent slight maximum around $\varepsilon = 1''$; we have some reasons to suspect technical and perhaps atmospheric origins, which are being explored.

To summarize this point, work remains to be done for ascerting quantitatively the δ versus m calibration curve; but because of the advantages, which will be developed, of using individual fast scans, we are definitely to pursue it.

6. A Comparison with the SCLERA Experiment

The choice of the SCLERA group is based, on one side, upon their conclusion that the shift of the inflection point could not be corrected, in practice, an on the other side, upon the establishment of a particular algorithm, the Finite Fourier Transform, leading to the definition of the solar edge as the point where it vanishes (FFTD). As said above, any definition of the limb is conventional and permitted: it remains, afterwards, to look after its connections with other definitions, its observational behaviour and its solar significance.

The connection of the FFTD with the definition by the inflection point is clear. A Taylor expansion of the Finite FT integral readily shows that all the coefficients af the odd derivatives vanish, and therefore that both definitions are equivalent at the approximation of the second derivative, and differ only at the level of the *fourth* and higher even ones.

The peculiar property of the FFTD demonstrated by the SCLERA Group is that the zero defined in that way is stationary for the small values of the spreading function: in other words, no 'shift' correction has to be applied, as is the case with the inflection point definition. Indeed, the value published by the SCLERA Group for the solar oblateness found by this method is given with a very small r.m.s. error.

One should notice that the Finite FT is obtained by a scan of period 0.6 s, and that the adjustment to the zero takes much more time. Therefore, a considerable smoothing in time occurs, including image-motion and the fluctuations of the spreading function. It may work. However, one may have a different methodological philosophy, as was once nicely expressed by Lallemand, the father of the electron camera: 'You can either weight a sack of apples as a whole, or weight each apple and compute the sum'. He neglected to add that the great advantage of the second procedure is that you can choose the apples one by one, throw away the bad ones, and weight only the good ones. This applies very easily to our experiment, since for *each* scan, the quality is measured by the maximum slope, and discarding the scans with low slope is fully permitted because they are known to be the bad ones.

Still another great advantage of fast scans with short slit is that any local brightness accident on the Sun appears on the signal. Dicke and Goldenberg have been criticized because their observations integrated the flux over a 30° sector of the limb, so that faculae could induce apparent pole-equator differences. The FFTD method as used at SCLERA integrates over the total scanning amplitude ($\pm 6\overset{''}{.}8$ to $\pm 27\overset{''}{.}2$) and over the length of the slit (100″). A fast scan with short slit allows for discarding not only the tracings having a low maximum gradient, but also those showing a definite brightness excess, like a small facula, at a given position angle of the diameter under measurement.

In fact, the FFTD method and ours can overlap. The drawback of the long time-constant does not result from the FFT definition, but from the technical constraints of its application. By using diode-arrays, the problems of the time-constant and of the length of the slit should vanish. We have explained above why we sticked to direct fast scans, but we do not mean that diode-arrays could not serve in such measurements.

Conversely, we have undertaken to compute FFTDs' from the signals we have sampled and stored, since the analysis of how the FFTD works on a frozen fast scan, and how it compares with the inflection point definition, would be extremely instructive. But it could be still more useful: in his presentation at the Sacramento-Peak Meeting, Hill (1980) mentioned, on the basis of the latest observations of the SCLERA Group (Knapp *et al.*, 1980), that the second derivative $\mathrm{d}^2I/\mathrm{d}r^2$ at the limb is modified in the course of oscillations. Indirectly, we are reaching the same conclusion, since we have found – see below – oscillations in the value of the maximum gradient, which imply changes of the second derivative on both sides of its zero. Again, processing a pure fast scan of the limb could yield more directly the same information as a series of FFTD of varying amplitude.

7. The Solar Stability of These Definitions

As the solar problems we are dealing with imply small variations of the local physical parameters, either in time (oscillations) or from one region to another (e.g. pole/equator) one must question about the individual or combined influence of these parameters upon what is being measured.

It is interesting, at this point, to quote a section of the above mentioned paper by Hill (1980):

'It is important to note that these results (modifications of $\mathrm{d}^2I/\mathrm{d}r^2$) are based on observations rather than theoretical modeling, which has encountered some difficulty (Hill *et al.*, 1978). The above observational based value for the inflection point edge definition is to be contrasted with the much smaller results from theoretical modeling such as those from Rösch and Yerle (1980). This only serves to accent the vulnerability that exists when results from a theoretical analysis are used where theory itself does not properly describe the relevant observational phenomena.'

We can agree completely with the last sentence, and we cannot be surprised that the theoretical modeling by Hill *et al.* has encountered some difficulty. Precisely, what we did was to start from the HSRA model, which is semi-empirically built to 'properly describe' a large bulk of 'relevant observational' data, and then to introduce *small departures* from this model and see what happens to the limb.

Up to now, we have only modified the temperature distribution with height; thus, we found the differential coefficient $\Delta r/\Delta T$ (the drift of the inflection point with temperature) which appears to be quite small ($\simeq 0\overset{''}{.}006$ per $100°$). We have undertaken to compute the differential coefficient $\Delta r/\Delta \rho$ (the drift with density) which may well be larger and reconcile our results with the SCLERA ones. Of course, differential coefficients are computed for the brightness at any part of the limb, and will serve in the interpretation of the limb profile variations in terms of changes in the local physical parameters.

8. Fluctuations of the Maximum Brightness Gradient

During preliminary observations on *one limb* to calibrate the δ versus m correlation, we currently obtained daily series of several thousands of scans, over up to ten consecutive hours, showing oscillatory variations of the maximum brightness gradient (Yerle, 1981). Obviously, such oscillations could be due as well to fluctuations of the atmospheric spreading function as to real solar oscillations. Anyhow, the fact that part of the effect, at least, should be solar is to be expected since it converges with the results obtained by the SCLERA Group and by the global Doppler experiments. From this point of view, these observations provide a new approach to the general problem of solar oscillations, in detecting them through a directly observable parameter which had not yet been taken profit of, and which could give a new insight into the fluctuations of the physical conditions in the solar layers which contribute to the limb profile. On another hand, the discrimination between solar and terrestrial contributions, on this specific parameter, could be very useful in the ever lasting discussion of the subject.

A peculiar feature in these observations, as compared to others, is that, as they refer to a very limited length along the limb (less than 1″, as compared to 100″ for the

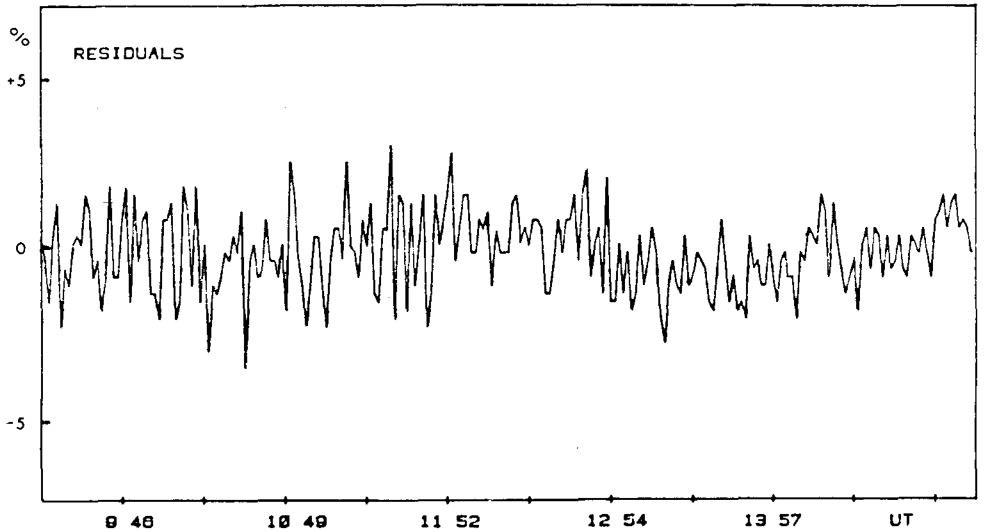

Fig. 5. Residuals of the maximum gradient (after compensation for the daily trend).

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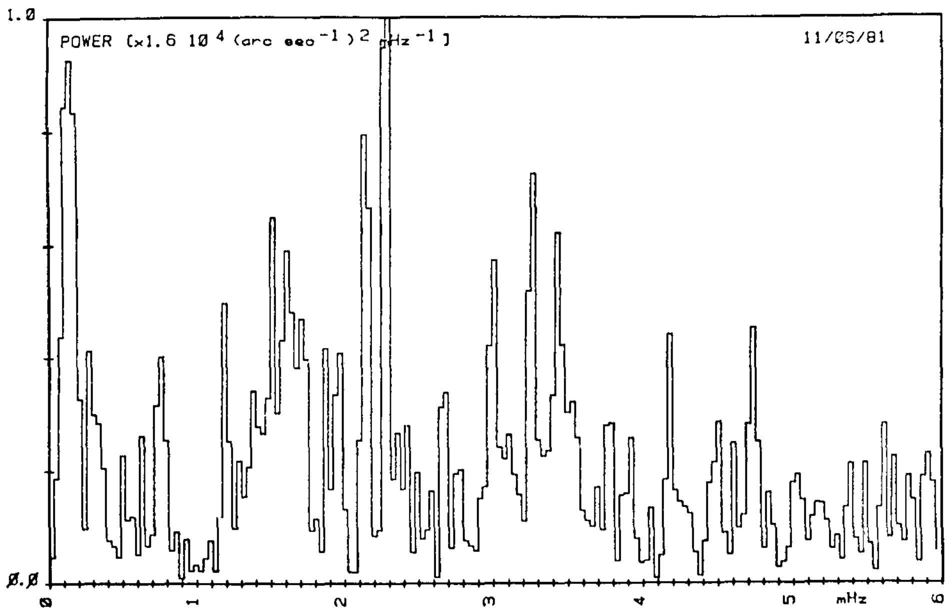

Fig. 6. Power spectrum of fluctuations of the maximum brightness gradient (observations over 8 hr).

SCLERA instrument) they practically integrate all the oscillating modes without filtering.

Figure 5 shows the residuals in *m* after correction for the trend due to the variation of the spreading function both with zenith distance and with the increasing image deterioration in day-time. A period of the order of the well-known 160 min oscillation, and which seems to be in phase with it, is readily visible, as well as fluctuations of much shorter period. Power spectra computed from data of that type show a number of peaks, as seen, for example, on Figure 6, particularly in the 2–4 mHz region. Because of the already mentioned integration of all modes, conclusions will be possible only on the basis of a large number of spectra.

9. Secular Oscillations and Angular Solar Diameter

The experiment we have undertaken as well as the SCLERA one offers a possibility for detection of secular changes of the solar diameter under the condition of an accurate *angular* scaling. Hill (1980) suggested to use as a reference the diffraction angle of a sharply monochromatic radiation by a grating the groove spacing of which has been controlled by interferences of the same radiation. One may express as a more general principle that, an angle being defined by the ratio of two lengths, it should be accurately known if both are measured in units of the same wave-length. Long ago we have been considering several designs following such a principle. The choice has to be done according to various practical problems arising for each of them.

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