RELATION MODULES OF AMALGAMATED FREE PRODUCTS AND HNN EXTENSIONS

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0. Introduction. Let G be a group and

$$1 \to R \to F \xrightarrow{\pi} G \to 1 \tag{1}$$

a free presentation of G, i.e. a short exact sequence of groups with F free. Conjugation in F induces on $\bar{R} = R/R'$, the abelianized normal subgroup R, the structure of a right G-module (if $r \in R$, $x \in F$ then $(rR')(x\pi) = x^{-1}rxR'$). The G-module $\bar{R}$ is called the relation module determined by the presentation (1). For a detailed discussion of this subject we refer to Gruenberg [**3**].

In this note, we study relation modules of free products with amalgamation and HNN extensions. We will assume that these group-theoretic constructions are given by certain canonical free presentations which are obtained from the defining data (free factors and amalgamated subgroup, base group and associated subgroups, respectively) in a natural way. In Section 1, we prove the exactness of a short sequence, characterizing the relation module of an amalgamated free product in terms of the relation modules of the free factors and the augmentation ideal of the amalgamated subgroup (Theorem 1). A similar result for HNN extensions (Theorem 2) is proved in Section 2. The short exact sequence obtained in Theorem 2 expresses the relation module of an HNN extension in terms of the relation module of the base group and the augmentation ideal of the associated subgroups. As an application of our results, we obtain the well-known Mayer–Vietoris sequence for the (co)homology of a free product with amalgamation and a similar sequence for HNN extensions. (This sequence is due to Bieri [**1**].)

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1. Free products with amalgamation. Let $G = G_1 *_U G_2$ be the free product of groups G_1, G_2 with amalgamated subgroup U and suppose that G_1, G_2 are given by free presentations

$$1 \to R_i \to F_i \xrightarrow{\pi_i} G_i \to 1 \qquad (i = 1, 2). \tag{2}$$

Then there is a canonical free presentation

$$1 \to R \to F_1 * F_2 \xrightarrow{\pi} G_1 *_U G_2 \to 1 \tag{3}$$

of G, where $F_1 * F_2$ is the free product of F_1 and F_2, and the epimorphism π is defined by

$$\pi|_{F_1} = \pi_1, \qquad \pi|_{F_2} = \pi_2.$$

We denote the relation modules associated with (2), (3) by $\bar{R}_i$ $(i = 1, 2)$ and $\bar{R}$, respectively. As usual, $\mathbb{Z}G$ denotes the integral group ring of G and IG the augmentation ideal, i.e. the kernel of the canonical homomorphism $\mathbb{Z}G \to \mathbb{Z}$.

THEOREM 1. *There is a short exact sequence of G-modules*

$$0 \to (\bar{R}_1 \otimes_{G_1} \mathbb{Z}G) \oplus (\bar{R}_2 \otimes_{G_2} \mathbb{Z}G) \to \bar{R} \to IU \otimes_U \mathbb{Z}G \to 0.$$

The proof will be given later in this section. Now let A be a right and B a left G-module. As a consequence of Theorem 1 we obtain the following result.

COROLLARY 1. *There are long exact sequences*

$$\ldots \to H^{n+1}(U, A) \to H^{n+2}(G, A) \to H^{n+2}(G_1, A) \oplus H^{n+2}(G_2, A) \to H^{n+2}(U, A) \to \ldots,$$
$$\ldots \to H_{n+2}(U, B) \to H_{n+2}(G_1, B) \oplus H_{n+2}(G_2, B) \to H_{n+2}(G, B) \to H_{n+1}(U, B) \to \ldots.$$

These are the well-known Mayer–Vietoris sequences for the (co)homology of an amalgamated free product (see, e.g., [**6**, p. 30]).

Proof of Corollary 1. Using the change of rings (see [**4**, p. 164, Proposition 12.2]) and the reduction theorems (see [**4**, p. 199, Corollary 6.5]), one gets the following isomorphisms:

$$\begin{aligned}
&\mathrm{Ext}^n_G(\bar{R}, A) = H^{n+2}(G, A) && (n \geq 1),\\
&\mathrm{Ext}^n_G(IU \otimes_U \mathbb{Z}G, A) = \mathrm{Ext}^n_U(IU, A) = H^{n+1}(U, A) && (n \geq 2),\\
&\mathrm{Ext}^n_G(\bar{R}_i \oplus_{G_i} \mathbb{Z}G, A) = \mathrm{Ext}^n_{G_i}(\bar{R}_i, A) = H^{n+2}(G_i, A) && (n \geq 1).
\end{aligned}$$

Applying $\mathrm{Ext}^n_G(-, A)$ to the short exact sequence of Theorem 1 together with these isomorphisms yields the first sequence. The exactness of the second sequence is proved in a similar way.

Now we list some facts which will be used later.

LEMMA 1. *Let a free presentation*

$$1 \to N \to Q \xrightarrow{\eta} H \to 1 \tag{4}$$

of a group H be given. Then

$$0 \to \bar{N} \xrightarrow{\kappa} IQ \otimes_Q \mathbb{Z}H \xrightarrow{\nu} IH \to 0,$$

where $(nN')\kappa = (n-1) \otimes 1_H$ *and* $((1-x) \otimes 1_H)\nu = 1 - (x\eta)$ $(n \in N,\ x \in Q)$ *is an exact sequence of right H-modules.*

A proof of this lemma can be found, e.g., in [**4**, p. 198, Theorem 6.3]. The exact sequence in Lemma 1 is usually referred to as the relation sequence stemming from the free presentation (4).

LEMMA 2. *Let* $H = \mathop{*_V}\limits_{i=1}^{n} H_i$ *be the free product of* $H_1, \ldots, H_n$ *with amalgamated subgroup V, i.e. the push-out of a family of embeddings* $\varphi_i : V \to H_i$ $(i = 1, \ldots, n)$ *in the* *category of groups. Then the augmentation ideal IH is the push-out of the family $\{IV\otimes_V \mathbb{Z}H \to IH_i \otimes_{H_i} \mathbb{Z}H\}$ in the category of H-modules.*

In other words, augmentation is a push-out preserving functor. For a proof of Lemma 2, see [**2**, p. 141, Proposition 8].

LEMMA 3. *Let $H = \mathop{*}\limits_{i=1}^{n} H_i$ be the free product of $H_1, \ldots, H_n$. Then the map*

$$\Phi : \bigoplus_{i=1}^{n} (IH_i \otimes_{H_i} \mathbb{Z}H) \to IH$$

given by

$$\bigoplus_{i=1}^{n} ((h_i - 1) \otimes k_i) \mapsto \sum_{i=1}^{n} (h_i - 1)k_i, \quad h_i \in H_i, \quad k_i \in H,$$

is a natural isomorphism of right H-modules.

A proof of this lemma can be found in [**2**, p. 140, Proposition 7].

Proof of Theorem 1. Let $F = F_1 * F_2$ and consider the following diagram.

$$\begin{array}{ccccccccc}
 & & & & 0 & & 0 & & \\
 & & & & \downarrow & & \downarrow & & \\
 & & & & (\bar{R}_1 \otimes_{G_1} \mathbb{Z}G) \oplus (\bar{R}_2 \otimes_{G_2} \mathbb{Z}G) & & 0 & & \\
 & & & & \downarrow & & \downarrow & & \\
 & & & & (IF_1 \otimes_{F_1} \mathbb{Z}G) \oplus (IF_2 \otimes_{F_2} \mathbb{Z}G) & \xrightarrow{\Psi} & IF \otimes_F \mathbb{Z}G & & \\
 & & & & \downarrow{\scriptstyle \psi} & & \downarrow{\scriptstyle \nu} & & \\
0 & \longrightarrow & IU \otimes_U \mathbb{Z}G & \xrightarrow{\mu} & (IG_1 \otimes_{G_1} \mathbb{Z}G) \oplus (IG_2 \otimes_{G_2} \mathbb{Z}G) & \xrightarrow{\varepsilon} & IG & \longrightarrow & 0 \\
 & & & & \downarrow & & \downarrow & & \\
 & & & & 0 & & 0 & &
\end{array}$$

By Lemma 1,

$$0 \to \bar{R}_i \to IF_i \otimes_{F_i} \mathbb{Z}G_i \to IG_i \to 0 \qquad (i = 1, 2)$$

is exact. Tensoring with $\mathbb{Z}G$ over G_i yields the exact sequence

$$0 \to \bar{R}_i \otimes_{G_i} \mathbb{Z}G \to IF_i \otimes_{F_i} \mathbb{Z}G \to IG_i \otimes_{G_i} \mathbb{Z}G \to 0$$

($\mathbb{Z}G$ is a free $\mathbb{Z}G_i$-module). Hence, the first column is exact. The second column is the relation sequence for (3) and therefore exact. Using Lemma 3, one gets an isomorphism

$$\Phi : (IF_1 \otimes_{F_1} \mathbb{Z}F) \oplus (IF_2 \otimes_{F_2} \mathbb{Z}F) \to IF.$$

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Applying $-\otimes_F \mathbb{Z}G$ to this yields the isomorphism $\bar{\Psi}$. It is given by

$$\bar{\Psi}:((f_1-1)\otimes g_1)\otimes((f_2-1)\otimes g_2))\rightarrow(f_1-1)\otimes g_1+(f_2-1)\otimes g_2,$$

where $f_i \in F_i$, $g_i \in G$.

By Lemma 2, IG is the push-out of the family $\{\varphi_i\otimes 1: IU\otimes_U \mathbb{Z}G\rightarrow IG_i\otimes_{G_i}\mathbb{Z}G\}$. Hence, the bottom row is exact (see [**2**, p. 142]). Here μ is defined as $\{\varphi_1\otimes 1, -(\varphi_2\otimes 1)\}$, and ε is given by

$$\varepsilon:(((g_1-1)\otimes h_1)\oplus((g_2-1)\otimes h_2))\mapsto(g_1-1)h_1+(g_2-1)h_2,$$

where $g_i \in G_i$, $h_i \in G$.

One has, obviously, $\bar{\Psi}\nu=\psi\varepsilon$. Hence,

$$\bar{R}\cong\ker\nu\cong\ker\psi\varepsilon.$$

Now the exactness of the first column and the bottom row imply that there is an exact sequence

$$0\rightarrow(\bar{R}_1\otimes_{G_1}\mathbb{Z}G)\oplus(\bar{R}_2\otimes_{G_2}\mathbb{Z}G)\rightarrow\ker\psi\varepsilon\rightarrow IU\otimes_U\mathbb{Z}G\rightarrow 0$$

and this completes the proof of the theorem.

Theorem 1 can easily be generalized to the case of more than two free factors, i.e. if G is the free product of groups $G_1, \ldots, G_n$ with amalgamated subgroup U, the G_i are given by free presentations with associated relation modules $\bar{R}_1, \ldots, \bar{R}_n$, and the presentation of G under consideration is constructed in the obvious way, then the result reads as follows.

THEOREM 1′. *There is a short exact sequence*

$$0\rightarrow\bigoplus_{i=1}^{n}(\bar{R}_i\otimes_{G_i}\mathbb{Z}G)\rightarrow\bar{R}\rightarrow\bigoplus_{i=1}^{n-1}IU\otimes_U\mathbb{Z}G\rightarrow 0.$$

The proof of Theorem 1 can be carried over without any difficulties, replacing the direct sums of two modules in the diagram by direct sums of the corresponding n modules. The exact sequence

$$0\rightarrow\bigoplus_{i=1}^{n-1}(IU\otimes_U\mathbb{Z}G)\rightarrow\bigoplus_{i=1}^{n}(IG_i\otimes_{G_i}\mathbb{Z}G)\rightarrow IG\rightarrow 0$$

replacing the bottom row can be found in [**2**, p. 142].

2. HNN extensions. Let G be a group given by a free presentation (1) and let A and B be subgroups of G with $\varphi: A\rightarrow B$ an isomorphism. Let

$$G^*=\langle G, t; t^{-1}at=a\varphi\ (a\in A)\rangle$$

be the HNN extension of G with stable letter t and associated subgroups A, B (see [**5**, Chapter IV.2, p. 178ff.] concerning the definition and standard notations). Let

$F^* = F * \langle t \rangle$ be the free product of F and the infinite cyclic group generated by t. Then

$$1 \to R^* \to F^* \xrightarrow{\pi^*} G^* \to 1, \tag{5}$$

where π^* is given by

$$\pi^*|_F = \pi, \qquad t\pi^* = t,$$

is a free presentation of G^*. Let $\bar{R}$, $\overline{R^*}$ denote the relation modules determined by (1), (5), respectively.

THEOREM 2. *There is an exact sequence of G^*-modules*

$$0 \to \bar{R} \otimes_G \mathbb{Z}G^* \to \overline{R^*} \to IB \otimes_B \mathbb{Z}G^* \to 0.$$

Now let C be a right and D a left G^*-module. Again, by applying the (co)homology functor as in Section 1, one easily gets the following corollary.

COROLLARY 2 (Bieri [**1**]). *There are long exact sequences*

$$\ldots \to H^{n+1}(B, C) \to H^{n+2}(G^*, C) \to H^{n+2}(G, C) \to H^{n+2}(B, C) \to \ldots,$$
$$\ldots \to H_{n+2}(B, D) \to H_{n+2}(G, D) \to H_{n+2}(G^*, D) \to H_{n+1}(B, D) \to \ldots.$$

In preparation for the proof of Theorem 2, we now start with some preliminary considerations. Recall that a normal form for the HNN extension G^* is a sequence

$$g_0, t^{\varepsilon_1}, \ldots, t^{\varepsilon_n}, g_n \qquad (n \geq 0),$$

where $\varepsilon_i = \pm 1$ and

(i) g_0 is an arbitrary element of G,

(ii) if $\varepsilon_i = -1$ then $g_i \in \mathscr{A}$, where $\mathscr{A}$ is a fixed system of representatives of the right cosets of A in G such that 1 is the representative of A,

(iii) if $\varepsilon_i = +1$ then $g_i \in \mathscr{B}$, where $\mathscr{B}$ is a fixed system of representatives of the right cosets of B in G such that 1 is the representative of B, and

(iv) there is no consecutive subsequence t^ε, 1, $t^{-\varepsilon}$.

The Normal Form Theorem for HNN extensions (see [**5**, p. 182]) states that any element ω of G^* has a unique representation as $\omega = g_0 t^{\varepsilon_1} \ldots t^{\varepsilon_n} g_n$, where $g_0, t^{\varepsilon_1}, \ldots, t^{\varepsilon_n}, g_n$ is a normal form. From now on we will not distinguish between a normal form and the corresponding element of G^*. It will be clear from the context what is actually meant.

LEMMA 4. *Let G^* be as above. Then the intersection of the right ideals generated by IG and $(t-1)$, respectively, in $\mathbb{Z}G^*$ is the right ideal generated by $(t-1)$. IB, i.e.*

$$IG \,.\, \mathbb{Z}G^* \cap (t-1) \,.\, \mathbb{Z}G^* = (t-1) \,.\, IB \,.\, \mathbb{Z}G^*.$$

Proof. Let Γ denote the set of all normal forms with $g_0 = 1$, Γ^- the set of normal forms with $g_0 = 1$ and $\varepsilon_1 = -1$, and Γ^+ the complement of Γ^- in Γ. For $\gamma = t^{-1} g_1 t^{\varepsilon_2} \ldots t^{\varepsilon_n} g_n \in \Gamma^-$, we define

$$\gamma^- = t^{\varepsilon_2} \ldots t^{\varepsilon_n} g_n, \qquad g(\gamma) = g_1.$$

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We define a partial ordering on Γ as follows: $\gamma_1 > \gamma_2$ if and only if there is a $\gamma_3 \in \Gamma$ such that $\gamma_1 = \gamma_3 \cdot \gamma_2$.

Now we can state the following obvious facts.

(I) Every element $\alpha \in \mathbb{Z}G$ has a unique expression as

$$\alpha = \sum_j \beta_j g_j, \quad \text{where} \quad \beta_j \in \mathbb{Z}B, \qquad g_j \in \mathscr{B}.$$

(II) Every element $\omega \in \mathbb{Z}G^*$ has a unique expression as

$$\omega = \sum_{i,j} \beta_{ij} g_j \gamma_i \tag{6}$$

with $\beta_{ij} \in \mathbb{Z}B$, $g_j \in \mathscr{B}$, $\gamma_i \in \Gamma$.

(III) An element $\omega \in \mathbb{Z}G^*$ is in $IG \, . \, \mathbb{Z}G^*$ if and only if in (6)

$$\sum_j \beta_{ij} g_j \in IG$$

for all i.

Now suppose that we are given an element $\omega \in \mathbb{Z}G^*$ such that

$$(t-1) \, . \, \omega \in IG \, . \, \mathbb{Z}G^*.$$

We denote by Γ_0 the set of all $\gamma \in \Gamma$ such that γ occurs with non-zero coefficient in (6). Then we have, by using the HNN relations $tb = b\varphi^{-1}t$ $(b \in B)$,

$$(t-1) \sum_{i,j} \beta_{ij} g_j \gamma_i = \sum (\beta_{ij}\varphi^{-1})(tg_j\gamma_i) + \sum (\beta_{ij}\varphi^{-1}) g(\gamma_i)\gamma_i^- - \sum_{i,j} \beta_{ij} g_j \gamma_i, \tag{7}$$

where the first sum runs over all i, j with $g_j \neq 1$ or $\gamma_i \notin \Gamma^-$, and the second sum runs over all i, j with $g_j = 1$ and $\gamma_i \in \Gamma^-$. φ denotes the ring isomorphism $\varphi : \mathbb{Z}A \to \mathbb{Z}B$, induced by the group isomorphism $\varphi : A \to B$.

Let γ_0 be maximal in Γ_0 with respect to the partial ordering introduced above. Then, if (case 1) γ_0 occurs in the first sum, $(tg_j\gamma_0)$ is maximal among the normal forms occurring in (7). Consequently, by (III), we have $\beta_{0j}\varphi^{-1} \in IG$ and this implies $\beta_{0j} \in IB$. If (case 2) γ_0 occurs in the second sum then the normal form (γ_0) occurs with coefficient $-\sum_j \beta_{0j} g_j$ in (7), because all normal forms $(tg_j\gamma_i)$ occurring in the first sum are in Γ^+ and γ_0 is greater than all normal forms occurring in the second sum. Hence, by (III), $\sum_j \beta_{0j} g_j \in IG$. But, by case 1, all β_{0j} with $g_j \neq 1$ are in IB. Consequently, all $\beta_{0j} \in IB$.

Now we can apply induction on the number of elements of Γ_0, and this completes the proof.

Proof of Theorem 2. Denote by T the infinite cyclic group generated by t and consider the following diagram.

$$
\begin{array}{ccccccccc}
 & & & & 0 & & 0 & & \\
 & & & & \downarrow & & \downarrow & & \\
 & & & & \bar{R} \otimes_G \mathbb{Z}G^* & & \bar{R}^* & & \\
 & & & & \downarrow & & \downarrow & & \\
 & & & & (IF \otimes_F \mathbb{Z}G^*) \oplus (IT \otimes_T \mathbb{Z}G^*) & \xrightarrow{\Psi} & IF^* \otimes_{F^*} \mathbb{Z}G^* & & \\
 & & & & \downarrow{\scriptstyle \psi} & & \downarrow & & \\
0 & \longrightarrow & \ker \varepsilon & \xrightarrow{\mu} & (IG \otimes_G \mathbb{Z}G^*) \oplus (IT \otimes_T \mathbb{Z}G^*) & \xrightarrow{\varepsilon} & IG^* & \longrightarrow & 0 \\
 & & & & \downarrow & & \downarrow & & \\
 & & & & 0 & & 0 & &
\end{array}
$$

By Lemma 1, $0 \to \bar{R} \to IF \otimes_F \mathbb{Z}G \to IG \to 0$ is exact. Tensoring with $\mathbb{Z}G^*$ over $\mathbb{Z}G$ yields the exactness of the first column ($\mathbb{Z}G^*$ is a free $\mathbb{Z}G$-module). The second column is the relation sequence for (5) and therefore exact. Using Lemma 3, one gets an isomorphism

$$\Phi : (IF \otimes_F \mathbb{Z}F^*) \oplus (IT \otimes_T \mathbb{Z}F^*) \to IF^*.$$

Applying $- \otimes_{F^*} \mathbb{Z}G^*$ to this yields the isomorphism Ψ. It is given by

$$\Psi : (((f-1) \otimes f_1^*) \oplus ((t-1) \otimes f_2^*)) \mapsto (f-1) \otimes f_1^* + (t-1) \otimes f_2^*,$$

where $f \in F$, $f_i^* \in F^*$.

Obviously, the bottom row is exact. Here ε is given by

$$\varepsilon : (((g-1) \otimes g_1^*) \oplus ((t-1) \otimes g_2^*)) \mapsto (g-1)g_1^* + (t-1)g_2^*,$$

where $g \in G$, $g_i^* \in G^*$.

Since $\Psi v = \psi \varepsilon$, one gets

$$\overline{R^*} \cong \ker v \cong \ker \psi\varepsilon.$$

Now the exactness of the first column and the bottom row imply that there is an exact sequence

$$0 \to \bar{R} \otimes_G \mathbb{Z}G^* \to \overline{R^*} \to \ker \varepsilon \to 0.$$

Consider $\varepsilon_1 : IG \otimes_G \mathbb{Z}G^* \to IG^*$ and $\varepsilon_2 : IT \otimes_T \mathbb{Z}G^* \to IG^*$, given by $\varepsilon_1 : ((g-1) \otimes g^*) \mapsto (g-1)g^*$ and $\varepsilon_2 : ((t-1) \otimes g^*) \mapsto (t-1)g^*$, respectively. Since ε_1 and ε_2 are injective, one gets $\ker \varepsilon = (\operatorname{Im} \varepsilon_1) \cap (\operatorname{Im} \varepsilon_2)$; hence

$$\ker \varepsilon \cong (IG \,.\, \mathbb{Z}G^*) \cap (t-1) \,.\, \mathbb{Z}G^*.$$

By Lemma 4, $\ker \varepsilon \cong (t-1) \,.\, IB \,.\, \mathbb{Z}G^*$. So it remains to show that $IB \otimes_B \mathbb{Z}G^*$ and $(t-1) \,.\, IB \,.\, \mathbb{Z}G^*$ are isomorphic as right G^*-modules.

The canonical map $IB \otimes_B \mathbb{Z}G^* \to IB \,.\, \mathbb{Z}G^*$, given by $(b-1) \otimes g^* \mapsto (b-1)g^*$, where $b \in B$, $g^* \in G^*$, is obviously an isomorphism. Now consider the map $\tau : \mathbb{Z}G^* \to$

$\mathbb{Z}G^*$, where τ is given by $\tau: g^* \mapsto (t-1)g^*$. The restriction of τ to $IB \,.\, \mathbb{Z}G^*$ yields an epimorphism $IB \,.\, \mathbb{Z}G^* \to (t-1) \,.\, IB \,.\, \mathbb{Z}G^*$. We claim that τ is injective. Then the restriction of τ to $IB \,.\, \mathbb{Z}G^*$ is also injective, and we are done.

To establish the claim, suppose that we are given $\alpha \in \mathbb{Z}G^*$ such that $(t-1) \,.\, \alpha = 0$, i.e. $\alpha = t^k \alpha$ for every integer k. Express α as

$$\alpha = \sum_j n_j \,.\, \gamma_j, \qquad n_j \in \mathbb{Z}, \qquad \gamma_j \in G^*. \tag{8}$$

Let g_0 be an element of G^* occurring in (8) with non-zero coefficient. The elements g_0, tg_0, $t^2 g_0, \ldots$ are pairwise distinct. Since only a finite number of elements of G^* occur in (8) with non-zero coefficient, there exists an integer k such that $t^k g_0$ and g_0 have distinct coefficients in (8). But this implies $\alpha \neq t^k \alpha$ for some k. Hence, τ is injective, and this completes the proof of Theorem 2.

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