STOCHASTICITY OF THE ORBIT OF THE ASTEROID 1989 AC

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ABSTRACT

Using mainly Lyapunov Characteristic Indicators, the
stochasticity of the orbit of the asteroid 1989 AC has been
investigated in the framework of three-body, five-body and
seven-body restricted models. From our first results, the
orbit of 1989 AC has been found fairly chaotic, through an
interplay of the 3/1 resonance with Jupiter and close appro-
aches with the inner planets.

1. INTRODUCTION

Numerous extensive studies on dynamics of planet-crosser
asteroids (for example by Milani et al., 1989) have shown a
great diversity among the computed orbits; this diversity re-
flects evidently the chaotic nature of this kind of dynamics.
Close approaches with the planets are now known to play a
great role in the coming out of the chaos. Our aim here is
to study the origin of the stochasticity for a recently dis-
covered asteroid, 1989 AC (lately numbered and named 4179
Toutatis), nowadays near the 3/1 resonance with Jupiter.

For this purpose, we compute the so-called Lyapunov Cha-
racteristic Indicators, quantities related to the now well-
known theory of Lyapunov Characteristic Exponents, which pro-
vide a quantitative measure of the stochasticity for a given
orbit. For the numerical integration of the orbit, we have
used first a seven-body model (Sun-Venus-Earth-Mars-Asteroid-
Jupiter-Saturn). Then, in order to trace back the origin of
the stochasticity, we have also computed truncated planetary
models: the first is the classical restricted three-body model

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(Sun-Jupiter-Asteroid), when the second model takes into account only the perturbations due to -and thus the encounters with- the three Inner Planets cited above (Venus-Earth-Mars). We present here our first results.

2. THE LYAPUNOV CHARACTERISTIC INDICATORS

It is well known since the classical study of Henon and Heiles (1964; see also Henon, 1981) that two orbits initially close diverge either linearly or exponentially depending on whether the initial points lie in an integrable or in a stochastic region of the phase space. This property has been extensively used as an indicator of stochasticity, through the socalled Lyapunov Characteristic Exponents (hereafter called the LCE's); initially introduced by Lyapunov for periodic orbits, and further developed by Oseledec, they indicate quantitatively how fast nearby orbits diverge and thus the degree of unpredictability of such orbits.

Generally, speaking, the LCE's are non-uniform integrals of motion; therefore, if the dynamical system is ergodic, all the LCE's are almost everywhere constant; a contrario, if all the LCE's are not almost everywhere constant, then the dynamical system is certainly not ergodic. Practically, when we compute the LCE's, if the maximal LCE tends to zero for $t \to \infty$, then the system is integrable; but if the maximal LCE tends to be constant for $t \to \infty$ (and a fortiori if several other LCE's have the same behaviour), then the dynamical system is in the ergodic region (for more details, see Froeschle, 1984, 1990).

We must keep in mind that the LCE's are stricto sensu limiting values at $t \to \infty$, which is evidently impossible to compute practically. We then define the Lyapunov Characteristic Indicators (hereafter called the LCI's) as the truncated values of the LCE's for a finite time t_M. When the largest LCI has a finite non-zero value at t_M, we may consider the orbit as stochastic with good confidence; but when it does not show any tendency to stop its monotonous decrease down to zero, we only may suppose that it has a given probability (depending on t_M) to be integrable. Previous computations have shown that a fairly good compromise between accuracy and reasonable computation time is 10^4 years $< t_M < 10^5$ years; nevertheless, when we suspect that a given orbit, a given orbit, although looking integrable at t_M, could be in fact chaotic, we can continue the computation to greater values of t_M.

3. RESULTS

The asteroid 1989 AC was discovered with the 0.9 m Schmidt Telescope at Caussols (Observatorire de la Cote d'Azur) January the 4th. The fast-moving object was soon (I.A.U. Circular 4715, 1989 Jan. 13) suspected to be at or near the 3/1 resonance with Jupiter. For our study, we have taken as initial conditions the elements computed by Green (1989).:

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a = 2.499213 AU
e = 0.639804
ω = 274.9901°
i = 0.4650°    (1950.0)
Ω = 127.1834°
T = 1988 Nov. 22.6040
P = 3.951 years = P_JUPITER/3.004
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Elements of the planets at the initial time T were taken from
the Astronomical Ephemeris 1988.
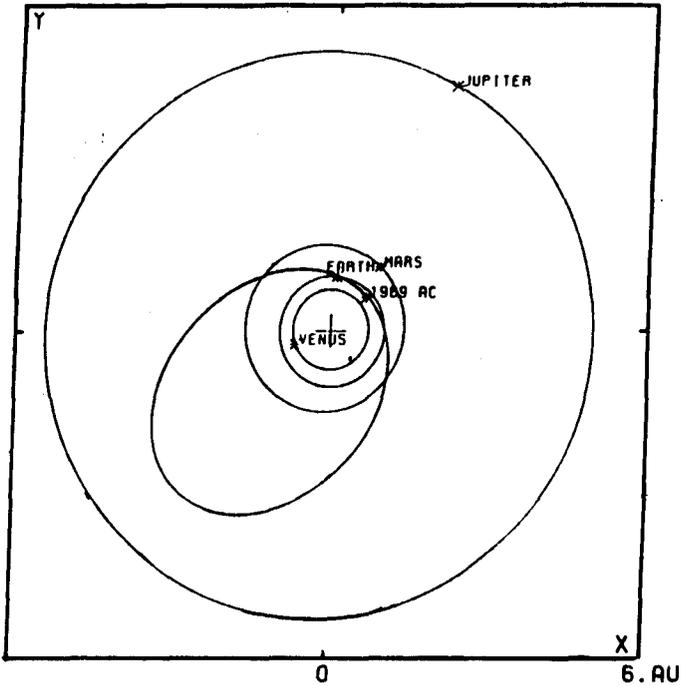
Fig. 1: Heliocentric orbit of Asteroid 1989 AC, together
with Jupiter, Mars, Earth and Venus; the asterisks
indicate the initial positions at epoch T = 1988
Nov. 22.6040.

3.1 The Sun-Jupiter-Saturn-Inner Planets Model

As the osculating orbit of 1989 AC is located between
the orbits of Venus and Jupiter (see Fig. 1), our main compu-
tation is made within the frame of a restricted seven body model,
where the heliocentric orbit of 1989 AC (with an infinitesi-
mal mass) is perturbed by Venus, Earth, Mars, Jupiter and
Saturn; this latter acts rather through its influence on
Jupiter than directly on 1989 AC. All bodies are taken as
point masses, submitted only to the newtonian gravitational

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force law. The equations of motion are integrated using the classical Bulirsch - Stoer method, which, through a controlled variable step size, allows to handle close approaches with planets much more accurately than with a fixed step size, as used for example by Milani et al.(1989).

The evolution of the orbital elements and the LCI's is computed for 50000 years backward and 100000 years forward from the epoch T. The orbit appears to be strongly governed by the 3/1 resonance with Jupiter, as the semi-major axis stays around 2.5 AU during a long time (see Fig. 2); but,due to repeated relatively close approaches with the inner planets, the value of the eccentricity progressively increases, so that finally the asteroidal path plunges catastrophically inside the Sun. As regards the LCI's, the non-zero value of all of them indicates that the orbit is stochastic (see Fig. 3), and that no quasi-integral remains.
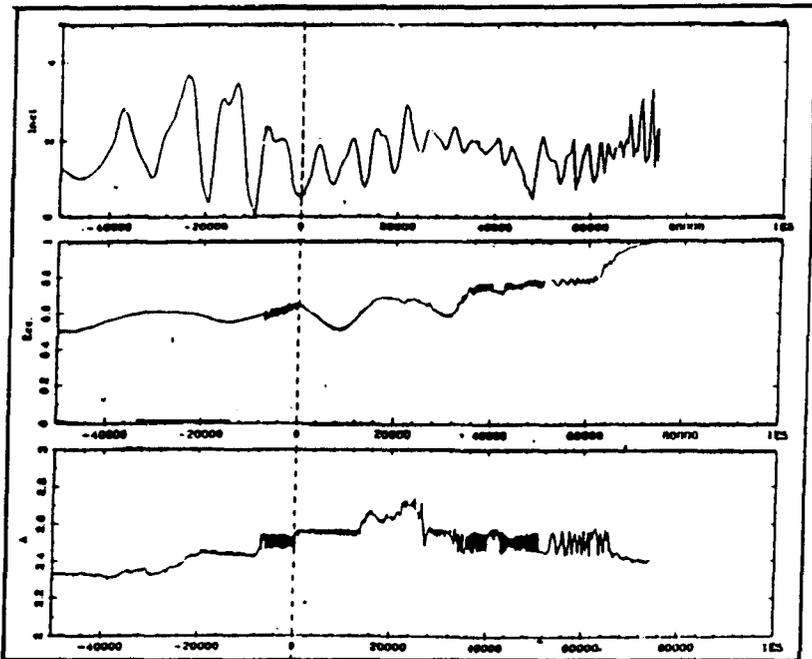
Fig. 2: Evolution of the semi-major axis A and the eccentricity e of the "true" orbit of 1989 AC for 50000 years backward and 100000 years forward;seven-body model, Bulirsch-Stoer integrator;the vertical dashed line indicates the starting date T.

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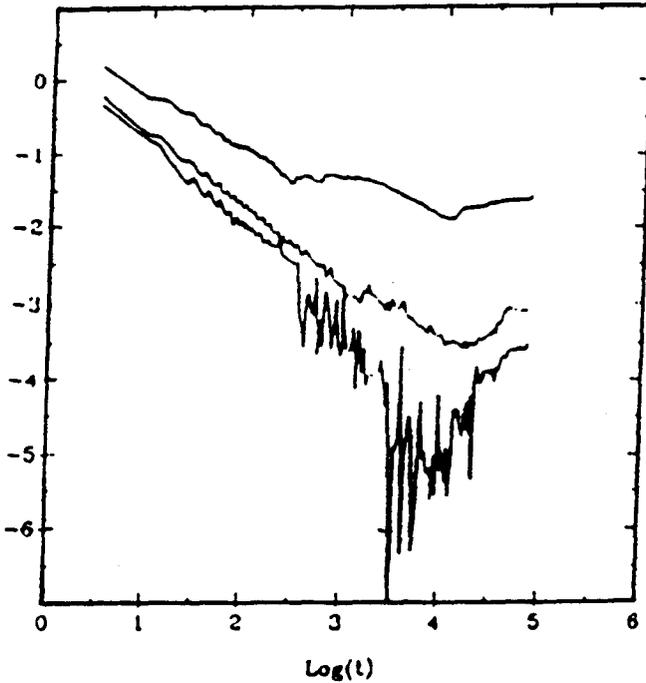
Fig. 3: Variation of the LCI's forward in time for the "true"
       orbit of 1989 AC; seven-body model, Bulirsch-Stoer
       integrator.

        In this numerical experiment, we used the osculating
elements of 1989 AC at the epoch T, with a zero initial value
of the mean anomaly: M_0 = 0. Then, another run was performed
with the same seven-body model for an orbit initially very
close to the former one (the "true" orbit): we changed only
the initial value of the mean anomaly, by half a degree (M_0 =
0.5°). This varied orbit stays in a first stage near the
3/1 resonance with Jupiter, but suffers at around +50000
years a very strong encounter with the Earth, at a distance
of 8×10^{-4} AU; this encounter induces an increase of the
semi-major axis, which allows later two very close encounters
with Jupiter, at distances respectively of 6×10^{-2} AU and
3×10^{-2} AU; a large jump in the semi-major axis up to great
values occurs then, and the asteroidal orbit becomes an
escape trajectory (see Fig. 4).

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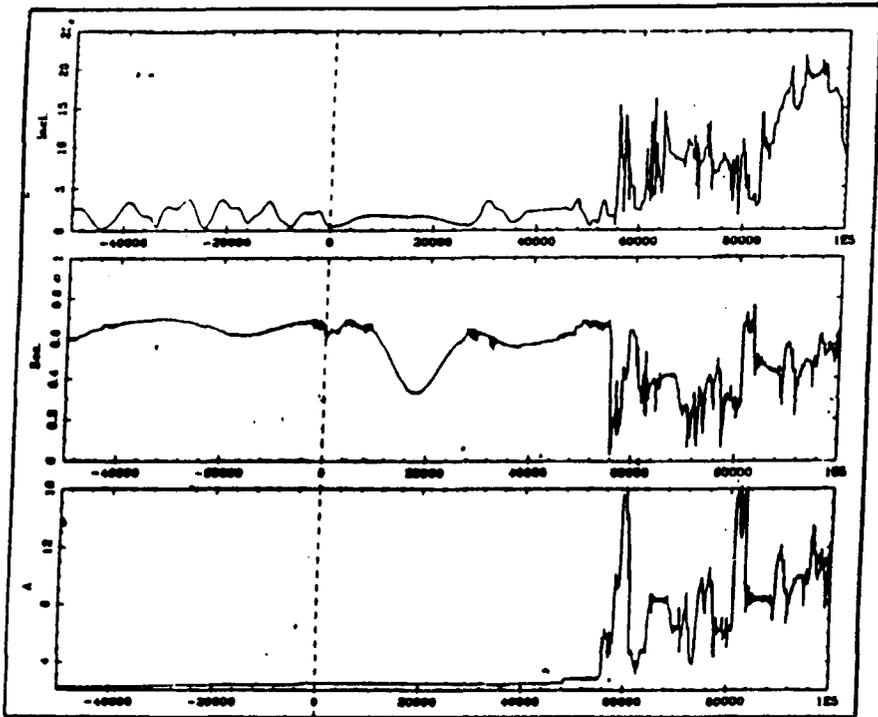
Fig. 4: Evolution of the semi-major axis A and the eccentri-
city e of the varied orbit of 1989 AC; same notations
as Fig. 2.

Therefore, the "true" and the varied orbits, although
initially very close, diverge relatively rapidly and have
very different final destiny; this is the charaxteristic of
a chaotic behaviour, and it not astonishing that the LCI's
of the "true" orbit (see Fig. 3) as well as the LCI's of the
varied orbit (see Fig. 5) indicate this stochasticity. Never-
theless, the LCI's have the same values for the two orbits,
which confirms their validity as quantitative indicators of
the stochasticity.

On the other hand, the different orbital evolutions
of these two initially close orbits show that the motion of
1989 AC is under the combined influences of the 3/1 resonance
with Jupiter and the close approaches and encounters with the
inner planets, which give the chaotic character cited above.
In the following two sub-sections, 3.2 and 3.3, we thus study
separately the influence of the major planet Jupiter, and the
influences of the inner planets, namely Venus, Earth and
Mars.

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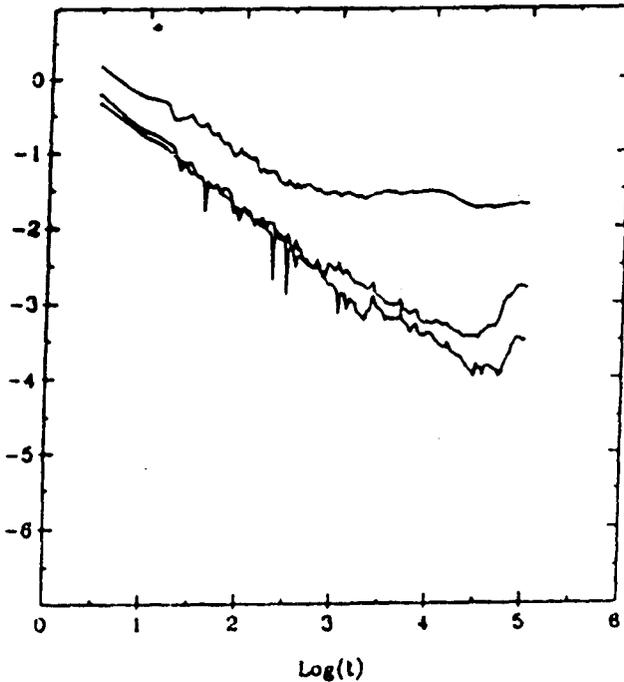
Fig. 5: Variation of the LCI's forward in time for the varied
orbit of 1989 AC; same notations as for Fig. 3.

3.2 The Sun-Jupiter model

The evolution of the orbital elements of 1989 AC, compu-
ted for 200000 years backward and forward using a restricted
three-body model (Sun-Jupiter-Asteroid) with the same inte-
gration technique as for the seven-body model, shows that the
asteroid is locked in the 3/1 resonance with Jupiter, at
least during the considered interval of time (see Fig. 6).The
LCI's do not show any non-zero limit (see Fig.7), as expected
for such a near-integrable case. Therefore, Jupiter alone is
not responsible for any tendency to stochasticity within the
time span of 400000 years.

3.3 The Inner Planets model

In order to track the origin of stochasticity, we have
performed computation using a restricted five-body model (Sun-
Venus-Earth-Mars-Asteroid) with the same integration techni-
que as above. Results (see Figs. 8 and 9) show a fair stoc-
hasticity: the largest LCI is less than the one with the
seven-body integration, and the two others do not show any
tendency to non-zero limit (Fig. 9). Multiple close appro-
aches with the inner planets thus appear to play a non-negli-
gible role in the stochasticity of the orbit of 1989 AC.

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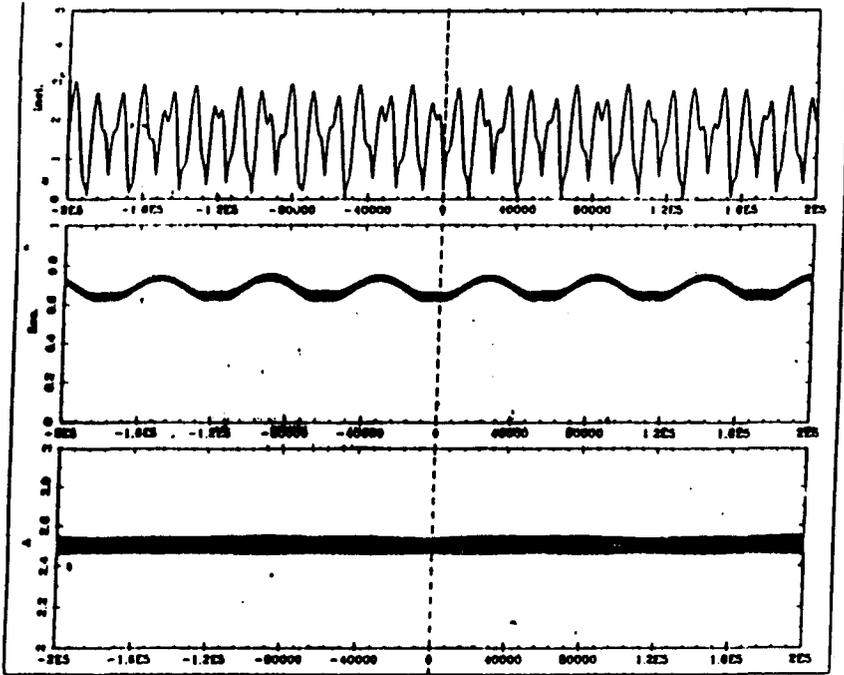
Fig. 6: Evolution of the semi-major axis A and the eccentricity e of the orbit of 1989 AC for 200000 years backward and forward; three-body model; same notations as for Fig.2.
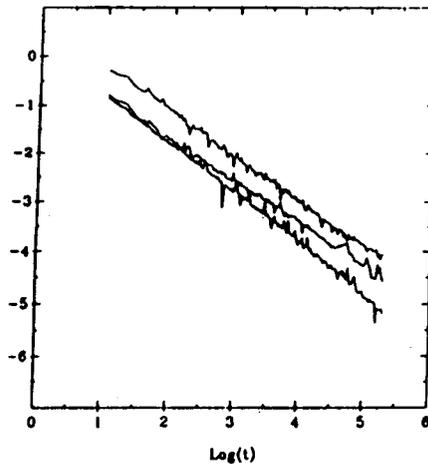
Fig.7: Variation of the LCI's forward in time for the orbit of 1989 AC; three-body model; same notations as for Fig. 3.

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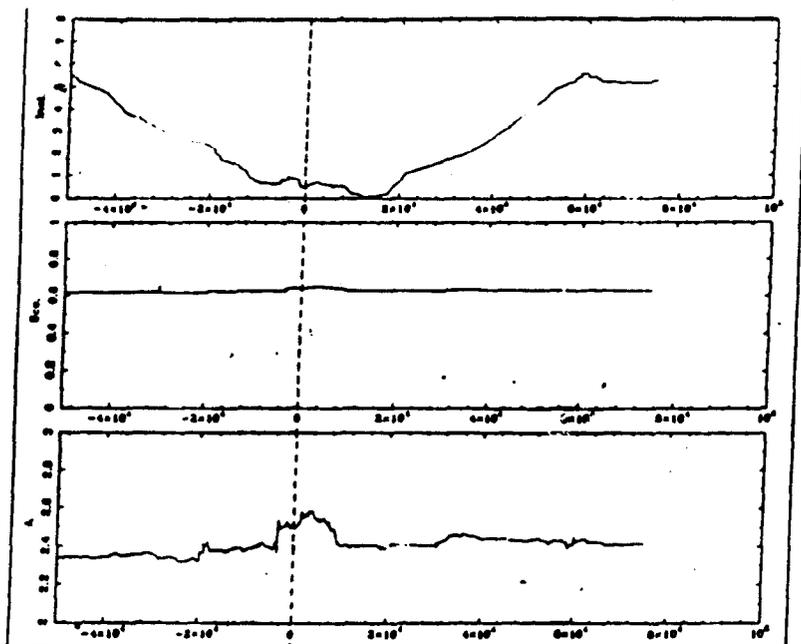
Fig.8: Evolution of the semi-major axis A and the eccentri-
city e of the "true" orbit of 1989 AC for 50000 years
backward and 100000 years forward;five-body model;
same notations as for Fig.2.
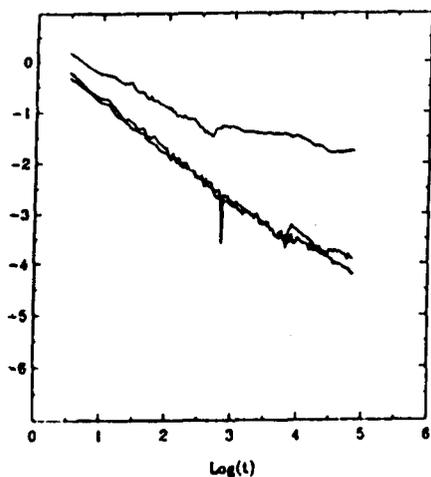
Fig.9: Variation of the LCI's forward in time for the orbit
of 1989 AC; five-body model;same notations as for Fig.3.

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4. CONCLUSION AND PERSPECTIVES

Our first results indicate that the orbit of the asteroid 1989 AC is chaotic. This stochasticity seems to be determined mainly by close approaches and encounters with the inner planets Venus, Earth and Mars. Nevertheless,Jupiter keeps some control on the evolution of the asteroid's orbit through the 3/1 resonance.

We intend now to study the effects of the cares takes in the computation of close approaches by using different integration schemes, and through statistical analysis on the variations of the orbital elements.

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