

Building Planets with Dusty Gas

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Abstract. We have developed a new numerical technique for simulating dusty-gas flows. Our code incorporates gas hydrodynamics, self-gravity and dust drag to follow the dynamical evolution of a dusty-gas medium. We have incorporated several descriptions for the drag between gas and dust phases and can model flows with submillimetre, centimetre and metre size “dust”. We present calculations run on the APAC¹ supercomputer following the evolution of the dust distribution in the pre-solar nebula.

1. Introduction

Up until recently we had only one observation to test our theories of planet formation against - our own Solar System. Now however with new planets and solar systems being both identified and parameterised at a rate approaching one a month, the observational constraints are much tighter and our lack of understanding of many aspects of the planet formation process is all too obvious. At the most basic level we know that micron size grains of dust in the pre-solar nebula clump and coagulate together to form planets, objects 10^{13} – 10^{14} times larger. Planet formation is a multi-stage process, taking us from dust grain to boulder to planetesimal to planetary embryo. Analytical arguments (Goldreich & Ward 1973) have presented us with constraints on the time scales for each stage but little more. It is the very first stage of the process that we are concerned with in this paper - from micron scale dust to metre sized boulders.

Theoretical models have changed much in recent years and the simple picture of a thin dust layer accumulating at the disk midplane, becoming gravitationally unstable, and breaking into planetesimals (Safronov 1969; Goldreich & Ward 1973) now seems unlikely.

¹Australian Partnership for Advanced Computing <http://nf.apac.edu.au/>

Recently Goodman & Pindor (2000) showed that turbulent drag causes radial instabilities in the dust layer, even if the disk self-gravity is negligible. In their steady state models, grains in a uniform dust layer experience radial drift as expected. However, for perturbed disks they predict that over-dense rings form within an orbital period, with the ring thickness similar to the thickness of the dust layer. These rings eventually collapse into planetesimals in the kilometre size range.

In this paper we present the first three-dimensional numerical simulations that include the effects of hydrodynamical forces, self-gravity and gas drag upon an evolving dusty gas disk. We describe a new numerical code, based upon the smoothed particle hydrodynamics (SPH) technique which uses a collection of particles to approximate a fluid. At present we run simulations with uniform grain size and do not allow dust particles to have individual or time varying chemical and physical properties. Adding this level of sophistication to the model would require only minor changes to the code.

2. The Planet Building Code

We have developed a new parallel two-phase (dust & gas) hydro + tree code by merging a dusty gas SPH code (Maddison 1998) with an MPI parallel Hashed Oct Tree N-body + SPH code (Humble 1999). Following Monaghan & Kocharyan (1995), the dusty gas is approximated by two inter-penetrating flows that interact via a drag force.

The particles are tagged as being gas or dust and are then allowed to interact according to gas-gas, gas-dust and dust-dust interactions. Only gas particles feel a pressure force and are affected by turbulent viscosity, which we parameterise with the standard SPH artificial viscosity terms. The gas-dust combination feel the drag force and also the mixed term seen in both the dust and gas particle acceleration equations. The only dust-dust interaction is through gravity.

The two fluids are coupled by gravity and drag, and the equation of motion is given by:

$$\begin{aligned}\frac{d\mathbf{v}_g}{dt} &= -\frac{1}{\rho_g} \nabla P + \frac{K}{\hat{\rho}_g} (\mathbf{v}_d - \mathbf{v}_g) - \nabla \Phi, \\ \frac{d\mathbf{v}_d}{dt} &= -\frac{1}{\rho_d} \nabla P - \frac{K}{\hat{\rho}_d} (\mathbf{v}_d - \mathbf{v}_g) - \nabla \Phi,\end{aligned}$$

where the terms on the right hand side are gas pressure, drag force and gravity respectively. The density $\hat{\rho}$ is the mass density per unit volume, while ρ is the local density (related via $\hat{\rho} = \Theta\rho$, where Θ is the volumetric void fraction such that $\Theta_g + \Theta_d = 1$). The functional form of K depends upon the drag regime (e.g. Epstein or Stokes) being considered. In the simulations we present, we have implemented Epstein drag which is appropriate for protoplanetary disks. Thus $K = (\rho_g \Theta_d c) / r_{\text{dust}}$, where c the local sound speed, ρ_g the local gas density, Θ_d the dust void fraction and r_{dust} the dust grain size.

The drag term is calculated using a pairwise implicit backward Euler scheme, while the hydrodynamics and self-gravity are solved explicitly. Operator splitting is used to integrate particle orbits under the influence of the un-softened

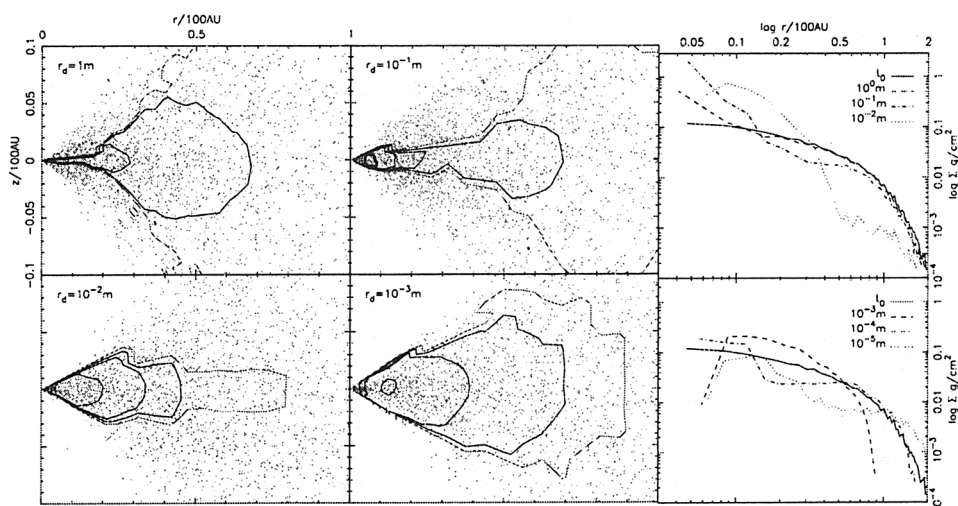


Figure 1. The 4 left hand panels show final gas (dots) and dust (contours) density distribution for 12.5 K gas + 12.5 K dust models of various grain sizes after $\sim 10^4$ years. Five contour levels (10^{-13} to 10^{-17} g/cm^3) are plotted. The 2 right hand panels show the final dust surface density distributions of all 6 models (initial distribution labelled t_0), demonstrating inward radial drift.

gravity of the central star. The code has been tested for stability and accuracy with standard periodic box simulations to ensure that the drag terms are correct, and collapsing sphere simulations to test the gravity. The code scales linearly to more than 32 processors. For simplicity we assume that the dust grains do not evaporate or coagulate and that the gas does not condense. We take the dust grains to be incompressible.

3. The Simulations

We are particularly interested in the effects of grain size upon disk morphology and global disk dynamics. We present the results of 6 simulations of gas disks laden with dust of different grain sizes. All simulations start with a prolate spheroid of rotating, self-gravitating gas (with a star at the centre) that spins down to near-equilibrium. The dust is then added (overlaid on the 3D flared gas disk) and the simulations are then followed for approximately 10^4 years (which corresponds to 11 orbits at 100 AU).

The model parameters used were: $M_\star = 1.0 M_\odot$, $M_{\text{disk}} = 0.01 M_\star$, $M_{\text{dust}} = 0.01 M_{\text{disk}}$ and $R_{\text{disk}} = 100$ AU. The grain parameters were: $\rho_d = 2.4 \text{ g}/\text{cm}^3$ and $r_d = 1, 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}$ and 10^{-5} m. The α -disk parameters used were: $c(R) = c_0(R/100\text{AU})^{-3/8}$, $T(R) \propto (R/100\text{AU})^{-3/4}$, $H/R = 0.1$ at 100 AU, and $\gamma = 5/3$ with an isothermal equation of state. Low resolution runs used 2×12500 particles, and high resolution runs used 2×125000 particles. The self-gravity softening $\varepsilon = 1.0$ AU, and the SPH artificial viscosity parameters $\alpha_{\text{SPH}} = 0.1$ and $\beta_{\text{SPH}} = 0.0$.

4. Discussion

For large (10 m) and small (micron) dust sizes, we expect that the dust distribution will stay close to the initial flared disk. The largest grains (not shown) are weakly coupled to the gas, and if started in Keplerian motion, they will remain there. On the other hand, the tiny grains are so strongly coupled to the gas that they are essentially co-moving (on the timescales we examine). For both extremes, we see little evolution of the dust distribution. It is for the regime $0.1 \text{ mm} < r_{\text{dust}} < 1 \text{ m}$ that the most significant disk evolution occurs.

In the $r - z$ plots of Fig. 1, significant deviation from the initially flared disk occurs in the inner regions of the 1m and 10 cm plots, in the mid regions (from $r/100\text{AU} = 0.6$ to $r/100\text{AU} = 0.9$) of the 1 cm plot, and in the outer regions of the 1 mm and smaller plots. In these regions, the dust is moving at close to Keplerian speeds, whereas the gas is (as always) sub-Keplerian. As the velocity difference is sustained, the Epstein drag is optimal, and the energy loss rate is high – allowing a thin layer of dust to form in the midplane and migrate radially. These thin dense dust disks are those that Goodman & Pindor (2000) suggest have global turbulent instability modes.

While the 10 cm and 1 cm dust exhibits the highest surface density (top right of Fig. 1), the volumetric density is largest in the inner regions of the 1m and 10 cm disks. Therefore these size ranges are probably the most interesting from a planet formation viewpoint.

The Lagrangian nature of the code means that it is trivial to add empirical grain growth models, and to follow the grain temperature and density histories and hence generate chemical compositions. The equations of state and drag term can easily be altered on a per-region or per-particle basis to account for local disk conditions.

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References

- Goldreich, P., & Ward, W. R. 1973, *ApJ*, 183, 1051
Goodman, J., & Pindor, B. 2000, *Icarus*, 148, 537
Humble, R. J. 1999, “Parallel N-body+SPH”, PhD Thesis, Monash University
Maddison, S. T. 1998, “Gravitational Instabilities in Protostellar Disks”, PhD Thesis, Monash University
Monaghan, J.J. & Kocharyan, A. 1995, *J. Comp. Phys.*, 87, 225
Safronov, V. S. 1969, “Evolution of the Protoplanetary Cloud and Formation of Earth and the Planets”, (Engl. transl. NASA TTF-677 1972)