ON GLACIER ENERGY BALANCE, ABLATION, AND AIR TEMPERATURE

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ABSTRACT. The paper tries to reconcile the facts that there are often useful correlations between ablation or run-off and air temperature while net radiation is usually the major source of ablation energy. Equations are derived from the energy balance to describe statistics for the ablation–temperature relation in terms of statistics for the relations between individual energy fluxes and air temperature. As examples, statistics are evaluated for four published series from Arctic Canada. Although the net radiation is the largest energy source in all four cases, the ablation rate is moderately well correlated with temperature and poorly correlated with net radiation. This is because the sensible heat flux is more variable than the radiation in three cases and is itself better correlated with temperature in all four cases. The major contributions to the increase of ablation rate with temperature (on average 6.3 kg m^{-2} d^{-1} deg^{-1}) are due to sensible heat, followed by latent heat with a small contribution from net radiation. The resulting ablation–temperature model explains about half the variance of ablation rate. The main application of such a simple model is for the estimation of ablation totals in areas where glaciological and hydrological data are sparse.

RÉSUMÉ. *Sur le bilan d'énergie d'un glacier, l'ablation et la température de l'air.* On tente de concilier le fait qu'il y a souvent une corrélation utilisable entre l'ablation ou l'écoulement et la température de l'air, avec celui que le bilan radiatif net est d'ordinaire la principale source d'énergie disponible pour l'ablation. On a tiré du bilan énergétique des équations pour décrire statistiquement la relation ablation–température à partir des relations statistiques entre les flux individuels d'énergie et la température de l'air. Par exemple, on analyse statistiquement quatre séries de données publiées concernant l'Arctique Canadien. Bien que le bilan radiatif net soit dans les quatre cas la source principale d'énergie, la vitesse de l'ablation est relativement bien corrélée avec la température mais mal avec la radiation nette. Ceci tient à ce que le flux de chaleur sensible est plus variable que la radiation dans trois cas et est lui-même mieux corrélé avec la température dans les quatre cas. Les principales contributions à l'augmentation de la vitesse d'ablation avec la température (d'environ 6,3 kg m^{-2} d^{-1} deg^{-1}) sont dues à la chaleur sensible, suivie par la chaleur latente avec une petite contribution de la radiation nette. Le modèle résultant ablation–température explique à peu près la moitié de la variance de la vitesse d'ablation. La principale application d'un modèle aussi simple est l'estimation des totaux d'ablation dans les régions où les données glaciologiques et hydrologiques sont rares.

ZUSAMMENFASSUNG. *Über die Energiebilanz, die Ablation und die Lufttemperatur an Gletschern.* Die Arbeit versucht die Tatsachen, dass häufig nützliche Korrelationen zwischen Ablation oder Abfluss und Lufttemperatur herangezogen werden, während die Netto-Strahlung gewöhnlich die Hauptquelle der Ablationsenergie bildet, in Übereinstimmung zu bringen. Aus der Energiebilanz werden Gleichungen zur Beschreibung des statistischen Verhaltens der Beziehung zwischen Ablation und Temperatur aus der Statistik für die Beziehung zwischen den einzelnen Energieflüssen und der Lufttemperatur hergeleitet. Als Beispiele werden die Statistik für 4 veröffentlichte Beobachtungszeiten aus der Kanadischen Arktis bearbeitet. Obwohl die Netto-Strahlung in allen 4 Fällen die grösste Energiequelle darstellt, ist die Ablationsrate mit der Temperatur recht gut korreliert, mit der Netto-Ablation dagegen nur schwach. Der Grund dafür liegt darin, dass der Fluss der fühlbaren Wärme in 3 Fällen stärker schwankt als die Strahlung und mit der Temperatur in allen 4 Fällen besser korreliert ist. Den grössten Beitrag zum Anwachsen der Ablation mit der Temperatur (im Mittel 6,3 kg m^{-2} d^{-1} deg^{-1}) liefert die fühlbare Wärme, gefolgt von der latenten Wärme mit einem kleinen Anteil aus der Netto-Strahlung. Das sich ergebende Modell für Ablation und Temperatur erklärt etwa die Hälfte der Schwankungen in der Ablationsrate. Die Hauptanwendung eines so einfachen Modells dürfte in der Abschätzung von Ablationsmengen in Gebieten, für die nur wenige glaziologische und hydrologische Daten vorliegen, zu sehen sein.

INTRODUCTION

The fact that the energy used for ablation is provided by a number of distinct energy sources has been known since the last century. For example, Mousson (1854, p. 90–101) mentions the effects of solar radiation, contact with the air, and rainfall as the three ways in which ablation energy is delivered to the glacier surface whilst Hess (1904, p. 209–19) clearly stated his belief that radiation is the most important factor for ablation, i.e. the largest source of energy. Since the 1930's numerous attempts have been made to measure the components of the glacier surface energy balance, usually only for short periods. Paterson (1969, p. 58–61) gives a partial summary of such studies which shows that the earlier presumption of the

importance of radiation was essentially correct. The net radiation actually constitutes more than half of the total energy supply in 24 of the 32 cases listed by Paterson although his figures are naturally somewhat influenced by errors and differences in definition. Generalizations are still difficult but the relative importance of radiation as a heat source appears to increase with altitude on glaciers as well as with the continentality of the climate, see, e.g. Tvede (1974, p. 86). However, the corresponding absolute values of the ablation decrease, a fact which is obscured by expressing the energy sources as percentages of the total energy supply.

It has often been assumed that air temperature is an important element in explaining variations in ablation and mass balance. Finsterwalder and Schunk (1887) already assumed a relation between ablation and temperature for their study of the variation of the Suldenferner in the Eastern Alps. More recently, a number of workers have compared mass-balance series with temperature at neighbouring weather stations, often in combination with precipitation. Examples of different approaches to this are by Liestøl (1967), Martin (1974), Hoinkes and Steinacker (1975) and Braithwaite (unpublished). Interpretations of the moderate correlations between mass balance and temperature vary. For example, Wallén (1949) suggests that it mainly reflects changes in heat supply from the air whilst Ahlmann (1953) puts more emphasis on the change in length of the ablation season with temperature. On the other hand, Hoinkes (1955) and Hoinkes and others (1968) suggest that temperature should be mainly regarded as an index of radiation and of the probability that summer precipitation will fall as snow which will increase the surface albedo. Hoinkes (1955) explicitly rejects the notion that greater heat exchange from air to ice during a hot summer is sufficient to account for the greater ablation.

In connection with the development of short-term forecasting models, attempts have been made to relate glacier run-off to meteorological elements using multiple regression models, e.g. by Goodison (1972), Jensen and Lang (1973), and Østrem (1973). Such studies have often found that meteorological elements like air temperature, wind speed, and vapour pressure contribute significantly to the overall correlation whilst the correlations with radiative elements like global radiation, sunshine duration, and cloudiness are often surprisingly small, see the discussion following Østrem (1973). This cannot be explained by saying that temperature affects ablation because it is an index of radiation as, if it were, the radiative elements would be selected in preference to temperature in any stepwise regression scheme.

The present paper developed out of an attempt to reconcile the facts that radiation is usually the major source of ablation energy whilst there are often useful correlations between ablation or run-off and temperature.

THEORY

The daily ablation rate at any location and at time t is given by the energy balance equation:

$$a_t = \frac{1}{L} \sum_{i=1}^{i=M} q_{it} \tag{1}$$

where L is the latent heat of fusion and q_{it} is the ith energy source at time t. There are M energy sources. It will be assumed that the ablation rate and the energy fluxes are all stationary random variables fluctuating around mean values for a period of length N days, for example

$$a_t = \bar{a} + a_t' \tag{2}$$

where the overbar ⁻ denotes averaging over the period of N days and the prime ' denotes departure from the mean on any day where the mean is defined as

$$\bar{a} = \frac{1}{N} \sum_{t=1}^{t=N} a_t.$$ (3)

The mean ablation rate will be given by

$$\bar{a} = \frac{1}{L} \sum_{i=1}^{i=M} \bar{q}_i,$$ (4)

whilst the deviation from the mean will be given by:

$$a_t' = \frac{1}{L} \sum_{i=1}^{i=M} q_{it}'$$ (5)

where $\bar{q}_i$ and q_{it}' are respectively the mean and the deviation from the mean of the ith energy source.

Instead of representing the daily ablation rate in terms of the sum of M energy sources as in Equation (1) it is chosen to represent it by a regression equation in terms of T_t the daily mean temperature as might be measured in a standard meteorological screen at a height of one to two metres above the glacier surface:

$$a_t = \alpha_0 + \beta_0 T_t + e_t$$ (6)

where the intercept α_0 is the mean ablation rate at an air temperature of $0°C$, the slope β_0 is the temperature response for ablation, and e_t is the residual of the regression equation with a standard deviation of γ_0. Equation (6) is assumed to be only valid for $T_t \geqslant 0°C$ whilst the ablation rate is assumed to be zero for negative temperatures. However, as none of the series analysed in the following section contains negative temperatures this point is not very important for the present study.

If the values of α_0 and β_0 are chosen to make the mean of the residuals e_t zero and to minimize their variance, i.e. according to the ordinary least-squares method (Kreyszig, 1970, chapter 17), they are given by

$$\beta_0 = \frac{1}{S_T{}^2} \frac{\sum_{t=1}^{t=N} a_t' T_t'}{(N-1)}$$ (7)

and

$$\alpha_0 = \bar{a} - \beta_0 \bar{T}.$$ (8)

The corresponding correlation coefficient for the regression equation (Kreyszig, 1970, chapter 18) is given by:

$$\rho_0 = \frac{1}{S_a S_T} \frac{\sum_{t=1}^{t=N} a_t' T_t'}{(N-1)}$$ (9)

where S_a and S_T are the standard deviations of a_t and of T_t whilst $\bar{T}$ and T_t' are respectively the mean temperature and deviation from the mean at time t. The correlation ρ_0 is a measure of the usefulness of the representation of ablation rate in terms of temperature alone as in Equation (6).

Substitution of Equation (5) into (7) with re-arrangement gives:

$$\beta_0 = \frac{1}{L} \sum_{i=1}^{i=M} \beta_i \tag{10}$$

where β_i is the temperature response for the ith energy source or, alternatively, it is the slope of the regression equation relating q_{it} to T_t and is defined in an analogous way to Equation (7). Substitution of Equations (4) and (10) into Equation (8) gives:

$$\alpha_0 = \frac{1}{L} \sum_{i=1}^{i=M} \alpha_i \tag{11}$$

where α_i is the value of the ith energy source at an air temperature of $0°C$ or, alternatively, it is the intercept of the regression equation relating q_{it} to T_t. Substitution of Equation (5) into Equation (9) gives:

$$\rho_0 = \frac{1}{L} \sum_{i=1}^{i=M} \frac{S_i}{S_a} \rho_i \tag{12}$$

where S_i is the standard deviation of the ith energy source and ρ_i is the correlation between the ith energy source and temperature defined in an analogous way to Equation (9).

Equation (10) expresses the simple fact that the response of the ablation rate to variations in temperature is simply the sum of the responses of the individual energy sources divided by the latent heat of fusion. Equation (11) states that the ablation at an air temperature of $0°C$ is the sum of the values of the individual energy sources at $0°C$ divided by the latent heat of fusion. Equation (12) expresses the correlation between ablation rate and temperature as a weighted sum of the correlations between the individual energy sources and temperature. The weighting factor is S_i/LS_a which expresses the variability of the ith energy source relative to that of the ablation. From Equation (12) it can be seen that any particular source should fulfil two conditions if it is to make a substantial contribution to the correlation between ablation and temperature; the energy source must be quite variable compared to the variability of the ablation and it must also be strongly correlated itself with temperature.

Equations (10) to (12) are still very general as no numerical values have been specified for the various quantities, in particular it should be noted that values of zero for ρ_0 and β_0 are not excluded at this stage (as they stand, the equations should be valid for any arbitrary variable). The statistics in the equations must be calculated separately on a case-by-case basis by statistical analyses of field data, whilst the equations themselves merely define the relationships that the statistics will have with each other.

The αs and βs in Equations (10) and (11) are defined in statistical terms but they must be closely related to physical processes. There will generally be four significant heat sources; the flux of net radiation Q_R, sensible heat flux Q_S, latent heat flux Q_L, and the heat of conduction into ice Q_I which is actually a heat sink but can be regarded as a negative source. If any of these terms vary with temperature, their temperature responses will contribute to β_0 while their mean values at $0°C$ will contribute to α_0. It may be expected *a priori* that the short-wave component of the net radiation and the heat of conduction into ice will be more or less independent of temperature so they will mainly contribute to α_0. The long-wave component of the net radiation will generally be negative at $0°C$ and will make a negative contribution to α_0. However, long-wave radiation will increase with temperature so that it will also contribute to β_0. The sensible heat flux and, to lesser extent, the latent heat flux should both contribute to β_0 while contributing little to α_0. These are expectations that are falsifiable by experience although they may seem reasonable. For example, a strong correlation even

between the sensible heat flux and air temperature is not at all self-evident because sensible heat flux is actually a function of many variables (including temperature) so that any relationship with temperature alone could be "masked" by variations in other elements, e.g. the wind speed.

The equations presented here are theoretical and the various terms must be calculated for real data which may not exactly fulfil the assumptions made. For example, measurements of ablation rates and of the energy sources will involve errors, so that Equation (1) will not be satisfied exactly by measured ablation data, denoted by A_M in the following section, nor will Equations (10) to (12). On the other hand, the ablation calculated from the measured energy sources according to Equation (1), denoted by A_C in the following section, will satisfy all the equations but will not be the "true" ablation. The assumptions of randomness and stationarity in the data, implicit in the use of the ordinary least-squares formulae in Equations (7) and (8), may be violated by autocorrelation and seasonal trends in the field data. These assumptions could, perhaps, be avoided by a more complicated mathematical treatment but it hardly seems worthwhile at present as the effects of errors in the data are probably more serious.

FOUR EXAMPLES

Three of the series studied are from an altitude of 210 m on the White Glacier, Axel Heiberg Island, and one is from 300 m on the Sverdrup Glacier, Devon Island. The sources of data are Andrews (1964) for White Glacier 1960 (16 periods of irregular duration between 8 July and 10 August 1960), Müller and Roskin-Sharlin (1967) and Müller and Keeler (1969) for White Glacier 1961 (63 days between 12 June and 18 August 1961), Havens and others (1965) for White Glacier 1962 (11 periods of irregular duration between 16 July and 31 July 1962) and Keeler (1964) for Sverdrup Glacier 1963 (33 days between 9 July and 10 August 1963). In the cases of the 1960 and 1962 series from the White Glacier, the energy fluxes for periods of irregular duration were scaled to daily equivalents for the purpose of the present study. The air temperature data originate from instruments at different heights above the glacier surface; at 1.7 m for White Glacier 1960, 1.5 m for White Glacier 1961 and 1962, and 2.2 m for Sverdrup Glacier 1963.

The data set for each series comprises net radiation Q_R, sensible heat flux Q_S, latent heat flux Q_L, heat conduction into ice Q_I, and heat supplied by rainfall Q_P together with two different estimates of ablation A_C and A_M. The first of these is the value calculated from the measured energy fluxes according to Equation (1) whilst A_M was determined by direct measurement of mass loss in the surface layer of the glacier. Naturally the two ablation estimates are seldom in perfect agreement; errors in A_C reflect the combined effects of error in the individual energy terms whilst A_M will involve various errors discussed by Müller and Keeler (1969). For the purposes of the present analysis, the two ablation estimates are treated as being equally good (or bad). The surface material in all four cases was melting ice.

Results from the statistical analyses are presented in Table I. They comprise means and standard deviations of the various terms as well as the coefficients of correlation between the terms. Asterisks in the table denote correlations which are significantly different from zero at less than the 5% level according to a Student's t-test (Kreyszig, 1970, p. 343). Correlations involving Q_P have been omitted as the heat supply by rainfall is too sporadic for valid statistical treatment; in two of four cases it is zero whilst in the other two cases it only averages 0.03 MJ d⁻¹ m⁻².

There are several interesting results in Table I. First, although there is fair agreement between the means of A_C and A_M with differences ranging from −6 to +5 mm d⁻¹, the correlations between the ablation estimates can be quite bad, 0.60 in the worst case. This is a sure indication of substantial random errors in one or both of the ablation estimates. Second, it is noteworthy that the net radiation Q_R is the major heat source in all four cases, followed by

Q_S whilst Q_L varies greatly between cases and Q_I is fairly constant. Third, Q_S and Q_L are highly correlated in only two of the cases. Fourth, the sensible heat flux Q_S correlates better with the ablation in three cases than does Q_R although the latter is the most "important" or largest heat source in all four cases. The reason for this is that the sensible heat flux is more variable than the net radiation in the three cases, i.e. it has a larger standard deviation. Fifth, there is generally only a weak correlation between net radiation Q_R and air temperature T. Sixth, the air temperature correlates generally better with ablation than does the net radiation Q_R although, naturally, not as highly as the sensible heat flux Q_S does.

TABLE I. MEANS, STANDARD DEVIATIONS, AND CORRELATION COEFFICIENTS FOR FOUR ENERGY BALANCE SERIES FROM ARCTIC CANADA

	Q_R MJ m⁻² d⁻¹	Q_S MJ m⁻² d⁻¹	Q_L MJ m⁻² d⁻¹	Q_I MJ m⁻² d⁻¹	A_C kg m⁻² d⁻¹	A_M kg m⁻² d⁻¹	T °C
White Glacier 1960—Sample size 16							
Mean	6.80	5.40	3.22	−1.12	43	38	5.9
Standard deviation	±1.71	±3.73	±2.42	±0.13	±20	±17	±2.0
Correlation with							
Q_R	1.00	0.11	0.11	−0.33	0.35	0.27	0.12
Q_S		1.00	0.96*	0.38	0.97*	0.86*	0.82*
Q_L			1.00	0.33	0.95*	0.81*	0.74*
Q_I				1.00	0.27	0.19	0.36
A_C					1.00	0.87*	0.78*
A_M						1.00	0.67*
T							1.00
White Glacier 1961—Sample size 63							
Mean	5.39	4.16	0.04	−1.90	23	21	3.2
Standard deviation	±2.30	±4.38	±1.77	±0.77	±16	±17	±2.2
Correlation with							
Q_R	1.00	−0.06	−0.12	−0.35*	0.28*	0.11	−0.06
Q_S		1.00	0.12	0.14	0.86*	0.87*	0.79*
Q_L			1.00	0.32*	0.43*	0.36*	0.43*
Q_I				1.00	0.21*	0.16	0.23*
A_C					1.00	0.91*	0.80*
A_M						1.00	0.76*
T							1.00
White Glacier 1962—Sample size 11							
Mean	8.68	3.84	0.79	−1.55	35	41	5.5
Standard deviation	±3.56	±3.18	±0.66	±0.30	±17	±20	±2.1
Correlation with							
Q_R	1.00	0.53*	−0.06	−0.40	0.88*	0.19	0.37
Q_S		1.00	−0.05	−0.34	0.86*	0.88*	0.81*
Q_L			1.00	−0.74*	0.01	0.07	0.34
Q_I				1.00	−0.46	−0.24	−0.49
A_C					1.00	0.60*	0.69*
A_M						1.00	0.84*
T							1.00
Sverdrup Glacier 1963—Sample size 33							
Mean	4.43	2.69	1.23	−0.97	22	22	4.0
Standard deviation	±1.60	±2.31	±1.33	±0.31	±11	±17	±1.5
Correlation with							
Q_R	1.00	−0.10	−0.16	−0.34*	0.27	−0.22	0.13
Q_S		1.00	0.92*	0.05	0.92*	0.76*	0.80*
Q_L			1.00	0.10	0.87*	0.80*	0.76*
Q_I				1.00	0.01	0.34*	−0.32*
A_C					1.00	0.70*	0.79*
A_M						1.00	0.49*
T							1.00

* Correlation significant at the 5% level according to Student's *t*-test.

The contributions by the different energy sources to the correlation ρ_0 between the ablation A_C and temperature were computed according to Equation (12) from the results in Table I and are listed in Table II. For comparison the corresponding correlations between ablation A_M and temperature are given in brackets in the table. On average, the contribution from Q_S is highest (68% of the total on average), followed by Q_L (23%) and Q_R (9%). The dominant role of the sensible heat flux compared to the net radiation arises because it is more strongly correlated with temperature in all four cases (average correlation of 0.81 ± 0.01 compared to 0.14 ± 0.18 for the four cases) while it has a greater standard deviation in three of the cases, i.e. it is more variable. This result falsifies the notion that ablation is correlated with temperature because temperature is an index of radiation.

TABLE II. CONTRIBUTIONS TO ρ_0, THE CORRELATION BETWEEN ABLATION AND TEMPERATURE, DUE TO THE DIFFERENT ENERGY SOURCES

Sample	ρ_0	Q_R	Q_S	Q_L	Q_I
White Glacier 1960	0.78 (0.67)	0.03	0.46	0.27	0.01
White Glacier 1961	0.80 (0.76)	−0.03	0.65	0.14	0.03
White Glacier 1962	0.69 (0.84)	0.23	0.45	0.04	−0.03
Sverdrup Glacier 1963	0.79 (0.49)	0.06	0.50	0.27	−0.03
Mean of four cases	0.77 (0.69)	0.07	0.52	0.18	−0.01

Note: ρ_0 refers to ablation A_C while figures in brackets refer to ablation A_M.

The contributions by the different energy sources to β_0, the temperature response for ablation A_C, were computed according to Equation (10), i.e. the contributions are simply slopes (in MJ m^{-2} d^{-1} deg^{-1}) of regression equations for the individual energy sources divided by the latent heat of fusion (0.335 MJ kg^{-1}). Results are listed in Table III which also includes the corresponding slope for A_M in brackets. Once again, the major contribution to β_0 is due to the sensible heat Q_S which contributes an average of 4.2 ± 0.6 kg m^{-2} d^{-1} deg^{-1} to the average β_0 value of 6.3 ± 1.0 kg m^{-2} d^{-1} deg^{-1}. The agreement between the latter and the corresponding value of 6.3 ± 1.1 for A_M is also encouraging. Both these figures are in rough agreement with values in the literature, e.g. 4.5 from Zingg ([1952]), a range of 5.1 to 7.0 from Kasser (1959) and two values of 6.1 and 6.5 from Orheim (1970). The contributions by Q_S in Table III are equivalent to an average temperature response for sensible heat flux of 1.41 ± 0.20 MJ m^{-2} d^{-1} deg^{-1} which can be compared with 1.68 ± 0.23 MJ m^{-2} d^{-1} deg^{-1} found by Kuhn (1979) on the Hintereisferner using a different method. The contributions by Q_R in Table III are equivalent to an average temperature response for net radiation of 0.20 ± 0.30 MJ m^{-2} d^{-1} deg^{-1} which is about two-thirds of the value suggested by Kuhn (1979) for long-wave radiation with an effective emissivity of 0.7.

TABLE III. CONTRIBUTIONS TO TEMPERATURE RESPONSE β_0 OF ABLATION DUE TO THE DIFFERENT ENERGY SOURCES. ALL UNITS ARE kg m^{-2} d^{-1} deg^{-1}

Sample	β_0	Q_R	Q_S	Q_L	Q_I
White Glacier 1960	7.8 (5.7)	0.3	4.6	2.7	0.1
White Glacier 1961	5.8 (5.7)	−0.2	4.7	1.0	0.2
White Glacier 1962	5.5 (7.9)	1.9	3.6	0.3	−0.2
Sverdrup Glacier 1963	6.0 (5.8)	0.4	3.8	2.1	−0.2
Mean of four cases	6.3 (6.3)	0.6	4.2	1.5	0.0

Note: β_0 refers to ablation A_C while figures in brackets refer to ablation A_M.

The contributions by the different energy sources to α_0, the ablation A_C at $0°C$, were computed according to Equation (11), i.e. the contributions are simply intercepts (in $MJ\ m^{-2}\ d^{-1}$) of regression equations for the individual energy sources divided by the latent heat of fusion. Results are listed in Table IV which also includes the corresponding intercepts for A_M in brackets. In this case the main contribution to α_0 is by net radiation Q_R which contributes an average of $16\pm3\ kg\ m^{-2}\ d^{-1}$ to the ablation rate at $0°C$ which is equivalent to a mean net radiation value of $5.36\pm1.01\ MJ\ m^{-2}\ d^{-1}$. However, it is puzzling that the contributions by Q_S and Q_L are generally negative with averages of -8 ± 3 and $-3\pm3\ kg\ m^{-2}\ d^{-1}$ respectively. It would be tempting to dismiss these values as being merely errors of measurement or of sampling, i.e. to postulate that the average combined error in the contributions by Q_S and Q_L amounts to at least $-11\ kg\ m^{-2}\ d^{-1}$. However, the good agreement of α_0 with the corresponding intercept for A_M (averages of 1 ± 4 and $1\pm3\ kg\ m^{-2}\ d^{-1}$) makes this implausible as errors in A_C and A_M should be independent of each other. If accepted as true, the negative contributions to α_0 by Q_S and Q_L would suggest that the melting glacier surface already acts as a sensible and latent heat source for the overlying air with an air temperature of $0°C$. This also seems puzzling. There are no individual events in the four cases with an actual air temperature of $0°C$ so the problem cannot be examined in more detail and the α_0 values found must be regarded as extrapolations.

TABLE IV. CONTRIBUTIONS TO THE ABLATION α_0 AT $0°C$ DUE TO THE DIFFERENT
ENERGY SOURCES. ALL UNITS ARE $kg\ m^{-2}\ d^{-1}$

Sample	α_0	Q_R	Q_S	Q_L	Q_I
White Glacier 1960	-3 (4)	19	-11	-6	-4
White Glacier 1961	4 (2)	17	-3	-3	-6
White Glacier 1962	5 (-3)	16	-9	$+1$	-3
Sverdrup Glacier 1963	-2 (-1)	11	-7	-5	-2
Mean of four cases	$+1$ $(+1)$	16	-8	-3	-4

Note: α_0 refers to ablation A_C while figures in brackets refer to ablation A_M.

The correlations in Table II give an idea of the relative contributions of the different energy sources to the overall correlations between ablation and temperature. However, if one were to attempt the prediction of ablation using only temperature, the standard deviation γ_0 of the residuals in Equation (6) would be an absolute measure of the accuracy of prediction. The values of γ_0 are listed in Table V for both ablation estimates A_C and A_M, whilst the figures in brackets are the corresponding percentages of explained variance obtained by squaring the correlation coefficients. The average value of γ_0 for the eight cases together is

TABLE V. STANDARD DEVIATIONS OF THE RESIDUALS γ_0 FOR
CORRELATIONS BETWEEN ABLATION AND TEMPERATURE. ALL
UNITS ARE $kg\ m^{-2}\ d^{-1}$

Sample	For A_C	For A_M
White Glacier 1960	±13 (61%)	±13 (45%)
White Glacier 1961	±10 (64%)	±11 (58%)
White Glacier 1962	±12 (48%)	±11 (71%)
Sverdrup Glacier 1963	±7 (62%)	±15 (24%)
Mean of four cases	±11 (59%)	±13 (50%)

Note: Figures in brackets are the corresponding percentages of explained variance.

± 12 kg m^{-2} d^{-1} which corresponds to an average explanation of 54% of the variance of ablation. By contrast, the use of net radiation Q_R as the sole predictor of ablation only explains 15% of the variance. This comparison shows once again that temperature is a better predictor of ablation than net radiation in the cases considered. However, the prediction of only a little more than half of the variance of the ablation by temperature will still be inadequate for many practical purposes.

Discussion

The results from the previous section are summarized in Table VI which shows the average contributions by the different energy sources to the various statistics for ablation A_C. The figures refer to the average values for the four cases with the convention that total positive contributions in any case are 100%. For comparison with the other figures which are based on Tables II to IV, the contributions to mean ablation are also given. The figures show that net radiation is on average the major energy source for mean ablation as it constitutes 55% of the total energy income of which 87% is available for ablation. Furthermore, the net radiation provides all of the energy income at $0°C$ but, because the other terms are all negative, only 6% of this energy income is available for ablation at $0°C$. By contrast, the major contributions to the correlation between ablation and temperature and to the temperature response of ablation are due to the sensible heat flux, followed by the latent heat flux with only a minor role for net radiation. This can be explained by the fact that the sensible heat flux is more variable than the net radiation in three of the cases while it is more strongly correlated with temperature in all four cases.

TABLE VI. PERCENTAGE CONTRIBUTIONS TO VARIOUS ABLATION STATISTICS DUE TO THE DIFFERENT ENERGY SOURCES AS AVERAGES FOR THE FOUR CASES

		Q_R	Q_S	Q_L	Q_I
Mean ablation	$\bar{A}_C$	55%	34%	11%	-13%
Correlation	ρ_0	9%	68%	23%	-1%
Temperature response	β_0	9%	67%	24%	0%
Ablation at $0°C$	α_0	100%	-50%	-19%	-25%

Note: All figures are expressed as percentages of total positive contributions.

The above results are based upon the four series from Arctic Canada. The question arises as to whether they are applicable to glaciers in other areas, e.g. Scandinavia or the Alps. This can only be answered by a similar analysis of energy balance series from other areas (the theory and method used here are universal while the numerical results *sensu stricto* apply only to the studied series). It is hoped that this paper will prompt others to make similar analyses. If this is done for sufficient glaciers under various conditions, a rational classification of glacier–climatic regimes may emerge. Although a broad agreement has been found here between the inferences based upon the two different ablation estimates, analyses of data from more modern and, it is to be hoped, more accurate series would also be desirable. Despite the fact that the suggested analyses have not yet been made for other areas, there is already a hint that day-to-day variations of net radiation are less than those of sensible heat flux on glaciers in south Norway (Tvede, 1974, p. 81) while Lang (1978) has now questioned the old assumption that air temperature is a simple index of radiation under Alpine conditions. Lastly, the results of this study, if they were valid for other areas also, would provide an explanation of why run-off is often poorly correlated with radiation in comparison to temperature or other elements.

The modelling of ablation in terms of temperature alone achieves simplicity at the cost of accuracy, for example the average correlation between ablation and temperature in the four cases is only 0.73 ± 0.11 (this includes correlations for both A_C and A_M) which corresponds to an "explanation" of slightly more than half of the variance of daily ablation rate. The main application of such a simple model is for the estimation of seasonal or annual ablation totals in areas where glaciological and hydrological measurements are sparse but where, nevertheless, there is an economic need for rough estimates of glacier run-off. For such applications, air temperature can be extrapolated relatively easily from low-lying weather stations whilst the substantial random errors involved in estimating daily ablation rates from the model will tend to compensate over periods of many days or of a few months. The approach is used by Braithwaite (1980) as part of a programme of regional water-resource assessment in west Greenland. By contrast, the model is probably not accurate enough for short-term forecasting of run-off because no compensation of errors will occur in this case. For such purposes, multiple regression models involving a number of variables may be preferred.

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