THE NUMERICAL SOLUTION OF
FREDHOLM INTEGRAL EQUATIONS
OF THE SECOND KIND

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This thesis tackles some problems encountered in the numerical
solution of Fredholm integral equations of the second kind. We are
concerned specifically with the applicability and numerical performance of
algorithms for these equations, and are guided by the existence of the
following problems.

(i) Theoretically, the applicability of many algorithms often depends
on certain highly abstract assumptions being satisfied. These assumptions
are often difficult to verify in practice.

(ii) Error analyses for certain algorithms have tended to assume that
the given information and the solution are smooth, and hence predict a
higher order of convergence than that obtained in practice (where there are
usually singularities present).

In Chapter 2 we develop practical methods for deciding whether a given
integral operator is compact as an operator between certain spaces of
functions. This solves a problem of type (i), since compactness is an
abstract assumption used in the analysis of many algorithms for integral
equations. In Chapter 3 we look at a class of weakly singular convolution
type equations (typical of many that arise in practice), and answer the
question: What kind of singularities arise in the solutions to such
equations? In Chapter 4, the results of Chapter 3 are used to give a

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realistic error analysis (that is, one which takes account of the
singularities in kernel and solution) for Galerkin type methods for the
class of equations introduced in Chapter 3, hence solving a problem of type
(ii) for that class. The results of Chapters 3 and 4 concern only one
dimensional integral equations. An analysis of collocation methods for two
dimensional equations is given in Chapter 5. Convergence rates are
obtained for the cases of equations with both smooth kernels and weakly
singular kernels. The analysis in the latter case depends on a
characterisation of the properties of the solution to a typical two
dimensional weakly singular equation. This characterisation is also given
in Chapter 5. The methods proposed in Chapter 5 are illustrated in Chapter
6 by the numerical solution of a two dimensional equation arising in
electrical engineering.

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