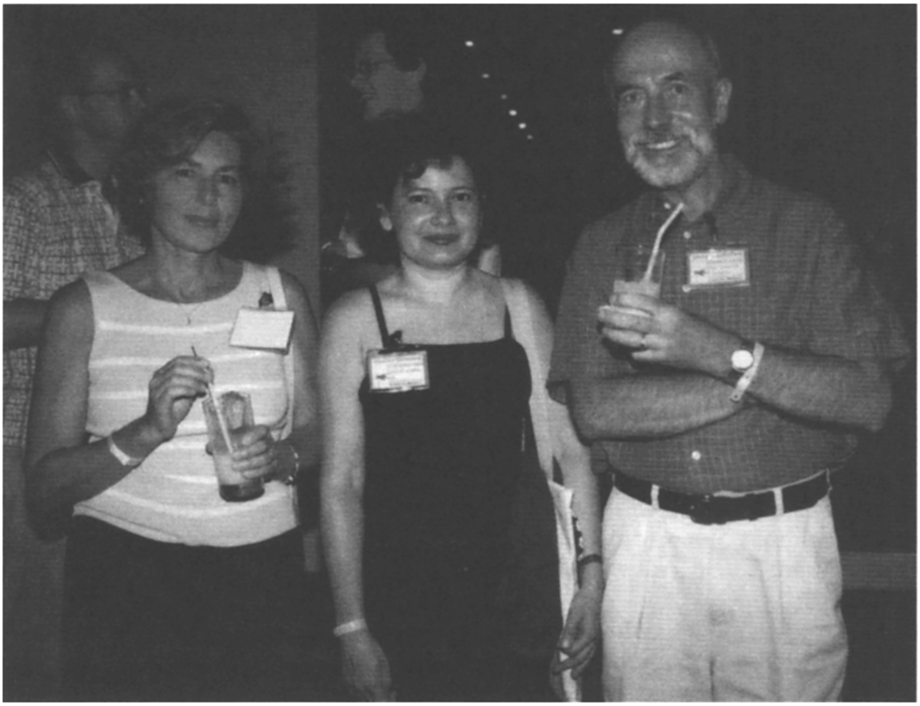
SESSION 3

Rotation, Solar and Stellar Physics



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The Internal Solar Rotation from Helioseismology

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Abstract. Helioseismology has provided detailed inferences about the internal rotation of the Sun, and hence stringent constraints on any attempt to understand the properties and evolution of stellar rotation. Here I briefly discuss the techniques used in the analysis and review the results that have been obtained. Strikingly, these results are markedly different from the predictions made before the helioseismic data became available, emphasizing the difficulties in the modelling of phenomena as complex as stellar internal rotation.

1. Introduction

The proximity of the Sun gives it a very special place in all aspects of stellar astrophysics. Here we can directly observe in detail phenomena, such as surface differential rotation and magnetic activity, that can only be inferred more or less indirectly in other stars. Also, the very extensive data on solar oscillations that have been acquired in the last decade allow inferences to be made about the detailed internal properties, both structure and rotation, of the Sun. The results of these helioseismic investigations provide a crucial test that must be applied to any theory purporting to describe stellar interiors and their evolution. In the case of solar structure, this applies to the details of the modelling of solar evolution and the physics of the solar interior. For solar rotation, even the basic physical processes responsible for the present internal rotation of the Sun remain uncertain.

The general understanding of solar and stellar rotation and its evolution seems well established, as amply documented elsewhere in these proceedings. Stars are formed with relatively rapid rotation. In solar-like stars angular momentum is lost in a magnetic stellar wind, coupled to the outer convection zone. Rotation within the convection zone is controlled by angular-momentum transport by the convective motions which establish the surface differential rotation, with the equator rotating more rapidly than the poles. Also, the convective transport ensures that the spin-down by the stellar wind is transmitted to the entire convection zone. The effect on the stellar interior is far more uncertain and depends on processes, perhaps hydrodynamic or magnetic, which can transport angular momentum from the deep interior towards the surface.

Pinsonneault et al. (1989) and Chaboyer, Demarque, & Pinsonneault (1995) modelled the evolution of solar internal rotation, assuming transport described as a diffusive process, the diffusion coefficients being estimated from rotationally induced instabilities. These calculations predicted that the Sun at the present age retained a core rotating at several times the surface rate. Similarly Zahn (1992), based on a model involving also transport by meridional circulation, noted that "we anticipate that the Sun has kept a rapidly rotating core". Hydrodynamical models of the rotating convection zone (Glatzmaier 1985; Gilman & Miller 1986) found that the angular velocity depended largely on the distance to the rotation axis (often described as 'rotation on cylinders'); this is indeed predicted for an inviscid flow in an adiabatic region by the Taylor-Proudman theorem. Consequently, the observed surface differential rotation would correspond to a decrease of angular velocity with depth at the equator; this gradient caused substantial problems in early models of the generation of the Sun's magnetic activity through dynamo action within the convection zone (for a review, see Gilman 1986).

The helioseismic investigations of solar rotation have provided the means for detailed tests of these theoretical predictions, by yielding accurate and detailed inferences of rotation as a function of position within the Sun. Here I briefly describe the techniques and data that have made this possible, and discuss how the resulting inferences have affected our understanding of the properties of stellar rotation. A much more detailed general review of helioseismology was given by Christensen-Dalsgaard (2002), while helioseismic investigations of solar rotation were also described by Thompson et al. (1996) and Christensen-Dalsgaard & Thompson (2003).

2. Seismology of Solar Rotation

The observed modes of solar oscillations have frequencies between around 1 and 5 mHz. They correspond to standing acoustic waves, or p modes, and, at relatively short surface wavelength, to surface gravity waves, known as f modes. The dependence of a mode on colatitude θ and longitude ϕ is given by a spherical harmonic Y_l^m, of degree l and azimuthal order m. For example, the radial component v_r of velocity, as a function of distance r to the centre, θ, ϕ and time t may be written as

$$\begin{aligned} v_r(r,\theta,\phi,t) &= \mathrm{Re}[\tilde{v}_r(r) Y_l^m(\theta,\phi)\exp(-i\omega t)] \\ &= \tilde{v}_r(r) c_{lm} P_l^m(\cos\theta)\cos(m\phi - \omega t + \delta) \ . \end{aligned} \tag{1}$$

Here P_l^m is a Legendre function and c_{lm} is a normalization constant; also, $\tilde{v}_r(r)$ is the radial eigenfunction, ω is the (angular) frequency and δ is a phase. Thus, in particular, modes with $m \neq 0$ are running waves in the longitude direction. It follows from the behaviour of the Legendre functions that the mode is essentially confined between latitudes $\pm\cos^{-1}[|m|/(l+1/2)]$; sectoral modes, with $m = \pm l$, are confined close to the equator, when l is moderate or large. In the Sun modes with degree from 0 and to several thousand are observed, although most of the analyses have been based on modes with $l \lesssim 250$. In addition to l and m a mode is characterized by its radial order n; for the f modes $n = 0$ whereas for p modes n generally corresponds to the number of nodes in the radial direction.

With the exception of radial modes, with $l = 0$, the p modes are characterized by a lower turning point where the mode, regarded as an interference of acoustic waves, undergoes total internal refraction. The distance r_t from the centre to the turning point satisfies

$$\frac{c(r_t)}{r_t} = \frac{\omega}{\sqrt{l(l+1)}} , \tag{2}$$

where c is the adiabatic sound speed. Thus modes of low degree extend over most of the stellar interior, whereas modes of high degree are confined near the surface. The f modes are predominantly located near the surface, increasingly so with increasing degree. Together with the variation with m in latitude extent this variation in the radial extent of the modes provides the basis for obtaining localized information about the properties of the stellar interior when, as in the solar case, modes covering an extensive range in l and m are observed.

In a non-rotating star the frequencies are independent of the azimuthal order m. This degeneracy is lifted by rotation. For the p and f modes the effect is, to a first approximation, simply the result of the advection of the longitudinally running waves described by equation (1) by a suitably averaged angular velocity $\langle\Omega\rangle$; thus the frequency $\omega_{nlm} \simeq \omega_{nl0} + m\langle\Omega\rangle$. More precisely, the *rotational splitting* can be written

$$\delta\omega_{nlm} \equiv \omega_{nlm} - \omega_{nl0} = m \int_0^R \int_0^\pi K_{nlm}(r,\theta)\Omega(r,\theta) r \mathrm{d}r \mathrm{d}\theta , \tag{3}$$

where R is the surface radius of the star, and the kernel $K_{nlm}(r,\theta)$ can be determined from the spherically symmetric structure of the star and its eigenfunctions; as indicated, the angular velocity Ω must in general be regarded as a function of r and θ. Since solar structure has been inferred with high accuracy from helioseismic inversion, the kernels can be assumed to be known. In the special case where $\Omega = \Omega(r)$ depends only on r, the rotational splitting is proportional to m,

$$\delta\omega_{nlm} = m\beta_{nl} \int_0^R K_{nl}(r)\Omega(r)\mathrm{d}r , \tag{4}$$

where β_{nl} is defined such that $\int K_{nl}(r)\mathrm{d}r = 1$.

It is evident that the information contained in the rotational splitting on the angular velocity depends on the structure of the kernels. In particular, few modes of only the lowest degrees extend to the solar core, and, as illustrated in Figure 1, the kernels are comparatively small in the central region. This makes inferences of the core rotation especially difficult; also, since only a few values of m are available, essentially no information can be obtained about the latitude variation in the core. On the other hand, the convection zone is probed by modes of degree $l \gtrsim 40$, allowing good resolution in both the radial and the latitude directions.

An important goal of the analysis of the helioseismic data is to obtain localized information about the properties of the solar interior. In order to illustrate such inverse analyses, I write equation (3) as

$$\Delta_i = \int_0^R \int_0^\pi K_i(r,\theta)\Omega(r,\theta) r \mathrm{d}r \mathrm{d}\theta , \tag{5}$$

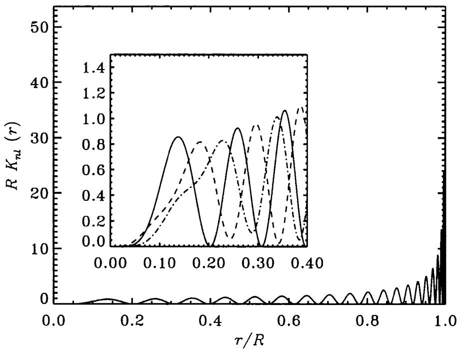

Figure 1. Rotational kernels for spherically symmetric rotation, for modes near a frequency of 3 mHz. The main panel shows an $l = 1$ mode. The insert shows expanded views of the behaviour in the core for $l = 1$ (solid), $l = 2$ (dashed), and $l = 3$ (dot-dashed).

where $i \equiv nlm$ labels the modes, and $\Delta_i = m^{-1}\delta\omega_{nlm}$. In many cases, the result of the analysis, aiming at inferring Ω at some location (r_0, θ_0), say, can be expressed as a linear combination of the data:

$$\overline{\Omega}(r_0, \theta_0) = \sum_i c_i(r_0, \theta_0)\Delta_i \; ; \tag{6}$$

using equation (5) this can be written as

$$\overline{\Omega}(r_0, \theta_0) = \int_0^R \int_0^\pi \mathcal{K}(r_0, \theta_0; r, \theta)\Omega(r, \theta) r \mathrm{d}r \mathrm{d}\theta \; , \tag{7}$$

where the *averaging kernel* is given by

$$\mathcal{K}(r_0, \theta_0; r, \theta) = \sum_i c_i(r_0, \theta_0) K_i(r, \theta) \; , \tag{8}$$

and is typically required to have unit integral over (r, θ). Thus if $\mathcal{K}(r_0, \theta_0; r, \theta)$ is suitably localized near (r_0, θ_0) $\overline{\Omega}(r_0, \theta_0)$ provides an average of $\Omega(r, \theta)$ in the vicinity of that location. Given the *inversion coefficients* $c_i(r_0, \theta_0)$, the variance on the inferred angular velocity can be obtained as

$$\sigma^2[\overline{\Omega}(r_0, \theta_0)] = \sum_i c_i(r_0, \theta_0)^2 \sigma^2(\Delta_i) \; , \tag{9}$$

where for simplicity I assumed that the data errors, with variance $\sigma^2(\Delta_i)$, are uncorrelated.

The details of the inversion method determine how the inversion coefficients are calculated. In one commonly used technique, the so-called regularized least-squares (or RLS) technique the solution is obtained as a parametrized least-squares fit to the data, regularized by limiting also the integral of, e.g.,

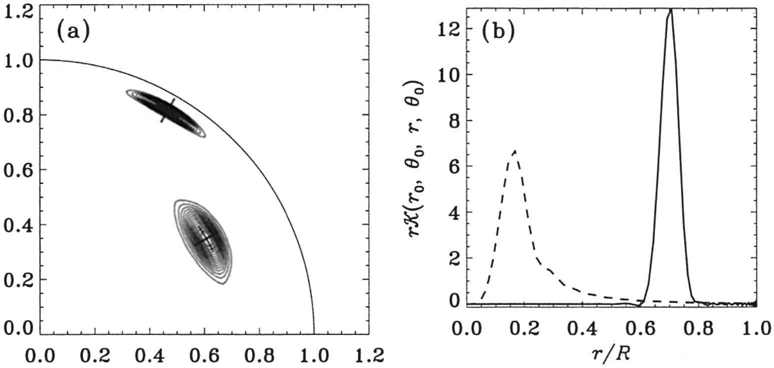

Figure 2. Averaging kernels $\mathcal{K}(r_0, \theta_0; r, \theta)$ for rotational inversions using OLA techniques, on datasets described below. (a) Contour plots, in a solar quadrant, of kernels from analysis of MDI data, at $(r_0, \theta_0) = (0.7R, \pi/3)$ and $(0.94R, \pi/6)$ (the target locations are indicated by crosses). (b) Radial equatorial cuts through kernels targeted at the equator and $r_0 = 0.7R$ (solid line; MDI data) and in the core (dashed line; BiSON and LOWL data). The curves have been normalized such that their integrals with respect to r/R are unity.

the square of the second derivative of the solution. From this fit the inversion coefficients may be calculated, and thence the averaging kernels and errors. In the optimally localized averages (OLA) techniques, the inversion coefficients are explicitly calculated to achieve the desired localization of the averaging kernels, while limiting the errors. Details on these techniques, as applied to inversion for solar rotation, were provided by Schou et al. (1998). To illustrate the potential for obtaining localized information, Figure 2 shows selected averaging kernels for inversions of solar data, discussed in more detail below.

3. The Solar Internal Rotation

Already the early results of global helioseismology provided evidence against the prevailing theoretical ideas on solar internal rotation. Using an ingenious observational technique, Duvall & Harvey (1984) obtained rotational splittings of sectoral modes; the analysis of these data by Duvall et al. (1984) showed no indication of rapid rotation of the radiative interior, even quite close to the solar centre. Also, the initial observations and analyses of the m dependence of rotational splittings (e.g., Brown & Morrow 1987; Christensen-Dalsgaard & Schou 1988; Libbrecht 1988; Brown et al. 1989) indicated that rotation in the convection zone was roughly independent of depth, with a transition near the base of the convection zone to nearly uniform rotation in the radiative interior; this was quite unlike the rotation on cylinders predicted theoretically.

In the last decade very extensive data have become available on rotational splittings. Data on low-degree modes have been obtained with disk-averaged observations from the BiSON (Chaplin et al. 1996) and IRIS (Fossat 1991) ground-based networks, as well as from the GOLF instrument (Gabriel et al.

1997) on the SOHO spacecraft. Also, the LOWL instrument (Tomczyk et al. 1995) has provided data on modes of low and moderate degree, whereas the GONG network (Harvey et al. 1996) has yielded very detailed data for a broad range of modes. An even more extensive mode set has been covered by the MDI instrument (Scherrer et al. 1995) on the SOHO spacecraft. In the following I summarize the results on solar rotation that have been obtained from these observations.

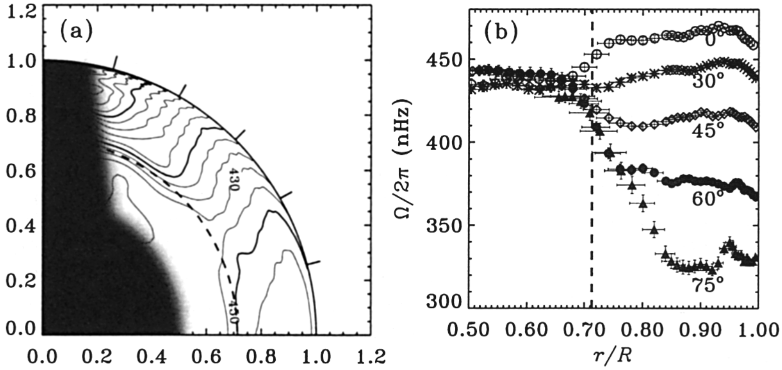

Figure 3. Inferred solar rotation rate $\Omega/2\pi$, from 144 days of observations with the MDI instrument on the SOHO spacecraft, analyzed with OLA inversion. (a) Contour plot in a quadrant of the Sun; the equator is at the horizontal axis and the pole along the vertical axis, both axes being labelled with fractional radius. Some contours are labelled in nHz, and selected contours are shown as bold. The tick marks at the edge of the outer curve are at latitudes 15°, 30°, 45°, 60°, and 75°. The shaded area indicates the region where no reliable inference can be made with these data. (b) Rotation rates as functions of fractional radius, at the latitudes indicated. The vertical error bars show 1-σ errors in the inferred values (cf. eq. 9) and the horizontal bars indicate the radial resolution of the inversion, as determined by the averaging kernels. The dashed circle in (a), and the vertical dashed line in (b), mark the base of the convection zone. Adapted from Schou et al. (1998).

The rotation rate in the convection zone and the outer parts of the radiative interior is illustrated in Figure 3, based on analysis of MDI data (Schou et al. 1998); selected averaging kernels for this inversion were shown in Figure 2. The near-surface rate essentially reproduces the known surface rotation rate; it is perhaps worth pointing out, however, that no information about the surface rate was used in the analysis. As indicated by the earlier investigations, this latitude variation is largely maintained throughout the convection zone. Near the base of the convection zone there is a region of strong rotational shear, in the so-called *tachocline* (Spiegel & Zahn 1992) which provides the transition to the nearly uniform rotation in the radiative interior. The apparent extent of the tachocline, as shown in Figure 3b, includes the effect of the finite resolution of the inference, as determined by the averaging kernels (cf. Fig. 2). By correcting for this it has been estimated that the actual width of the tachocline is less than approximately $0.05R$ (e.g., Corbard et al. 1998; Charbonneau et al. 1999). The strong radial gradient of rotation in the tachocline makes this a likely location for the operation of a dynamo mechanism which may be responsible for the 11-year solar magnetic cycle (e.g., Gilman, Morrow, & DeLuca 1989).

Closer inspection reveals further details in the rotation within the convection zone. Particularly striking is the near-surface shear layer, such that the maximum rotation rate occurs at a depth of around $0.05R$ (see also Corbard & Thompson 2002). The existence of this variation had previously been suspected on the basis of analysis of high-degree modes as well as differences in the rotation rates obtained from different surface tracers likely to be anchored at different depths (Korzennik et al. 1990). Also, the rotation rate at high latitudes shows more complex structure, possibly involving confined jet-like features (Schou et al. 1998; Howe et al. 1998). However, the inferences in this region appear to be affected by systematic errors, as reflected in differences between results obtained from different datasets or using different inversion methods (Schou et al. 2002).

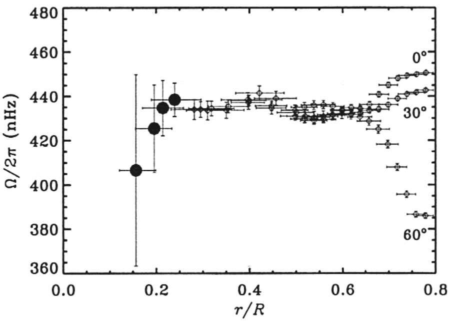

Figure 4. Inferred rotation rate $\Omega/2\pi$ in the solar interior, from OLA analysis of combined BiSON and LOWL observations, as a function of fractional radius. The small symbols show rotation at the equator and latitudes 30° and 60°. The large symbols show results of attempting to confine the averaging kernels to the core in the radial direction, with no confinement in latitude. The vertical bars are 1-σ errors, whereas the horizontal bars indicate resolution in the radial direction. Adapted from Chaplin et al. (1999).

To investigate the rotation of the solar core, data on low-degree modes are required. Since only few m components are available in this case, the total splitting being often comparable with the natural line width of the modes, great care is required in the analysis (e.g., Appourchaux et al. 2000; Chaplin et al. 2001). However, observations extending over several years have yielded relatively precise measurements of the splittings, from the BiSON (Chaplin et al. 2001), the GOLF (Gelly et al. 2002) and, with impressively small formal errors, the IRIS (Fossat et al. 2003) experiments. To illustrate the results of analyzing such data, Figure 4 shows the inferred rotation rate obtained by Chaplin et al. (1999) from a combination of BiSON and LOWL data; the averaging kernel for the most deeply localized inference was shown in Figure 2b. As already discussed above, the central regions make a very small contribution to the rotational splittings, even for the modes of lowest degree. As a result, it is extremely difficult to obtain localized information about the core rotation, and attempts to do so lead to rather large errors in the inferences, as also reflected in the figure. Even so, there is clearly no evidence for a rapidly rotating core; strict constraints on the possible higher rotation rate in the core were obtained by Charbonneau et al. (1998), by applying genetic forward modelling to LOWL data. Indeed, although the results are consistent with constant rotation in the radiative interior, it is tempting to speculate about the possibility of a *slowly* rotating core, relative to the rest of the radiative zone. Indications of slow core rotation, also based on BiSON data, had previously been found by Elsworth et al. (1995), while Corbard et al. (1997) and Eff-Darwich, Korzennik, & Jiménez-Reyes (2002) found a similar tendency from analyses of LOWL, and LOWL, GONG and MDI, data respectively. It should be pointed out, however, that the rotation of the deep solar interior is still uncertain, different datasets yielding rather different results, although none showing the rapid rotation that had been predicted theoretically (for a review, see Eff-Darwich & Korzennik 1998).

The GONG and MDI observations have provided data continuously spanning the period since the previous minimum in solar activity, and hence allow a search for time variations in the solar internal rotation. To search for such variations, the data are analyzed in segments of a few months; the average over time of these results, as a function of (r_0, θ_0) is computed, and the residuals obtained by subtracting the average from the inferred rotation rate for each segment are investigated. In the outer parts of the convection zone the results show bands of slightly more rapid and slower rotation, often described as zonal flows, which converge towards the solar equator as time progresses, with an apparent 11-year periodicity (e.g., Antia & Basu 2000; Howe et al. 2000a; Vorontsov et al. 2002). At the surface, these flows correspond to the so-called *torsional oscillations* previously identified in Doppler observations of solar surface rotation (Howard & LaBonte 1980; Ulrich 2001; for a comparison between the surface and helioseismic results, see Howe, Komm, & Hill 2000b). However, the helioseismic results show that these variations extend through a substantial fraction of the convection zone. Interestingly, the bands appear to follow approximately the location of emerging sunspots as the solar cycle progresses; however, the causal relation between the flows and the sunspots is not understood.

A possibly even more remarkable variation was inferred by Howe et al. (2000c) near the equator at and below the base of the convection zone. At $r = 0.72R$ the residuals showed a roughly periodic variation, with a period of 1.3 year and an amplitude corresponding to around $10\,\mathrm{m\,s^{-1}}$. An oscillation with a similar period but opposite phase was weakly present at $r = 0.63R$, while the variations at higher latitudes were less regular. The oscillations were seen in analyses of both GONG and MDI data using different inversion techniques. Also, the period was shown to be significantly different from 1 year. The physical origin of this oscillation, or its possible relation to the dynamo mechanism that may be operating in the same region, is so far not understood. It should be noted that Basu & Antia (2001) have questioned the statistical significance of this effect. Also, unfortunately, the oscillations seem to have died out around the beginning of 2001 (Howe 2003; Toomre et al. 2003). Although perhaps not unexpected in a such a complex dynamical system, the absence of further data clearly makes it difficult to establish beyond doubt the reality of the phenomenon, or to study it in more detail. It remains to be seen whether the oscillations resume in a later phase of the solar cycle.

4. Discussion

It is striking that the helioseismic inferences are completely at variance with the theoretical predictions on the solar internal rotation, made before helioseismology. In this the case of rotation differs from the case of solar structure, where the theoretical models have been found to agree reasonably well with the inferred sound speed and density (e.g., Gough et al. 1996; Basu et al. 1997; Turck-Chièze et al. 2001). This is perhaps not surprising. The physics of stellar rotation, involving the rather uncertain rate of spin-down, and the transport of angular momentum by turbulent convection in the convection zone and by the still uncertain transport mechanisms in the radiative interior, is substantially more complex than the description of the hydrostatic evolution of the spherically symmetric component of solar structure. Indeed, a major uncertainty in solar structural evolution is the possible presence of mixing processes which may well be related to the evolution of solar rotation. In particular, it is likely that a prominent difference between the solar and the model sound speeds just below the convection zone is caused by mixing related to the tachocline (e.g., Brun, Turck-Chièze, & Zahn 1999; Elliott & Gough 1999).

Modelling of rotation in the convection zone is discussed in these proceedings by Toomre & Brun (see also Brun & Toomre 2002). It is encouraging that the increase in numerical resolution, and the resulting turbulent character of the simulations of convection, is acting to bring the computed rotation profile into closer agreement with the observations. Also, the simulations allow identification of the mechanisms that dominate the redistribution of angular momentum. With further development of computational resources and numerical techniques, and the possible inclusion of magnetic effects, we may hope to be able to account fully for the helioseismically inferred rotation profile and perhaps understand the origin of the zonal flows and their relation to solar magnetic activity. Indeed, in a solar mean-field dynamo model Covas, Tavakol, & Moss (2001) found flow patterns similar to those observed; they also noted a tendency for period halvings to appear in the simulations which might account for the 1.3-year oscillation. It should be noted, however, that a full understanding of these dynamic phenomena will also require further observations, ideally over several solar cycles; this is true, in particular, for the 1.3-year oscillation if this is restricted to certain phases of the solar cycle, as appears to be the case.

From the earlier models of the rotation in the radiative interior it is evident that mechanisms beyond simple instabilities or transport by meridional circulation are required to account for the present nearly uniform rotation. Mestel & Weiss (1987) noted that a weak magnetic field in the radiative interior would suffice to provide the coupling needed to maintain uniform rotation. Numerical modelling by Charbonneau & MacGregor (1993) of the evolution of rotation in the presence of a primordial magnetic field showed that near-uniform rotation of the radiative interior would in fact be achieved over a broad range of initial magnetic parameters. Alternatively, it has been proposed that gravity waves, generated at the base of the convection zone and dissipated in the interior, could provide the required angular-momentum transport (e.g, Kumar & Quataert 1997; Zahn, Talon, & Matias 1997). Gough & McIntyre (1998), with reference to laboratory experiments and the Earth's atmosphere, pointed out that the simple model for gravity-wave transport would not lead to the required reduction of the angular momentum of the deep interior. Instead, they proposed a model where a weak magnetic field, and circulation near the base of the convection zone, established the tachocline and the nearly uniform rotation of the radiative interior; they noted that the field was needed to restrain the tachocline from extending more deeply than observed, the required field strength being entirely consistent with a primordial field in the solar interior. However, Talon, Kumar, & Zahn (2002) further developed the gravity-wave model in a form that overcame the original objections; they found that the low angular momentum in the core led it to be spun down rapidly, potentially leading to a slow core as may have been observed. At present it seems difficult to decide which of these two models are more likely to be correct. If further helioseismic data definitely support the slow core, transport by gravity waves may be favoured; however, it is possible that magnetic coupling between the core and the polar regions of the convection zone could also lead to a slow core.

It is evident that any model attempting to explain the evolution of stellar internal rotation must as a minimum account for the detailed profile observed in the Sun. This constraint has certainly been applied in many of the contributions to these proceedings; an interesting example is the modelling by Talon & Charbonnel (these proceedings) of rotation and lithium abundances in lower-main-sequence stars, assuming angular momentum transport by gravity waves as applied in the solar case by Talon et al. (2002). Similarly strong constraints on internal rotation are unlikely to be ever obtained for other stars. However, there is no doubt that the ongoing efforts towards asteroseismic investigations of a broad range of stars, particularly given upcoming space missions, will provide further observations relevant to the internal rotation of stars. In a distant future, it may become possible to carry out such observations interferometrically from space, providing some resolution of the stellar disks and hence the ability to determine rotational splittings of modes of moderate degree. In this way one may hope, for example, to be able to study the properties of tachoclines in other stars.

However, the detailed results available for the Sun will undoubtedly remain a reminder of the well-established fact that even such a simple thing as a star looks a lot less simple at close range.

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References

Antia, H.M., Basu, S. 2000, ApJ 541, 442

Appourchaux, T., Chang, H.-Y., Gough, D.O., Sekii, T. 2000, MNRAS 319, 365

Basu, S., Antia, H.M. 2001, MNRAS 324, 498

Basu, S., Chaplin, W.J., Christensen-Dalsgaard, J., Elsworth, Y., Isaak, G.R., New, R., Schou, J., Thompson, M.J., Tomczyk, S. 1997, MNRAS 292, 243

Brown, T.M., Morrow, C.A. 1987, ApJ 314, L21

Brown, T.M., Christensen-Dalsgaard, J., Dziembowski, W.A., Goode, P., Gough, D.O., Morrow, C.A. 1989, ApJ 343, 526

Brun, A.S., Toomre, J. 2002, ApJ 570, 865

Brun, A.S., Turck-Chièze, S., Zahn, J.P. 1999, ApJ 525, 1032 (Erratum: ApJ 536, 1005)

Chaboyer, B., Demarque, P., Pinsonneault, M.H. 1995, ApJ 441, 865

Chaplin, W.J., Elsworth, Y., Howe, R., Isaak, G.R., McLeod, C.P., Miller, B.A., van der Raay, H.B., Wheeler, S.J., New, R. 1996, Solar Physics 168, 1

Chaplin, W.J., Christensen-Dalsgaard, J., Elsworth, Y., Howe, R., Isaak, G.R., Larsen, R.M., New, R., Schou, J., Thompson, M.J., Tomczyk, S. 1999, MNRAS 308, 405

Chaplin, W.J., Elsworth, Y., Isaak, G.R., Marchenkov, K.I., Miller, B.A., New, R. 2001, MNRAS 327, 1127

Charbonneau, P., MacGregor, K.B. 1993, ApJ 417, 762

Charbonneau, P., Tomczyk, S., Schou, J., Thompson, M.J. 1998, ApJ 496, 1015

Charbonneau, P., Christensen-Dalsgaard, J., Henning, R., Larsen, R.M., Schou, J., Thompson, M.J., Tomczyk, S. 1999, ApJ 527, 445

Christensen-Dalsgaard, J. 2002, Rev. Mod. Phys. 74, 1073

Christensen-Dalsgaard, J., Schou, J. 1988, in: V. Domingo & E.J. Rolfe (eds), Seismology of the Sun & Sun-like Stars (Noordwijk: ESA Publications Division), ESA SP-286, 149

Christensen-Dalsgaard, J., Thompson, M.J. 2003, in: B.N. Dwivedi (ed.), The Dynamic Sun (Cambridge: Cambridge University Press), in the press

Corbard, T., Thompson, M.J. 2002, Solar Physics 205, 211

Corbard, T., Berthomieu, G., Morel, P., Provost, J., Schou, J., Tomczyk, S. 1997, A&A 324, 298

Corbard, T., Berthomieu, G., Provost, J., Morel, P. 1998, A&A 330, 1149

Covas, E., Tavakol, R., Moss, D. 2001, A&A 371, 718

Duvall, T.L., Harvey, J.W. 1984, Nature 310, 19

Duvall, T.L., Dziembowski, W.A., Goode, P.R., Gough, D.O., Harvey, J.W., Leibacher, J.W. 1984, Nature 310, 22

Eff-Darwich, A., Korzennik, S.G. 1998, in: S.G. Korzennik & A. Wilson (eds), Structure and dynamics of the interior of the Sun and Sun-like stars; Proc. SOHO 6/GONG 98 Workshop (Noordwijk: ESA Publications Division), ESA SP-418, 685

Eff-Darwich, A., Korzennik, S.G., Jiménez-Reyes, S.J. 2002, ApJ 573, 857

Elliott, J.R., Gough, D.O. 1999, ApJ 516, 475

Elsworth, Y., Howe, R., Isaak, G.R., McLeod, C.P., Miller, B.A., New, R., Wheeler, S.J., Gough, D.O. 1995, Nature 376, 669

Fossat, E. 1991, Solar Physics 133, 1

Fossat, E., Salabert, D., Cacciani, A., et al. 2003, in: A. Wilson (ed.), Proc. SOHO 12 / GONG+ 2002. Local and Global Helioseismology: The Present and Future (Noordwijk: ESA Publications Division), ESA SP-517, in the press

Gabriel, A.H., Charra, J., Grec, G., et al. 1997, Solar Physics 175, 207

Gelly, B., Lazrek, M., Grec, G., Ayad, A., Schmider, F.X., Renaud, C., Salabert, C., Fossat, E., 2002. A&A 394, 285

Gilman, P.A. 1986, in: P.A. Sturrock, T.E. Holzer, D. Mihalas, & R.K Ulrich (eds), Physics of the Sun, Vol. 1 (Dordrecht: Reidel), 95

Gilman, P.A., Miller, J. 1986, ApJS 61, 585

Gilman, P.A., Morrow, C.A., DeLuca, E.E. 1989, ApJ 338, 528

Glatzmaier, G. 1985, ApJ 291, 300

Gough, D.O., McIntyre, M.E. 1998, Nature 394, 755

Gough, D.O., Kosovichev, A.G., Toomre, J., et al. 1996, Science 272, 1296

Harvey, J.W., Hill, F., Hubbard, R.P., et al. 1996, Science 272, 1284

Howard, R., LaBonte, B.J. 1980, ApJ 239, L33

Howe, R. 2003, in: A. Wilson (ed.), Proc. SOHO 12 / GONG+ 2002. Local and Global Helioseismology: The Present and Future (Noordwijk: ESA Publications Division), ESA SP-517, in the press

Howe, R., Antia, H.M., Basu, S., Christensen-Dalsgaard, J., Korzennik, S.G., Schou, J., Thompson, M.J. 1998, in: S.G. Korzennik & A. Wilson (eds), Structure and dynamics of the interior of the Sun and Sun-like stars; Proc. SOHO 6/GONG 98 Workshop (Noordwijk: ESA Publications Division), ESA SP-418, 803

Howe, R., Christensen-Dalsgaard, J., Hill, F., Komm, R.W., Larsen, R.M., Schou, J., Thompson, M.J., Toomre, J. 2000a. ApJ 533, L163

Howe, R., Komm, R., Hill, F. 2000b, Solar Physics 192, 427

Howe, R., Christensen-Dalsgaard, J., Hill, F., Komm, R.W., Larsen, R.M., Schou, J., Thompson, M.J., Toomre, J. 2000c. Science 287, 2456

Korzennik, S.G., Cacciani, A., Rhodes, E.J., Ulrich, R.K. 1990, in: Y. Osaki & H. Shibahashi (eds), Progress of seismology of the sun and stars, Lecture Notes in Physics, vol. 367 (Berlin: Springer), 341

Kumar, P., Quataert, E.J. 1997, ApJ 475, L143

Libbrecht, K.G. 1988, in: V. Domingo & E.J. Rolfe (eds), Seismology of the Sun & Sun-like Stars (Noordwijk: ESA Publications Division), ESA SP-286, 131

Mestel, L., Weiss, N.O. 1987, MNRAS 226, 123

Pinsonneault, M.H., Kawaler, S.D., Sofia, S., Demarque, P. 1989, ApJ 338, 424

Scherrer, P.H., Bogart, R.S., Bush, R.I., et al. 1995, Solar Physics 162, 129

Schou, J., Antia, H.M., Basu, S., et al. 1998, ApJ 505, 390

Schou, J., Howe, R., Basu, S., Christensen-Dalsgaard, J., Corbard, T., Hill, F., Larsen, R.M., Rabello-Soares, M.C., Thompson, M.J. 2002, ApJ 567, 1234

Spiegel, E.A., Zahn, J.-P. 1992, A&A 265, 106

Talon, S., Kumar, P., Zahn, J.-P. 2002, ApJ 574, L175

Thompson, M.J., Toomre, J., Anderson, E.R., et al. 1996, Science 272, 1300

Tomczyk, S., Streander, K., Card, G., Elmore, D., Hull, H., Cacciani, A. 1995, Solar Physics 159, 1

Toomre, J., Christensen-Dalsgaard, J., Hill, F., Howe, R., Komm, R. W., Schou, J., Thompson, M. J. 2003, in: A. Wilson (ed.), Proc. SOHO 12 / GONG+ 2002. Local and Global Helioseismology: The Present and Future (Noordwijk: ESA Publications Division), ESA SP-517, in the press

Turck-Chièze, S., Couvidat, S., Kosovichev, A.G., Gabriel, A.H., Berthomieu, G., Brun, A.S., Christensen-Dalsgaard, J., García, R.A., Gough, D.O., Provost, J., Roca-Cortes, T., Roxburgh, I.W., Ulrich, R.K. 2001, ApJ 555, L69

Ulrich, R.K. 2001, ApJ 560, 466

Vorontsov, S.V., Christensen-Dalsgaard, J., Schou, J., Strakhov, V.N., Thompson, M.J. 2002, Science 296, 101

Zahn, J.-P. 1992, A&A 265, 115

Zahn, J.-P., Talon, S., Matias, J. 1997, A&A 322, 320