THE STRUCTURE AND INTERNAL KINEMATICS OF OPEN CLUSTERS

Robert D. Mathieu
Center for Astrophysics
Cambridge, Massachusetts

I. INTRODUCTION

Open clusters provide an opportunity for observational study of several important facets of stellar dynamics which cannot easily be addressed in globular clusters for either fundamental or practical reasons. In particular, open clusters are case studies for:

1) The dynamics of small-N stellar systems. This aspect of open clusters has been excellently reviewed by King (1980), to which the reader is referred. The work of Retterer (1979) and Goodman (this volume) suggests that any differences in the velocity distributions of small-N systems as compared to large-N systems due to the increased importance of close stellar encounters will appear primarily in the high-energy tails of the distributions. This will be reflected in the structure of the cluster halos, where quality data are difficult to obtain. In order to guide observational study, more theoretical work is needed regarding the expected nature and magnitude of any observable deviations from large-N theory.

2) The effects of a large stellar-mass spectrum. While globular clusters probably also have a spectrum of masses spanning an order of magnitude or more, the extension of the mass spectrum to higher masses in open clusters permits the dynamical consequences of a mass spectrum to be more easily studied. In addition, many studies have pointed out the significance of stellar mass loss in massive stars for the evolution of clusters (e.g., Terlevich 1983). However, as yet the theory regarding the effects of such mass loss has not provided unique observational predictions which might act as critical tests, so this important issue will not be discussed further here.

3) A spectrum of cluster ages. Open clusters span the complete spectrum of age from birth to dissolution. Except for the classic work of Wielen (1971 and this volume) in which he analyses the observed distribution of cluster ages, this feature of open clusters has been little exploited. Yet by mapping appropriate diagnostics with cluster

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age open clusters have the potential to provide empirical measures of the fundamental dynamical time-scales, e.g. the relaxation time, equipartition time and mixing time. Also, study of the very youngest clusters provides appropriate initial conditions for evolutionary models as well as insight into star- and cluster-formation processes.

4) The effect of the external environment. Open cluster halos are molded by the galactic tidal field. In addition, open clusters are subject to impulsive encounters with clouds in the interstellar medium. The effects of these environmental influences can be examined in numerous ways, including detailed analyses of the structure of cluster halos and observations of the variation with radius of anisotropy in the cluster velocity distributions.

Finally, open clusters have the advantage that many lie within 1 kpc of the sun. Thus, in principle, for any given magnitude limit stars of lower mass can be studied in open clusters than in globular clusters. In practice membership difficulties in open clusters reduce this advantage. The proximity of open clusters also allows excellent spatial resolution; every star in an open cluster can be precisely placed in two dimensions. There is no need to assume a particular mass function in order to study integrated light distributions. In addition, in the nearest open clusters the dynamically active binaries are resolvable, providing the only opportunity to test the extensive theory concerning their role as an energy source for clusters.

Unfortunately, while open clusters provide exciting opportunities for stellar dynamical research, they are, from a practical point of view, difficult systems to study. The inherent low-number statistics are often a severe limitation. The low total masses imply small internal motions (usually less than 1 km/sec in one dimension) which are a technical challenge to measure. And, perhaps most frustrating, without a great deal of work one is not even certain which stars are cluster members due to the extensive background contamination from the galactic disk. Fortunately, because of the narrowly confined distribution of open cluster members in velocity space they can be isolated through precise velocity measurements (usually via proper motions, but the radial-velocity techniques of Gieseking (1981) have also been effective). Such membership studies are extensive projects, although technical advances are rapidly lowering this threshold. In addition, proper-motion studies are significantly limited by the areal coverage and magnitude limits of first epoch plates. Nonetheless, essentially every study of open cluster dynamics in the last decade has begun with a comprehensive membership study, and the fruitful results should provide the motivation for more extensive membership studies in the near future as more recent generation plates become ripe for measurement.

2. THE STRUCTURE OF OPEN CLUSTERS

Historically the study of cluster structure has preceded the study of internal motions, and so we begin the review here. Even casual visual inspection of a deep photograph of a young cluster reveals that the more luminous (and thus more massive) stars are more centrally concentrated than the fainter cluster members. It has long been realized that a reasonable explanation for this mass segregation lies in the equipartition of energy among the cluster members, where the more massive stars give up energy to the lighter stars and fall toward the center of the potential well. One primary goal of recent work on open cluster structure has been to study more precisely to what extent the mass segregation can in fact be ascribed to energy equipartition and to determine whether the equipartition is complete among all stellar species.

One of the first clusters analysed from this perspective was the Hyades cluster (Pels, Oort and Pels-Kluyver 1975). In Fig. 1a are shown the stellar surface-density distributions (filled circles) for two stellar mass groups with average masses of 1.36 $M_\odot$ (Group 1) and 0.61 $M_\odot$ (Group 2). The mass segregation is evident in the higher central concentration of the Group 1 stars. Pels et al. point out that there is also mass segregation within the Group 1 stars, as noted earlier by van Bueren (1952) who showed that the A stars and red giants are all located very near the cluster center. This is also evident in the cumulative radial distributions (Fig. 1a) where the stars have been more finely divided in stellar mass. It is interesting that the lowest two mass groups are not markedly different in their spatial distribution, despite having the same ratio in average mass as the two intermediate mass groups. This will be further discussed below.

Pels et al. fit to the data dynamical models based on a truncated Maxwellian velocity distribution; the best-fit models are indicated by the open symbols. The fit of the models to the data is reasonable. More significantly, the best-fit models have velocity distributions with moduli of precision related as the inverse square root of the average stellar mass of each group, as would be expected for a cluster in equipartition.[1]

Quite a few proper-motion membership studies of open clusters have been done in the last 15 years. One of the best such studies examined the rich intermediate-age (2×10^8 yrs) cluster M11 (McNamara, Pratt and Sanders 1977). The structure of this cluster has been studied in detail

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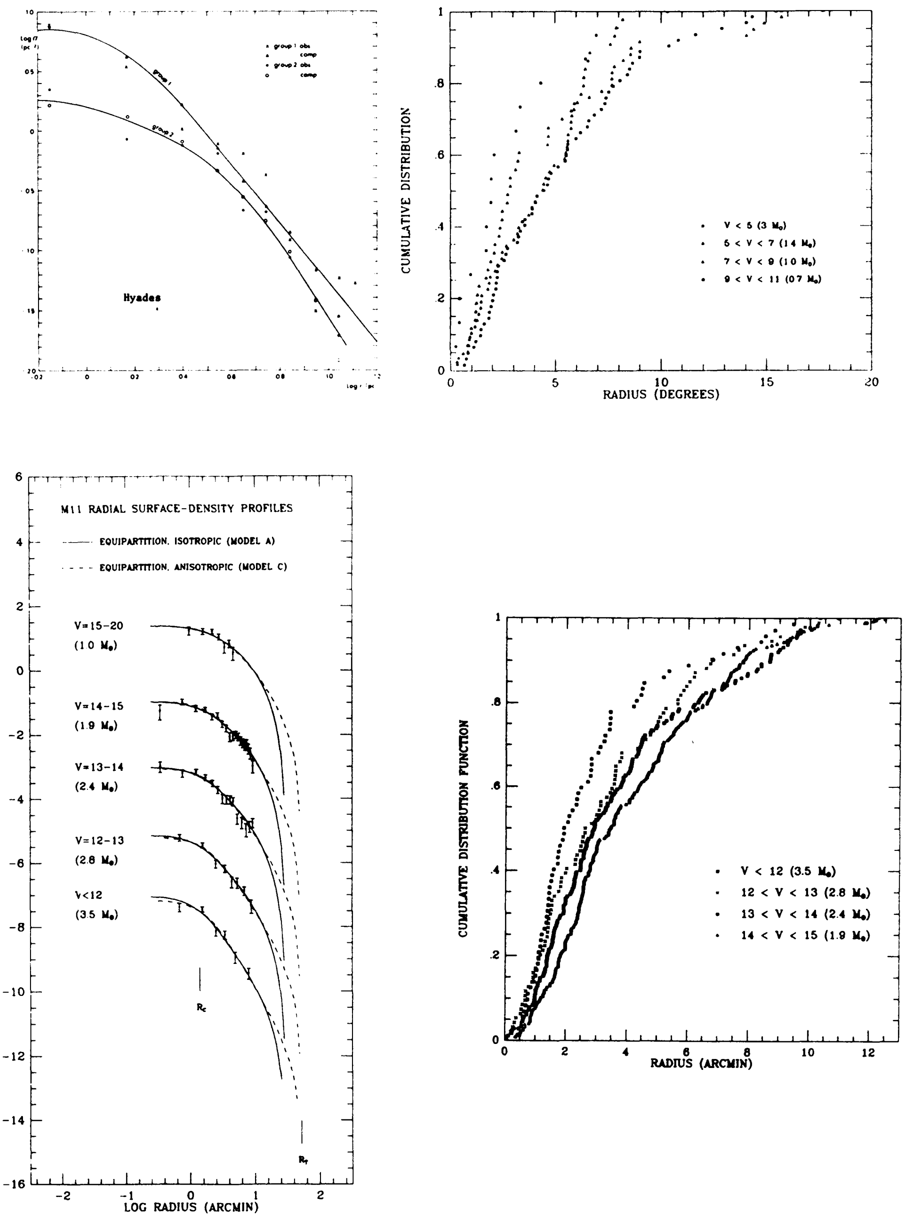
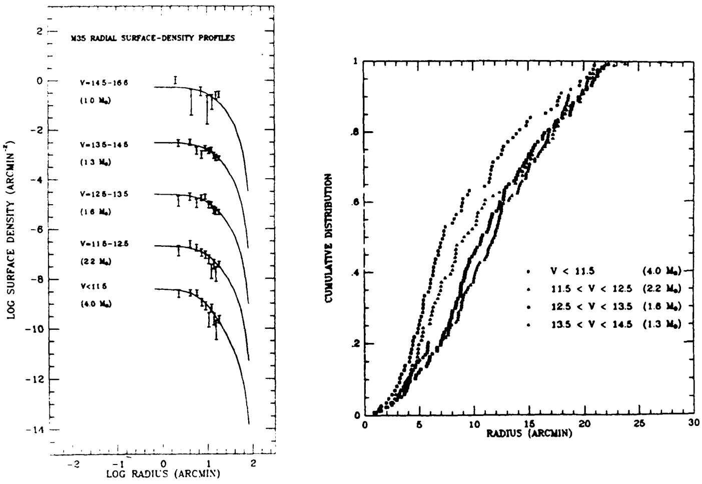
[1] Over the last decade Upgren and Weis have obtained precise photometry for roughly 800 stars with V < 16 in the Hyades field, including essentially every star on anyone's list of proper-motion candidates for Hyades membership. This work is now completed and will hopefully in the near future permit more extensive studies of the Hyades structure and luminosity function.



a)

b)

Fig. 1. Surface-density and cumulative radial distributions as a function of stellar mass. The assorted dynamical models shown are described in the text. a) Surface density taken from Pels *et al.* 1975, cumulative distribution from data in Oort 1979, b) taken from Mathieu 1984, c) and d) taken from Mathieu 1983b and e) taken from van Leeuwen 1983.



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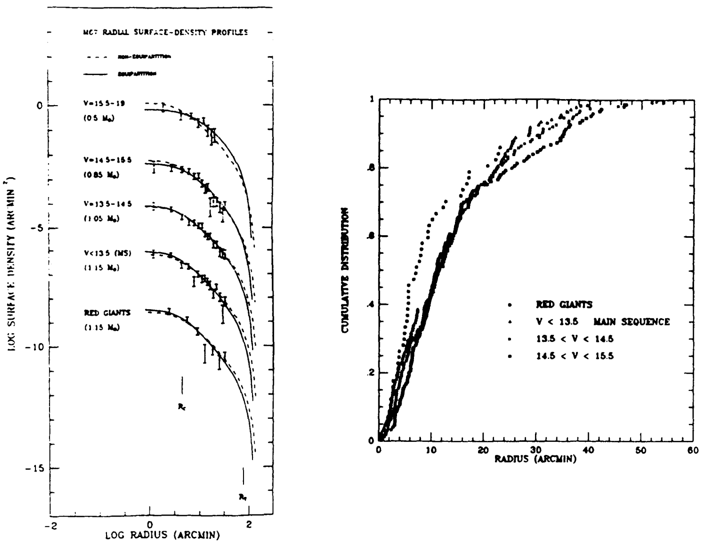

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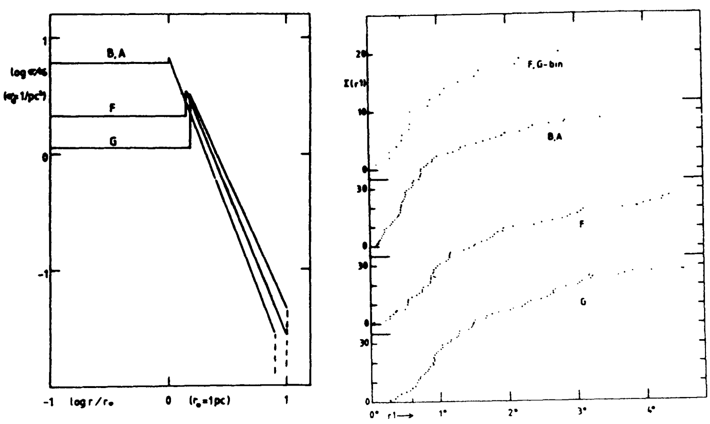

e)

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by Mathieu (1984) who also included star counts done on deep 4m plates in order to study the distribution of stars down to 0.7 $M_\odot$. The stellar surface-density distributions and the cumulative radial distributions (the latter for the proper-motion members only) are given as a function of stellar mass in Fig. 1b. Again mass segregation is evident; in fact the degree of central concentration is rather sensitive to stellar mass. The central concentration decreases continuously with lower stellar mass down to the lower mass limit of the data, although the star-count data cannot resolve the relative distributions of stars with masses between 0.7 and 1.9 $M_\odot$. In order to study in detail the cluster dynamics, multi-mass isotropic King models were fit to the data with varying assumptions concerning the dependence of the velocity distributions on stellar mass. Models without any energy equipartition were first considered (i.e. the velocity distributions were independent of stellar mass); no acceptable fits to the data were found. Next, models in complete equipartition were fit; such models provided excellent fits, as shown in Fig. 1b. Finally, models were fit in which only the two most massive stellar components were in equipartition while the lower three components were forced to have the same velocity dispersion. (The motivation here comes from Monte Carlo simulations of three-component models with mass ratios 6.25:2.5:1 (Spitzer and Shull 1975). In these models the most massive component quickly concentrated in the core, ejecting rapidly lower mass stars into the outer regions of the cluster where relaxation times are relatively long. Thus the spatial distribution of both low-mass components were very similar despite their large mass ratio.) These models were unable to fit the data well, although the best fit case could only be rejected at the 90% confidence level. The M11 data were thus able to distinguish between possible models and strongly suggest that, at least for stars with masses greater than roughly 1 $M_\odot$ within 7 core radii, the cluster is in a state of complete energy equipartition.

In Figs. 1c and 1d are shown the stellar surface-density and cumulative radial distributions of M35 and M67 (taken from Mathieu (1983b), using the proper-motion membership studies of Cudworth (1971; M35) and Sanders (1977; M67) as well as star counts done by the author in M35 and taken from van den Bergh and Sher (1960) in M67). The young cluster M35 (2×10^7 yr) appears structurally very similar to M11, with mass segregation again being present among all stars with masses greater than 1 $M_\odot$. Interestingly, the oldest cluster studied, M67 (5×10^9 yr), has the least evident mass segregation. (The apparent central concentration of the red giants is only significant at the 85% confidence level.) While the stellar mass range of the M67 proper-motion study is small, it still spans a ratio in stellar mass larger than that of the two most massive components in M11, where the segregation is readily apparent. Formally the data cannot distinguish between equipartition and non-equipartition models. However, the lowest mass stellar component (derived from star counts) does appear less centrally concentrated and models in complete equipartition provide somewhat better fits to the data than models without equipartition. (See further discussion of this point after the question of Bahcall.)

Finally, the stellar surface-density and cumulative radial distributions for the Pleiades, taken from van Leeuwen (1983), are shown in Fig. 1e. Interestingly, while the B-A stars are notably centrally concentrated, there is no statistically significant distinction between the spatial distributions of the F (1.2 $M_{\odot}$) and G (0.8 $M_{\odot}$) stars. This lack of evident mass segregation among the lower-mass stars is similar to the spatial distribution found in the Hyades and may be a general phenomena.

It is not obvious what is the expected degree of mass segregation among the lowest mass stars in a cluster. If a cluster is subject to the galactic tidal field, then in equipartition the velocity distributions of the lowest mass stars will be the most severely truncated. This will result in a lesser degree of mass segregation among these stars, but the magnitude of the effect is dependent on the severity of the tidal truncation. As already noted, Spitzer and Shull (1975) also found a lesser degree of mass segregation among lower mass stars in their simulations of isolated clusters. Finally, Wielen (private communication) finds in his N-body simulations that mass segregation becomes much less evident below the average stellar mass; however, Terlevich (1983) finds in her models that mass segregation remains among even stars of the lowest stellar masses. In all cases the precise nature of the mass function is important in that the degree of mass segregation is dependent on which stars play the dominant roles in relaxation processes and in determining the cluster potential.

Clearly we need to turn to the clusters themselves to find the answers. Unfortunately we immediately encounter one of the primary deficiencies in the data available for open clusters. In most cases the magnitude limits of open cluster membership studies exclude low-mass stars, in particular those with masses comparable to or less than the average stellar mass in the cluster. In order to be able to determine the extent of mass segregation among the lowest mass stars and thus obtain some insight into the physical processes determining their velocity distributions, we are badly in need of membership studies that extend to 0.5 $M_{\odot}$ ($M_V = 9$) or lower.

Returning again to the Pleiades (Fig. 1e), van Leeuwen has presented the cumulative radial distribution of binaries (spectral types F-G) in the Pleiades. (These binaries were selected photometrically and are thus biased toward systems of comparable mass components.) The binaries are centrally concentrated relative to "single" stars of similar spectral type. Similar results have been found in the Hyades (Mathieu, Stefanik and Latham 1985) and in several other clusters (Abt 1980), although only in the Pleiades are the relative central concentrations so distinct. Mathieu (1983b) has also found a pronounced central concentration of the blue stragglers in M67, which coincides with the central concentration of the spectroscopic binaries in the cluster (Mathieu and Latham, 1985a). All of these results point to the expected conclusion that binaries, being more massive, tend to be more centrally concentrated.

Leaving the subject of energy equipartition we next address the structure of cluster halos and processes which mold the high energy tails of the velocity distributions. Here we are hampered by the lack of proper-motion membership studies far from cluster centers, due primarily to limitations in coverage of the early epoch plates. The Hyades are in fact the only cluster which has been studied beyond the theoretical tidal radius (Pels et al. 1975). The findings in this one case are indeed interesting; consider again Fig. 1a. The theoretical tidal limit is 9.5 pc. Yet there are 9 stars beyond this limit ($10 < R < 14$ pc) which appear to be members; also, the outermost region was incompletely studied so several additional stars would be expected. Pels et al. suggest that these stars are in the process of escaping the cluster; in particular they suggest that these stars have escape energy but have not yet found the hole in the equipotential surfaces. (See also the paper by Seitzer in this volume.) On the other hand, it remains possible that these stars are members of the Hyades moving group rather that the cluster itself. More studies beyond cluster tidal radii are needed both to examine the process of stellar escape and to determine the frequency and nature of moving groups.

Both theory and numerical simulations suggest that in the presence of the galactic tidal field the outer regions of clusters should be flattened with axial ratios of 2:1.4:1 in the directions toward the galactic center, in the galactic plane perpendicular to the galactic center and perpendicular to the galactic disk, respectively (e.g., Wielen 1975). Interestingly, after a reanalysis of the Hyades in which photometric information was included in the membership selection, Oort (1979) found a significant flattening of the cluster outside a 4 pc radius in the expected orientation. However, the degree of flattening was significantly greater than predicted by theory. Marginal evidence for flattening along the galactic plane has also been found in the Pleiades (van Leeuwen 1983). NGC 3532 shows strong flattening (Gieseking 1981), but roughly orthogonal to the galactic plane. No significant flattening has been found in the clusters studied by Mathieu (1983b); however only in the case of M67 do the data extend beyond 5 pc radius. In summary, there may be evidence for the flattening of some clusters along the galactic plane; whether the degree of flattening is consistent with the tidal theory is not clear and better data than available at present will be required before a definitive comparison of the tidal theory and cluster halo structure is possible.

It is clear from these few studies of cluster halos that 1) the findings have been divergent from simple theoretical expectations and thus the subject merits further study and 2) the data available are entirely insufficient. As already mentioned, proper-motion studies are presently severely limited by the coverage of first epoch plates. This limitation can best be solved by the construction of an appropriate plate collection and the passage of time. Van Leeuwen is presently creating such a collection which will be of value for membership studies in 10-15 years. Until then, those working with proper motions should also examine existing collections for fortuitous coverage of regions around clusters. The radial-velocity membership techniques of Gieseking (1981) may provide an alternative, short time-scale method. In any case, the subject of cluster halos remains poorly understood, both observationally and theoretically, and we should direct our efforts toward a marked improvement in membership studies at a tidal radius and beyond.

3. THE INTERNAL KINEMATICS OF OPEN CLUSTERS

The dynamical models fit to the stellar spatial distributions in open clusters make very definite predictions concerning the internal kinematics of these clusters. Technical advances now permit the measurement of the internal motions of open clusters and within the next half-decade a wealth of data should become available to permit a detailed confrontation of observation and theory. Thus the review given here is, in many ways, only a preliminary report.

3.1 Radial Velocities

Radial-velocity measurements with single-measurement precisions of 0.5 km/sec for stars with $V < 15$ have become possible at several observatories around the world. Mathieu and Latham (1985b; see also Mathieu 1983b) have recently completed a kinematic study of the clusters M11 and M67. In M11 30 giants were observed, essentially a complete sample of the cluster giants. In M67 28 giants/subgiants were observed, selected from the cluster core. In addition, 25 giants in NGC 2506 have been observed, selected from the proper-motion study of Chiu and van Altena (1981). Spectra were obtained with the echelle spectrographs (10 km/sec resolution) behind the FLWO 1.5m and the MMT. The radial velocities were measured with cross-correlation techniques (see Tonry and Davis 1979). Single-measurement precisions were 0.6 km/sec, with 5-10 observations per star obtained over 3 years. (In the following, detected spectroscopic binaries will not be considered; see Mathieu 1983b for a complete discussion.)

The apparent velocity distributions are shown in Fig. 2, both in order to emphasize the degree to which the distributions are resolved and to present an essential difficulty in the use of radial velocities to study open cluster kinematics. The single-measurement precision is comparable to the bin size in Fig. 2. Thus in M11 and NGC 2506 the velocity distributions are well resolved. In M67 the cluster velocity dispersion is comparable to the measurement precision; indeed the narrowness of the M67 distribution is indicative of the resolution achieved for the other clusters.

Attention need be drawn to the highest velocity star in NGC 2506 and the lowest velocity star in M11. The former is a proper-motion probable member, but is 4σ off the cluster mean as determined by the remainder of the stars. The latter appears to be a binary on photometric grounds. However, neither star has yet been detected as a binary spectroscopically. Therein lies what may be the fundamental limitation in the use of radial velocities to study the internal kinematics of open clusters. The internal motions of open clusters are sufficiently small as to be comparable to the orbital velocities of long-period binaries. (For example, the primary of a 100 yr period binary can easily be observed to have a radial velocity due to orbital motion of 0.5-1.0 km/sec relative to the binary center-of-mass radial velocity, which is of course what one wishes to measure in order to study cluster kinematics.) However, the periods of such binaries are sufficiently long that their binary nature is not easily detectable. Since such long-period binaries will tend to overpopulate the tails of the observed cluster velocity distributions, measurements of the velocity dispersion will be inflated.

The biases due to binaries are discussed in detail in Mathieu (1983b), where it is shown that the root mean square is a poor measure of the cluster velocity dispersion in the presence of binaries. Instead, the inter-quartile-range (a statistic less sensitive to the extremes of a distribution) was chosen as a statistic by which to measure the true velocity dispersion. This statistic was calibrated against velocity dispersion with Monte Carlo techniques. This of course necessitates modeling the significant characteristics of a cluster binary population. Fortunately, several detailed studies suggest that cluster binary populations are similar to that in the field (Mathieu,

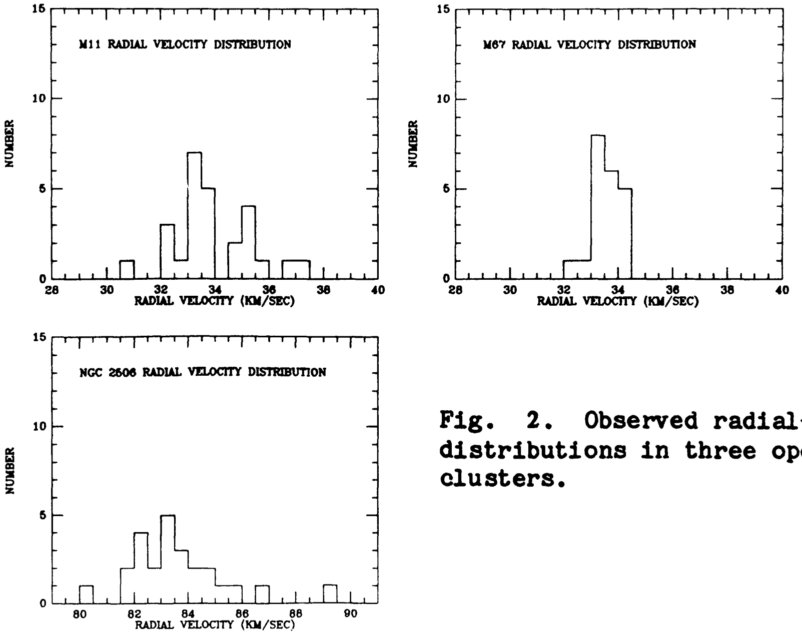

Fig. 2. Observed radial-velocity distributions in three open clusters.

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Stefanik and Latham 1985 (Hyades); Brosche and Hoffman 1979 (Pleiades); Bettis 1977 (Pleiades, Praesepe, Hyades)). In addition, Mayor and Mermilliod (1983) have found little variation in the spectroscopic-binary frequency among the Pleiades, Praesepe and Coma clusters. Nonetheless this analysis technique is susceptible to systematic errors if binary populations do vary from cluster to cluster.

The radial-velocity dispersions for M11 and M67 are given in Table I, both with and without the correction for binaries.[2] As expected, correction for the presence of binaries leads to systematically lower values for the velocity dispersion, but with larger errors since valuable information in the tail of the distribution is lost when using the inter-quartile-range.

Table I

Comparison of Theoretical and Observed Radial-Velocity Dispersions

	$\langle V_R^2\rangle^{1/2}$ (km/sec)		
	Theoretical	Observed	
		with binary correction	w/o binary correction
M11	1.09	1.21 ± 0.35	1.43 ± 0.20
M67	0.49	0.25 ± 0.18	0.48 ± 0.09

Also presented in Table I are the theoretically predicted velocity dispersions as derived from the best-fit dynamical models. Following Abt and Levy (1976) an additional 50% of the observed cluster mass has been assumed to be present in binary secondaries. Also, in the case of M11, an additional component of unseen low-mass cluster members has been included in the models, assuming a cluster mass function like the field initial mass function. This component produces only a 9% increase in the predicted velocity dispersion over that derived from models assuming no unseen mass.

The agreement between theory and observation is reasonably good. Thus the observed cluster structure and internal kinematics do appear properly related by the stellar dynamical theory. Such agreement is in fact not common. In clusters of galaxies, individual galaxies, some globular clusters and the disk of our own galaxy the particle motions

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[2] The data are presently being reanalysed and these numbers should be considered preliminary.

are notably larger than expected from the observed distribution of matter. The conclusion usually drawn is that there exists unseen "dark" matter in the system. That as much dark matter as observable matter seems to be present in the disk of our galaxy (Bahcall 1984) is particularly relevant here in that open clusters form in the galactic disk. It is thus of interest to ask what constraints the above results place on the presence of dark matter in open clusters. The errors in these observations are such that it is difficult to clearly discriminate the presence of unseen matter at the factor 2 level unless that matter is centrally concentrated, significantly contributing to the potential where the internal motions were measured. In other words, assuming energy equipartition, the observations can only detect dark matter made up of relatively massive particles, greater than a few solar masses in M11 or roughly one solar mass in M67. Assuming such particles, the M67 observations argue against unseen matter being present in an amount comparable to the observed stars; the M11 observations cannot rule out the presence of such a component of massive particles. As a case of particular interest, we note that if M67 is taken to be comprised of 50% (by mass) white dwarfs of 0.6 $M_{\odot}$, than the expected velocity dispersion is 0.6 km/sec, only marginally distinguishable with the data at hand from models without white dwarfs.

Gieseking (1981) has successfully measured the internal motions in NGC 3532, obtaining relative radial velocities with Fehrenbach objective-prism spectra of all stars of spectral types between B3 and F5 in a 4 square degree field. Typical single-measurement errors are on the order of 8 km/sec; with approximately 30 measurements per star Gieseking computes typical errors of 1.5 km/sec. For all cluster members Gieseking measures a velocity dispersion of 1.49$\pm$0.29 km/sec. He also suggests the presence of a cluster halo population with a velocity dispersion 2-3 times larger than that of the cluster stars. Finally, since NGC 3532 is markedly elliptical in shape, Gieseking analysed his data for evidence of rotation, finding none but placing an upper limit on systematic motions of 0.5 km/sec.

Looking beyond this completed work, a great deal of high-precision radial-velocity work in open clusters is in progress at several observatories and will soon reach fruition. McClure and Harris at the Dominion Astrophysical Observatory are completing an extensive study of NGC 188. The similarity of NGC 188 and M67 should provide for a very interesting detailed comparison of the two clusters. Mermilliod, Mayor and collaborators have in progress an extensive radial-velocity study of the giants in over 100 clusters, using the CORAVELs at Haute-Provence and La Silla. Several clusters included in their survey have sufficient numbers of giants to permit detailed kinematic studies. In addition they have observed dwarfs in the Pleiades (100 stars), Praesepe (100), Coma Ber (35) and α Persei (50). When completed these data will provide a superb basis for both dynamical analyses and studies of binary populations in clusters. Finally, at the Center for Astrophysics (Mathieu, Latham, Stefanik and Gordon) detailed studies of several clusters are in progress, as mentioned elsewhere in this review.

3.2) Proper Motions

In many ways proper-motion measurement is an inherently more powerful technique for the study of internal motions in open clusters than radial velocities. Proper motions provide velocity information in two orthogonal directions, allowing direct observation of anisotropy in clusters. They are not limited to late stars, as are most high-precision radial-velocity techniques, and can thus in principle study the cluster velocity distribution over the complete stellar mass range. Finally, except for occasional blended wide binaries, they are not affected by the presence of binaries in the cluster. (Note that this is only true for proper-motions measured over long time baselines. Proper motions determined from high-precision measurements over short time baselines have the same difficulties with binaries as do radial velocities.) The essential difficulty in the use of proper motions is simply precision. Measurement precision of 0.02 arcsec century^{-1} (achievable but not common) correspond to 0.5 km/sec at 500 pc and degrades linearly with distance. Thus, at the moment, for most clusters radial-velocity precisions are superior. However, the limitations of proper motions are purely technical and can thus in principle be overcome.

Jones' studies in the Pleiades (Jones 1970, σ = 0.2 km/sec; σ is the measurement precision) and Praesepe (Jones 1971, σ = 0.3 km/sec) first demonstrated the potential of high-precision proper-motion work. Since then several studies of kinematic precision have been done by McNamara and Sanders (1977, M11, σ = 1.8 km/sec, $\langle V^2\rangle^{1/2}$ = 1.7$\pm$0.4 km/sec; 1978, M67, σ = 2.9 km/sec, $\langle V^2\rangle^{1/2}$ < 1.5 km/sec; 1983, NGC 6494, σ = 0.8 km/sec, $\langle V^2\rangle^{1/2}$ = 0.8$\pm$0.1 km/sec). ($\langle V^2\rangle^{1/2}$ is the one-dimensional velocity dispersion.) In all of these studies the dynamical masses determined from the virial theorem were in reasonable agreement with the observed cluster mass in the form of stars, although it should be noted that the use of the virial theorem in itself introduces large uncertainties in relating cluster mass and observed velocity dispersion. Recently, Vasilevskis et al. (1979) published a proper-motion study of the inner region of the Pleiades with a measurement precision of 0.1 km/sec. These data have been analysed in detail by van Leeuwen (1980, 1983), who is presently expanding the proper-motion sample to fainter stars and regions more distant from the core. Van Leeuwen (1983) finds a significantly larger dynamical cluster mass than did Jones (1970), but argues that this mass is consistent with the stellar mass content of the Pleiades if one assumes a mass function similar to that in the field.

Certainly the most interesting finding of this proper-motion work has been the detection of anisotropy in both the Pleiades and Praesepe. In Fig. 3a we plot tangential vs. radial velocity dispersions in the Pleiades as a function of cluster radius, showing both the results of Jones (1970) and van Leeuwen (1983). (The difference in the absolute measures of the cluster velocity dispersion in these two studies is not

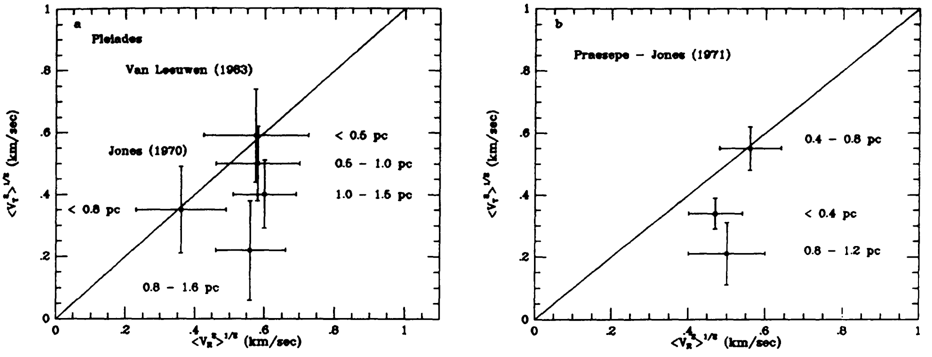

Fig. 3.a. Radial vs. tangential velocity dispersions in the Pleiades as a function of radius.
b. The same for Praesepe.

indicative of significant disagreements in the proper motions, but rather differences in models of the cluster structure used to remove the relative secular parallax [van Leeuwen, private communication]). Both studies show the velocity distribution in the central region to be isotropic. However, beyond a radius of roughly 0.5 pc (on the order of the core radius) the velocity distribution becomes anisotropic; while the radial velocity dispersion is isothermal with radius, the tangential velocity dispersion decreases. Van Leeuwen (1980) suggests that the tangential velocity dispersion decreases as 1/R beyond 0.5 pc. Thus the anisotropy in the Pleiades is present much closer to the cluster center than has been found for globular clusters (see review of Lupton in this volume and Cudworth (1976, 1979)). The results of Jones (1971) for Praesepe are shown in Fig. 3b. The degree of anisotropy in both clusters is similar.

Terlevich and Van Leeuwen are presently making detailed comparisons of her 1000-body simulations (Terlevich 1983) and the Pleiades data. One particularly interesting finding is that the rate of decrease of the tangential velocity dispersion with radius in the Pleiades appears to be less than that found in simulations of isolated clusters, but very similar to that found in simulations of clusters located in the galactic tidal field. They suggest that this result is indicative of tidal heating by the galactic field in both the simulations and the Pleiades. Such results argue eloquently for the need for additional accurate proper-motion studies outside of the cluster cores.

One of the most fundamental observable quantities for stellar dynamics is the behavior of the velocity distribution as a function of stellar mass. Unfortunately, very little data exist and, remarkably,

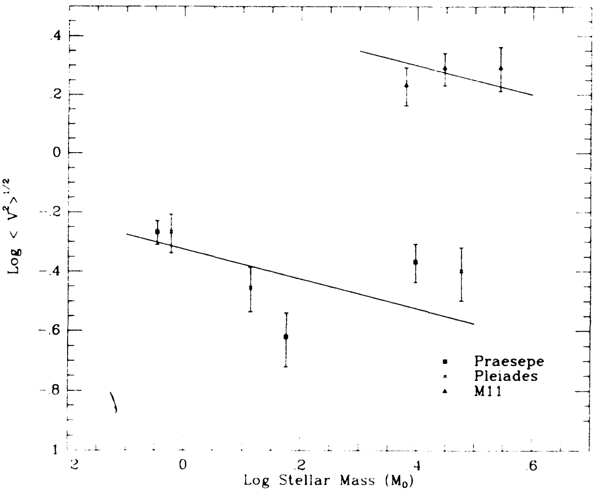

Fig. 4. The dependence of observed velocity dispersion on stellar mass. The solid line represents $\langle V^2 \rangle^{1/2} \propto M_*^{1/2}$.

the state of our knowledge has been little changed for well over a decade. The high-precision data available are, again, those of Jones' studies of the Pleiades and Praesepe and the work of McNamara and Sanders in M11, although in the latter case the data do not span a large range in stellar mass. The one-dimensional velocity dispersions as a function of stellar mass are shown in Fig. 4 for each cluster. No clear conclusions can be drawn from the data; neither the Pleiades data nor the M11 data can clearly distinguish between the $\langle V^2 \rangle^{1/2} \propto M_*^{1/2}$ relation (indicated by the solid lines in Fig. 4) or complete independence of the velocity distribution with stellar mass. The Praesepe data show an odd dependence of velocity dispersion on stellar mass, which is somewhat mimicked in the Pleiades data (although the latter are consistent with a less unusual dependence).

Hopefully the state of the observations will improve markedly in the next few years. Van Leeuwen is presently in the process of analysing his Pleiades data as a function of stellar mass, covering essentially the same stellar mass range as Jones' work. In addition, a program has been begun at the Center for Astrophysics to obtain high-precision radial velocities of Pleiades members with $10.5 < V < 16.5$ (roughly 1.0 $M_\odot$ to 0.4 $M_\odot$). The combined data set will span an order of magnitude in stellar mass and should provide a clearer picture of the dependence of the stellar velocity distribution on mass in the Pleiades. Finally, McNamara is in the process of obtaining precise proper motions in M35 which should be of kinematic accuracy and will span a factor 4 in stellar mass. The addition of another well-studied cluster will hopefully shed light on the generality of the Praesepe-like distribution.

4. BINARIES IN OPEN CLUSTERS

The final subject of this review has little to do with the structure and internal kinematics of open clusters but has been of central importance at this symposium. One of the more surprising findings in the study of clusters has been the lack of any detected binaries in globular clusters, other than those suggested by indirect evidence (e.g., x-ray sources or cataclysmic variables). However no such deficiency exists in open clusters; these systems have long been known to be rich in binaries. Indeed, several studies have shown that the binary populations in the Hyades, Pleiades and Praesepe are indistinguishable from that in the field where binaries are the dominant species (Mathieu, Stefanik and Latham 1985, Bettis 1977, Brosche and Hoffman 1979).

Using the simple formula of Heggie (1975) for binaries with 1 $M_\odot$ components, one finds that the boundary in binary period between hard and soft binaries in the Hyades is roughly 10^4-10^5 yr. In the Pleiades this boundary lies at 10^3-10^4 yr. Binaries in these period ranges are the most dynamically active and are significant in determining cluster evolution. The essential point here is that in open clusters many such binaries exist from the moment of cluster formation; assuming a field-like binary population (Abt and Levy 1976) roughly 10% of an initial cluster population are binaries with energies within an order of magnitude of the mean cluster kinetic energy. Furthermore, these binaries will rapidly concentrate toward the cluster core on an equipartition time scale. Thus binaries act as energy sources more or less continually throughout the lifetimes of open clusters.

The subject of binary encounters has to date been based entirely on analytic theory or numerical experiment with little observational constraint. It is natural to ask what observable consequences of such encounters might be expected in the nature of the binary population. Hut, in this symposium, has already pointed out that the spatial distribution of stars in a relaxed population of soft binaries would be indistinguishable from a random distribution. The theory thus provides little room for detailed observational confirmation with regard to soft binaries; nonetheless the distribution of wide pairs should be studied lest the observations contradict the theory. In addition, a comparison of the very youngest clusters with intermediate age clusters might provide evidence for the evolution of an initial soft binary population.

Study of the hard binary population should be more productive. A 100-body equal-mass numerical simulation run by Aarseth (1975) (see also Gianone, this volume) of a cluster with an 11% population of slightly hard binaries is instructive. After 24 crossing times, approximately two thirds of the binaries had had encounters which increased their binding energy (and significantly affected the cluster evolution). Nonetheless, none of the binaries suffered period changes of even an order of magnitude. Clearly, then, one cannot hope to detect any modification in the binary period distribution. However, exchange reactions are a common form of encounter, with massive single stars replacing lower mass binary components. Thus one might expect to detect a deficiency at the hard/soft boundary among binaries with lower-mass primaries, particularly in the cluster core. (Whether the corollary, an excess of binaries among massive stars, would be observable would depend on the population of invisible stellar remnants.) It is intriguing that in the Hyades there are several binaries with approximately solar-mass primaries and with periods near the hard/soft boundary, roughly in agreement with the number expected given a binary population similar to that in the field. However, three of these binaries are among the most distant cluster members found in Van Bueren's (1952) original survey; in fact they lie beyond the theoretical tidal radius. Have these binaries survived because they reside in the low-density halo? Or perhaps they are escaping the cluster as a result of encounters?

More study of wide binaries in other open clusters is very much needed. Clusters within 200 pc have dynamically active binaries with separations of $\gtrsim 1''$. Thus there are 3-4 relatively rich clusters, with membership studies, which can easily be studied. Indeed, as with our study of the Hyades, much or all of the necessary data may reside in the literature or plate stacks. The results of analyses of the binary populations in these clusters, combined with the Hyades study, will markedly improve statistics and should allow significant conclusions to be drawn with regard to the observable consequences of binary activity.

5. OUTSTANDING PROBLEMS AND NEW DIRECTIONS

Many of the essential problems in open cluster dynamics have received mention already, but two points merit repeated emphasis here. First, all stellar systems have a spectrum of masses to some degree and yet to date we have very little direct observational evidence of the dependence of velocity distributions on stellar mass. This knowledge is fundamental to any theoretical understanding of cluster dynamics and merits far more attention than it has been given to date. Hopefully the work in the Pleiades and M35 will provide a significant step forward in the near future.

Second, core collapse phenomena have been a primary focus of recent research in cluster dynamics, and with a substantial degree of success as has been evident at this symposium. However we should not entirely abandon the halos of clusters, where the remarkable success of the lowered Maxwellian velocity distribution may have lulled us into believing that we fully understand their dynamics, despite the fact that these outer regions are neither relaxed nor resident in a square-well potential. That the Hyades, the only open cluster whose outermost regions have been studied in detail, does not smoothly fit into the theoretical picture should motivate this direction of study. From an observational perspective the essential problem will be membership determination in that appropriate first-epoch plate material is rare.

Perhaps efficient combination of photometry and high-precision radial velocities will be a reasonable alternative in the short run.

These problems are classical ones in that they have been the subject of study since the first research into cluster dynamics; this of course makes them no less important. In the near future, however, strides will be taken in new directions as well, in particular in the study of very young clusters with ages of less than a relaxation time, less than an equipartition time, indeed less than a crossing time. The most basic questions need to be answered. What is the initial mass function? What is the initial stellar spatial distribution, in particular as a function of stellar mass? Is mass segregation the result of stellar dynamical processes or do clusters form with such structure? Burki (1978) has taken a first look at the very youngest clusters and suggests that the very massive stars form in the outer regions of clusters. However, Herbst and Miller (1982) find a marginally significant central concentration of the more massive stars in NGC 3293 (nuclear age of 6×10^6 yr). If we indeed find that the very youngest clusters are not mass segregated, then we have the means to directly measure the equipartition time-scale. If the very youngest clusters are mass segregated, we will have found an important insight into the ways stars form. Using similar reasoning it is clearly of interest to determine whether anisotropy exists in clusters younger than the Pleiades. This is a far more difficult observation, but is crucial in interpreting the observations of older clusters with regard to cluster evolution processes and time-scales.

Expanding our horizons a bit further, the dynamical principles developed for clusters can provide important insights into the star-formation process itself. Mathieu (1983a), Elmegreen (1983) and Lada, Margulis and Dearborn (1984) have applied simple stellar dynamics to forming clusters in order to argue for very high star-formation efficiencies. However, the detailed theoretical questions concerning the dynamical interaction of forming clusters with their parent molecular clouds remain unanswered. The observational side is equally untouched and promising; the stellar kinematics in star-forming regions are largely unknown. How do the stellar and gas kinematics compare? What implications does this have for magnetic fields in molecular clouds? How do the stellar kinematics of regions disturbed by shocks differ from quiescent regions? How many young clusters are in fact unbound? These questions have only begun to be answered; indeed many more still remain to be asked.

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DISCUSSION

BAHCALL: You gave a clear discussion of the mass segregation observed among the various clusters. However, there is one puzzling fact: the oldest cluster you discussed, M67, has the smallest amount of mass segregation, markedly less than for the youngest clusters. How do you understand this fact?

MATHIEU: The lowest mass component (derived from star counts) does appear to be somewhat less centrally concentrated and as a result equipartition models do seem to fit the data better than non-equipartition models. The data cannot distinguish the two models at a statistically significant level, however. So, you are right that the mass segregation in M67 is far less evident than in the younger clusters. The model which best fits the data has a shallower potential than for M11 and thus a more severely truncated velocity distribution, which lessens mass segregation. Also, in M67 the most massive stars are dominant contributors to its potential, which also lessens mass segregation among these stars. Thus, the lack of marked mass segregation is not necessarily inconsistent with equipartition in the cluster. Still, it is interesting that it is the oldest cluster which is the least mass segregated. I look forward to seeing whether studies presently being done on other equally old clusters tend to find similar cluster structure.

APPLEGATE: What mass functions do you find in the multimass models that you showed?

MATHIEU: The mass functions of M11 and M35 are not distinguishable from that found in the field by Miller and Scalo. However, in neither cluster do the data extend to masses significantly below 1 $M_{\odot}$, where the deficiencies in low mass stars that have been observed in several clusters first appear. In M67 the proper-motion membership data is consistent with star-count data of Van den Bergh and Sher, but again the proper-motion data do not extend faint enough to provide an independent confirmation of their observed deficiency. I should note that observational biases due to mass segregation cannot be invoked to explain away their results. Van Leeuwen argues that the luminosity function in the Pleiades is consistent with the field down to several tenths of a solar mass, while in the Hyades, Oort suggests that indeed there is a deficiency of the lowest mass stars beginning at $M_V = 9$.

LARSON: I would like to second Dr. Mathieu's final remark about the importance of young open clusters for the study of star formation. One of the most interesting questions concerns the stellar initial mass function (IMF) and whether it is the same everywhere or varies with location. There is some folklore to the effect that some young open clusters have a deficiency of low mass stars which is difficult to account for in terms of dynamical evolution, and there are even some hints that the IMF may vary with position within individual young clusters. As was pointed out several years ago by Freeman, if the IMF in some clusters or regions contains a much higher proportion of massive stars than a "standard" IMF, the evolution of the system can be radically altered. Effects of mass loss from massive stars will be of very major importance, and the remnants left by the massive stars can become an important or even dominant constituent of the system. The best place to look for possible variations in the IMF is in very young clusters where effects of dynamical evolution have not yet become important.

KING: The spatial distribution of the Hyades offers one additional opportunity. Because of the convergent effect in the proper motions, this is then one cluster that we can study in three dimensions rather than two.

APPLEGATE: I would like to reinforce Larson's comment on the importance of stellar evolution in clusters with flat mass functions. I find that the early evolution of clusters with IMF's flatter than $(N(M)\alpha m^{-2}$ can be completely dominated by mass loss; collisional evolution plays a very minor role. IMF's this flat are well within the range reported by Freeman for the young LMC clusters.

GOODMAN: Do you see any evidence in the open cluster radial-velocity data for motions in the atmospheres of giants, such as was found by Gunn and Griffin (1979)?

MATHIEU: No. The internal errors of our open cluster giant observations are essentially the same as our errors for all other observations, including for example the dwarfs in the Hyades.

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