Numerical Simulations of Dense Collisional Systems with Extended Distribution of Particle Sizes

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1. Introduction

The dynamical evolution of dense planetary rings, such as Saturn's rings, is mainly governed by the mutual impacts between macroscopic icy particles. The local equilibrium state is determined by the energy loss in partially inelastic impacts and the viscous gain of energy from the systematic velocity field. Due to frequent impacts the time-scale for the establishment of local energy equilibrium is very short, as compared to the time-scale for radial evolution, which is determined by viscous spreading, and in some cases also by the angular momentum exchange with external satellites. Therefore, local and radial behaviour can, to a large extent be studied separately. This fact is utilized by the local simulation method (Wisdom and Tremaine, 1988; Salo, 1991), following the orbital evolution in a small co-moving region inside the rings with periodic boundary conditions. Compared to previous simulation methods (Salo, 1987) this enables much higher surface density. With the presently attainable number of particles (up to several thousands), realistic modeling of dense regions is possible, taking simultaneously into account the particle size distribution, rotation of particles, as well as vertical self-gravity. By combining several local simulations with different surface densities, it is possible to deduce the expected radial behaviour as well.

According to laboratory measurements (Bridges *et al.*, 1984), impacts between icy particles are very dissipative already with impact speeds of a few mm/sec. This indicates that rings must be extremely flattened with geometric thickness of the order of few times the dominant particle size. In addition, friction should further reduce the equilibrium velocity dispersion (Salo, 1987; Araki, 1991). Strong flattening has several important implications for the dynamics of rings: for example effects due to finite size occupied by particles become important, as well as the influence of self-gravitation. These effects are most reliably treated by particle simulations.

2. Simulation Method

By assuming that the ring system possesses local azimuthal symmetry, one can limit the calculations to a local co-moving region, following the mean orbital motion of particles. This makes it possible to model dense portions of rings with reasonable number of particles. Boundaries are treated in terms of image particles: each time particle leaves the calculation box, one of its images enters the area with suitably modified position and velocity, taking into account the shearing motion. Various surface densities can be studied by adjusting the size of the calculation area. Numerical tests indicate that the method should be valid as long as mean free path between impacts stays safely below the dimensions of the calculation box.

S. Ferraz-Mello (ed.), Chaos, Resonance and Collective Dynamical Phenomena in the Solar System, 103–108.

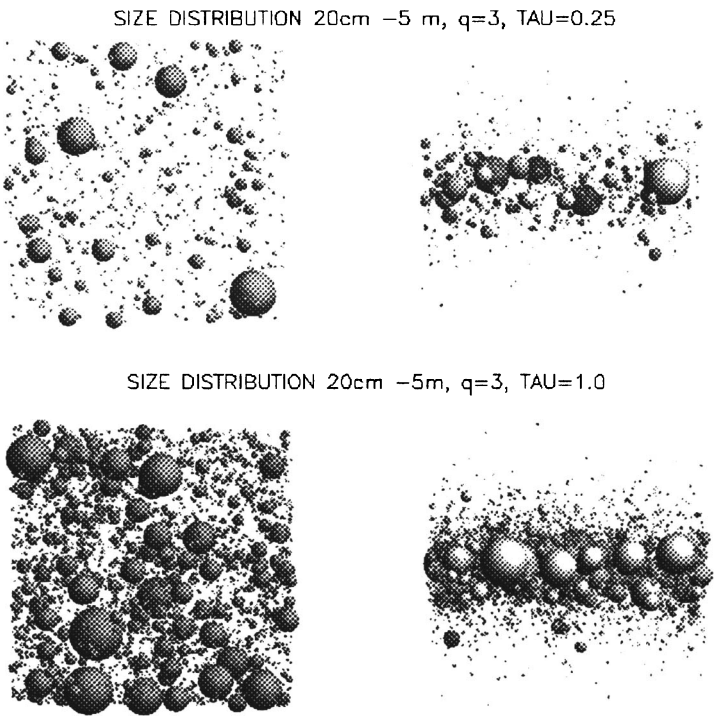

Fig. 1. The final distribution of particles at the end of two simulations performed for the optical thicknesses $\tau = 0.25$ and 1.0 ($N = 750$ and 3000). In the left-hand frames, view from above is shown, while in the right-hand frames, the system is seen from the side. The size of the simulation area is 50 meters and the size distribution corresponds to Voyager measurements, except that the distribution is truncated at 20 cm. Elastic model corresponds to Bridges *et al.* laboratory measurements, $\alpha = (v/v_c)^{-0.234}$, where v is the perpendicular component of impact velocity, and $v_c = v_B = 0.01$ cm/sec. Saturnocentric distance of 100 000 km is assumed. Self-gravity is not included.

Local nature of calculations facilites the use linearized dynamical equations where the systematic orbital motion is already eliminated: this leads to extremely fast orbital integrations. In addition, because of the high impact frequency, collisional calculations can be considerably speeded up by using an iterative method for search of impacts between close particle pairs: during each dynamical time-step each particle can experience several accurately calculated impacts. At the moment a 3000 particle simulation (200 impacts/particle to assure establishment of energy balance) can be performed in about 5-10 hours of CPU time in IBM ES/9000-260.

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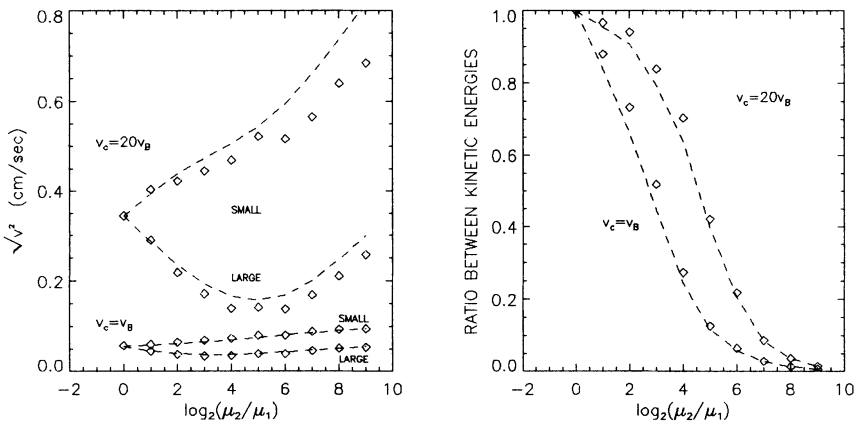

Fig. 2. The dependence of equilibrium velocity dispersion on mass-ratio in systems of two particle sizes (0.5 and 1.0 meters). Two elastic models are studied: standard model ($v_c = v_B$), as well as a hypothetical model of harder particles ($v_c = 20v_B$). Optical thickness of both components is 0.5. Simulation values (symbols) are shown together with theoretical calculations (Hämeen-Anttila and Salo, in preparation). Also shown is the ratio between kinetic energies.

3. Extended Size Distribution

Due to above mentioned inelasticity of icy particles, expected geometric thickness of ring systems is extremely low: if particles were described with mass-points, collisional balance would imply thickness of the order of about 5 meters for Saturn's rings (Bridges *et al.*, 1984), for all values of optical depth (compare to radial extent of 280 000 km). Therefore, for finite sized particles, minimum velocity dispersion and geometric thickness is in practice determined by the dominant particle size. According to Voyager measurements the distribution of particle sizes can be approximated with a power-law distribution with power-index $q \simeq 3$, extending from about 1 cm to 5 meters in radius. Fig. 1 displays the expected equilibrium behaviour of such distribution for low ($\tau = 0.25$) and moderate optical thickness ($\tau = 1.0$). In the presence of different particle sizes, some of the kinetic energy is transferred from large particles to small ones, although the system is far from energy equipartition: largest particles tend to form a near monolayer, while smallest particles (20 cm in Fig. 1) are confined to a layer of about 20 meters.

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4. No Energy Equipartition

The distribution of kinetic energy between various sizes is easiest to study with bimodal distributions of just two particle sizes (Fig. 2). Due to the dissipative nature of collisional interactions, equipartition generally takes place only for mass-ratios below about 10. Another possibility would be to have an excess of small particles so that the product $\mu\tau$ is the same for both populations (μ stands for particle mass). However, this is probably not possible for any realistic system: for example the power-index $q = 3$ most closely corresponds to equal optical thicknesses, the case which is studied in Fig. 2. The energy distribution depends to some degree on the elasticity model: for less dissipative, 'harder' particles ($v_c = 20v_B$) equilibrium velocity dispersions as well as their differences would be larger than for particles behaving according to laboratory measurements ($v_c = v_B$). In any case, maximum expected difference in equilibrium velocity dispersion stays below about 5 for all mass-ratios, indicating that kinetic energy is heavily concentrated on large particles. This is also in accordance with our theoretical models (Hämeen-Anttila and Salo, 1991).

5. Effects of Vertical Self-Gravity and Friction

An interesting problem in the dynamics of dense rings is the influence of self-gravity. Although the ring masses are too small to cause any large-scale instabilities, for example gravitational stirring by large ring particles (or small embedded moonlets) might be involved in the radial ringlet structure. Also, in the case of extremely flattened systems, the vertical component of the self-gravity can significantly exceed that of the central body. In simulations, although it is presently not yet practical to include all gravitational interactions, self-consistently calculated vertical self-gravity can be included by iterative methods, by tabulating the vertical distribution of mass-density over certain time period and then applying this new force field (in addition to central force). This process is repeated until convergence is achieved.

In general, the inclusion of vertical self-gravity leads to reduced geometric thickness, partly directly due to increased 'spring-constant', partly due to indirect effect via increased impact frequency and therefore enhanced energy dissipation. In the case of identical particles large optical depth can lead to interesting layering phenomena, as particles try to fill as much of available space as possible by forming regular lattice (Wisdom and Tremaine, 1988; Salo, 1991). In the presence of size distribution this regularity is generally avoided. Fig. 3 compares the vertical particle distribution in power-law simulations ($\tau = 1.0$) with and without self-gravity. As can be seen self-gravity causes the small particles to become embedded into the layer of largest particles, reducing their geometric thickness to about on half.

In Fig. 3 also the effects of friction are shown, for the case of maximal friction $\beta = 1$, implying that tangential velocity difference is completely lost in each impact. In this case geometric thickness is even further reduced. Significant amount of energy is stored to particle spins, about 40% of that in random velocities. Particles also acquire a residual prograde mean spin, $\bar{\omega}_z \simeq 0.25 - 0.30\Omega$, where Ω is the angular orbital velocity. This agrees with previous simulations of rarefied systems

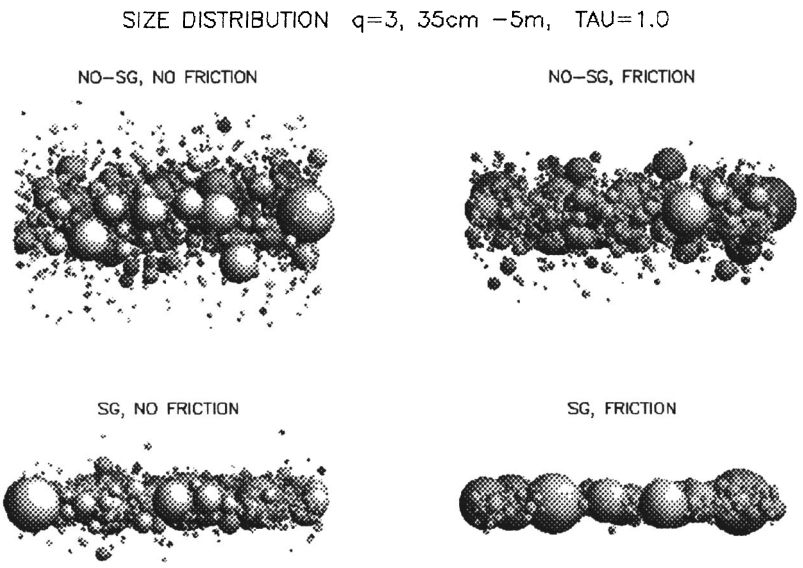

Fig. 3. Influence of vertical self-gravity and friction on the equilibrium state. Power-law size distribution with $q = 3$ is simulated, with standard elasticity model. In frame a) internal density of particles is zero while in b) density of $0.9gr/cm^3$ is assumed. In c) friction is included ($\beta = 1$), and in d) both self-gravity and friction. Optical thickness $\tau = 1$ in all cases.

(Salo, 1987) as well as with theoretical studies (Araki, 1991; Hämeen-Anttila and Salo, 1991). For small particles this mean rotation is generally insignificant as compared to the dispersion of spins, whereas for large particles clear alignment can be observed.

6. Future Studies

All the above presented models have assumed Bridges *et al.* (1984) elasticity model, which however can overestimate the true amount of dissipation and therefore lead to unrealistically flattened models. Indeed, if more recent laboratory measurements (Hatzes *et al.*, 1988) are taken into account, consirerably larger equilibrium thickness is obtained, of the order of 40-80 meters for cm-sized particles as compared to 20 meters in Fig. 1. Therefore, it would be extremely important to have better understanding of the elastic properties of ice at the conditions appropriate for ring systems. The amount of friction in impacts is also not known.

Another uncertain factor is the treatment of self-gravity. Simulations where self-gravity is properly included, by calculating all the gravitational forces between

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particles are much more time consuming than above models with iterated vertical field. Preliminary simulations with such improved code indicate that above results are still valid if only vertical components of mutual forces are included. However, inclusion of all components leads to much more complicated behaviour, with formation of wakes etc. In this case, also gravitational scattering is importantant, and tends to oppose the flattening due to vertical field. Future simulation studies will try to model ring self-gravity as realistically as possible, and also address larger range of elasticity models.

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