nur in Reihen von Reihen ordnen lassen, oder was dasselbe ist, bilden sie ein Mannigfaltigkeit von zwei dimensionen; verhält es sich dann mit den Relationen einer Reihe zu einer andern oder den Uebergängen aus einer in die andere auf eine ähnliche Weise wie vorhin mit den Uebergängen von einem Gliede einer Reihe zu einem andern Gliede derselben Reihe, so bedarf es offenbar zur Abmessung des Ueberganges von einem Gliede des Systems zu einum andern ausser den vorigen Einheiten $+1$ und -1 noch zweier andern unter sich auch entegegen-gesetzen $+i$ und $-i$. Offenbar muss aber dabei noch postulirt werden, dass die Einheit i allemal den Uebergang von einem gegebenen Gliede einer Reihe zu einem *bestimmten* Gliede der unmittlebar angrenzenden Reihe bezeichne. Auf dies Weise wird also das System auf ein doppelte Art in Reihen von Reihen geordnet werden können."—Gauss, *Werke*, vol. II, p. 176.

The "notion" appears also in Gauss' *Werke*, vol. VIII pp. 359–60, where the "combination" a, b, c, d is denoted by (a, b, c, d) and "we write"

$$(a, b, c, d) \ (\alpha, \beta, \gamma, \delta) \ = \ (A, B, C, D),$$

etc. According to the Editor (Stäckel), internal evidence assigns this fragment to about the year 1819, which is before the date of Hamilton's first published discussion of ordered pairs. That Gauss, in this equality, had broken away from geometrical intuition, seems evident, as he did not discuss 4-dimensional "space."

E. T. Bell.

California Institute of Technology.

A NOTE ON THE VALIDITY OF ARISTOTELIAN LOGIC

Dear Sir:

In a recent issue of this periodical,[1] there appeared an article by Mr. L. Kattsoff entitled "Concerning the Validity of Aristotelian Logic." By use of the formula for the A proposition, $A(ab) = (a < b) \ [(b < a) + (a < b')' \ (b' < a)']$[2] and corresponding formulae for the E, I and O types, Mr. Kattsoff gives the method of Dr. H. B. Smith of establishing the complete generality of Aristotelian logic, its consistency, and the validity of all the classical forms of inference. That Dr. Smith's

[1] *Philosophy of Science*, Vol. 1, No. 2, pp. 149–162, April, 1934.
[2] *Loc. cit.* p. 157. "$<$" and "$'$" have their usual meaning in the algebra of classes. "$+$" indicates a disjunction of propositions.

system is formally consistent there can be no doubt, that it permits all the forms of inference sanctioned by the classical treatment of the syllogism and immediate inference, is clear, but that it therefore constitutes a validation of Aristotelian logic depends on the highly dubious assumption that Aristotelian logic consists merely of these doctrines of syllogism, immediate inference and the square of opposition. In particular, it must be assumed that the laws of contradiction and of excluded middle are not a part of Aristotelian logic, since, as a slight examination will show, Dr. Smith's system assumes both of them to be false.

The law of contradiction may be stated in some such terms as the following: No proposition of the form "S is both P and non-P" can be true.[3] From this we might expect to infer for example, that "No entity is both red and non-red" and according to Dr. Smith's system this is a true proposition.[4] Any further specification, however, results in a false proposition. Thus the statements "No chair is both red and non-red," "No geranium is both red and non-red," "No round-square is both red and non-red" are all false and their contradictories true.[5] This not only involves a breakdown of one of the laws of logic most firmly established by tradition, but also is repugnant to common sense.

Further paradoxes await the enquirer who raises the question "If it is true that some chairs are both red and non-red, which chairs are?" For according to Dr. Smith's analysis, the propositions "This chair is both red and non-red" and "This chair is not both red and non-red" are both false however the chair in question be chosen.[6] This involves a denial of the law of excluded middle as well. Thus we are left with a situation in which no entities are both red and non-red, but some of every species of entity are both red and non-red and with regard to any individual entity, it neither is nor is not red and non-red.

[3] Some writers restrict the law to a relation between two contradictory propositions. *Cf.* Keynes, *Formal Logic*, 4th ed. p. 454. Sigwart, *Logic*. Translation by Dendy, Vol. I, p. 139. Most writers, however, admit some statement closely approximating the above as at least one if not the only formulation of the law. *Cf.* Bain, *Logic*, Vol. I, p. 16. Joseph, *Introduction to Logic*, 2nd ed. p. 16. Eaton, *General Logic*, p. 423. Stebbing, *Modern Introduction to Logic*, p. 469. In any case no writer would challenge the principle as stated above and it is definitely Aristotelian. Cf. Aristotle *Metaphysica* 1006 a-7.

[4] Since the class of entities is the universe class, the class of things which are both red and non-red, the null class; and $E(1, 0)$ is true. See the tables given on p. 160, *Loc. cit.*

[5] According to the formulation given, $E(ab)$ is false when $b = 0$ and $a \neq 1$. *Loc. cit.* p. 160.

[6] Assuming the traditional analysis of the singular proposition as a universal. If the singular be considered a particular, both the above propositions are true.

Because of the above paradoxes, it is difficult to see how Dr. Smith's system can be accepted as the equivalent of Aristotelian logic or how therefore it contributes to the establishment of the validity of that logic.

PAUL HENLE.

Smith College.

Dear Sir:

I am glad to reply to Dr. Henle's sympathetic criticism because, where issues can be so tightly drawn as in logic, there is great hope of coming to an understanding.

Dr. Henle's statement of the "law of contradiction," because of its ambiguity, leaves me somewhat puzzled. He states it thus:

"No proposition of the form 'S is both P and non-P' can be true."

(1) If S is a class with members the law holds,
(2) If S is an empty class the law fails,
(3) If S is a singular term (which seems to be implied by the reference to Aristotle) the meaning of the copula has changed to what is currently represented by ϵ.

In this last case the criticism would be irrelevant. Moreover the pretended cases, No entities are red and not red, No chairs are red and not red, are not instances of the law as stated.

Whether in the passage which Dr. Henle refers to, Aristotle has in mind a "law of contradiction" is, to say the least, problematical. At any rate the usual word for contradiction is not used. I quote the Organon in the Latin (Analyticorum Post. Lib. I, Cap. II, 12) because the word comes to us through the Latin:

"Contradictio autem est oppositio, cuius non est medium secundum ipsam. Pars vero contradictionis illa, quae aliquid de aliquo (enuntiat) est affirmatio; quae autem aliquid ab aliquo (removet) negatio."

The form of the "law" which Dr. Henle would have us accept may be used by writers who tacitly assume that the classes in question are not empty. In the original passage (Met. 1006 a) the reference is to a singular term, a *something* which cannot both be and not be (again a different use of the copula).

If the law of contradiction be stated:

P (is true) and P (is untrue) is impossible,
What is P and non-P is non-existent,

it does not break down on my interpretation of the categorical forms; and similarly for the law of excluded middle:

Everything (in the universe) is either P or non-P.