

MAGNETIC PERTURBATIONS TO STELLAR OSCILLATION EIGENFREQUENCIES

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ABSTRACT. Magnetic fields contribute to the splitting of the degeneracy of modes of like order and degree. The splitting is estimated for some simple hypothetical toroidal magnetic field configurations in the sun, and the results are compared with previous asymptotic estimates. Splitting by a field confined to a thin layer at the base of the convection zone is found not to agree with recent measurements.

1. INTRODUCTION

Our calculations are based on the formalism of Gough and Taylor (1984), except that whereas they obtained estimates from asymptotic theory we compute the perturbations from numerical eigenfunctions. With respect to spherical polar co-ordinates (r, θ, ϕ) all the field configurations we consider are of the form

$$\underline{B} = (0, 0, B(r) d P_k / d \theta) \quad (1)$$

where $P_k(\cos \theta)$ is a Legendre polynomial. If rotation is ignored, the resulting frequency ω_{nlm} of a mode of order n , degree l and azimuthal order m would be

$$\omega_{nlm} = \omega_{nl}^{(0)} + \sum_{s=0}^k K_{2s}^{(n,l)} Q_{2s,l,m} + O(B^4) \quad (2)$$

where $\omega_{nl}^{(0)}$ is the unperturbed frequency of the nonmagnetic stellar model;

$$Q_{2s,l,m} = \int_{-1}^1 P_{2s}^1(\mu) \{P_l^m(\mu)\}^2 d\mu / \int_{-1}^1 \{P_l^m(\mu)\}^2 d\mu \quad (3)$$

where P_l^m is the associated Legendre function of the first kind. The quantities $K_{2s}^{(n,l)}$ are functionals of the magnetic field, the equilibrium structure of the stellar model and the oscillation eigenfunctions. They are each composed of two parts: a contribution [which is K_{2s}^{mag} of Dziembowski and Goode (1985) and $I_{3,2s}$ of Gough and Taylor (1984)] from the magnetic distortion to the equilibrium structure of the star, and a contribution, related to I_4 of Gough and Taylor (1984), from the perturbed Lorentz force.

When rotation is also taken into account the frequency perturbation is quite complicated when the axis of field symmetry is inclined from the rotation axis by a nonzero angle β (e.g. Dicke, 1982; Gough and Taylor, 1984; Dziembowski and Goode, 1984, 1985). However, when the magnetic perturbation is much smaller than the effect of advection, it simply adds to the rotational frequency splitting a contribution which is approximately

$$\sum_{s=1}^k (Q_{2s,l,m} - Q_{2s,l,0}) K_{2s}^{(n,l)} P_{2s}(\cos\beta). \quad (4)$$

This formula is exact, to $O(B^2)$, when $\beta = 0$.

2. CORE FIELDS

We consider first field amplitudes of the form

$$B(r) = \begin{cases} (1+\alpha)(1+\alpha^{-1})^\alpha B_0 (r/r_c)^2 (1-(r/r_c)^2)^\alpha & r < r_c \\ 0 & r > r_c \end{cases} \quad (5)$$

whose influence was discussed by Gough and Taylor (1984). Here α is chosen to be $1 + 10r_c/R$, where R is the radius of the star. The maximum value of $B(r)$ is B_0 . In Figure 1 the magnetic frequency perturbation to oscillations of Christensen-Dalsgaard's (1982) solar model 1 for the case $r_c = 0.7R$, where R is the radius of the sun, is plotted against l for oscillations with cyclic frequency $\nu = \omega/2\pi$ nearest to 3.5 mHz. For each value of l the perturbations were obtained by linear interpolation between the two modes with frequencies closest to 3.5 mHz. Modes with $l < 10$ penetrate into the region where the magnetic field is strong (for these modes $r_t < 0.3R$, where r_t is asymptotic lower turning point which satisfies $c(r_t)/r_t = \omega/l$ where c is the sound speed), and for them the contribution from the perturbed Lorentz force dominates. As l increases, less of the mode experiences the direct effect of the field, and when l increases beyond about 20 ($r_t > 0.44R$) only the distortion of the star's hydrostatic structure outside the region where the field is large has a significant influence on the frequency.

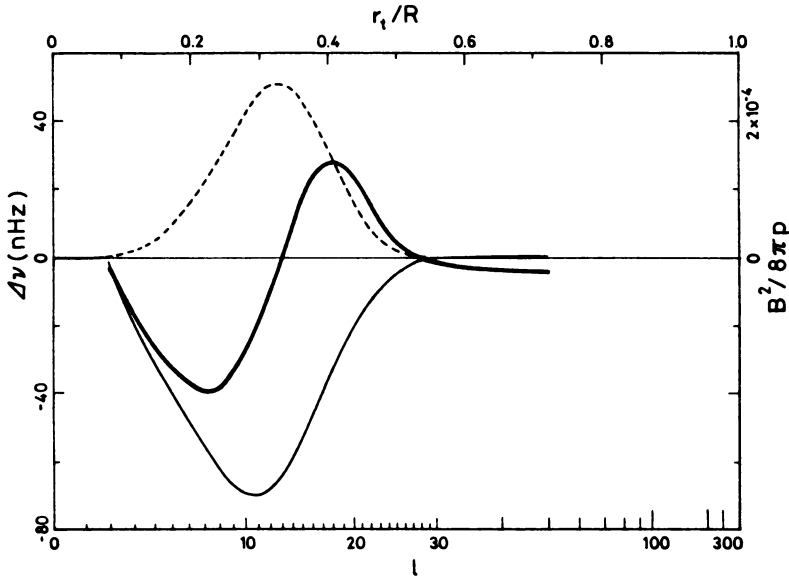


Figure 1. The thick solid curve is the frequency difference $\Delta\nu = \nu_{n11} - \nu_{n10}$ (in mHz) produced by the hypothetical toroidal magnetic field in the sun given by Equations (1) and (5) with $r_c = 0.7R$ for modes with frequencies 3.5 mHz. It was computed by linear interpolation between the modes with frequencies closest to 3.5 mHz. The thin solid curve is the contribution to $\Delta\nu$ from the perturbed Lorentz force. The nonuniform l axis has been chosen in such a way that $r_t(l)$ varies uniformly from 0 to R from left to right. The dashed curve is $B^2/8\pi\rho$, plotted against $r_t(l)/R$, where ρ is the gas pressure of the equilibrium state ($k=2$ and $B_0=10^7G$).

Table I shows that for the deeply penetrating quadrupole modes the asymptotic estimates of Gough and Taylor (1984) are in good agreement with the numerical results.

3. CONFINED FIELDS

When the field is confined to a range in r that is comparable with or less than the characteristic vertical wavelength of the mode, the integrals are sensitive to the phase of the spatial oscillations in the eigenfunction. In that case the magnetic splitting oscillates as frequency varies. This is exhibited by Vorontsov (these proceedings) and is also illustrated in Figure 2, where perturbation coefficients $K_2^{(n,l)}$ resulting from the magnetic field (1) with

$$B(r) = \begin{cases} B_0 (1 - (r-r_0)^2/d^2)^2 & |r-r_0| < d \\ 0 & |r-r_0| > d \end{cases} \quad (6)$$

representing a field concentrated at the base of the convection zone, are plotted for several values of l .

| n | $r_c = 0.3$ | | | $r_c = 0.5$ | | $r_c = 0.7$ | |
|----|-------------|------------|------------|-------------|------------|-------------|------------|
| | ν_{n1} | δ_2 | δ_4 | δ_2 | δ_4 | δ_2 | δ_4 |
| 16 | 2.47 | 1.94 | 0.88 | 5.09 | - 2.98 | 8.16 | - 13.0 |
| 18 | 2.74 | 1.43 | 0.22 | 5.15 | - 3.58 | 8.02 | - 13.7 |
| 20 | 3.01 | 1.20 | - 0.42 | 5.13 | - 4.02 | 7.88 | - 14.2 |
| 22 | 3.29 | 1.39 | - 0.87 | 5.08 | - 4.36 | 7.73 | - 14.7 |
| 24 | 3.56 | 1.85 | - 1.13 | 5.02 | - 4.64 | 7.58 | - 15.1 |
| GT | | 2.50 | - 2.54 | 4.53 | - 5.80 | 7.25 | - 16.3 |

Table 1. Coefficients $\delta_{2S} = 10^6 K_{2S}^{(n,l)} / \omega_{n1}^{(o)}$ for the relative magnetic frequency splitting of quadrupole ($l = 2$) modes due to magnetic fields given by equations (1) and (5) with $B_o = 10^7 G$ and $k = 2$. All splitting coefficients are proportional to B_o^2 . In the bottom line are the asymptotic values computed from Table 2 of Gough and Taylor (1984), taking due account of an erroneous factor of 2 in their coefficient I_4 . The geometrical factors $Q_{\lambda,1,m}$ and $S_{2,1,m}$ tabulated by Gough and Taylor are also a factor 2 too large, and the distortion factors h_Ω, h_λ are misprinted).

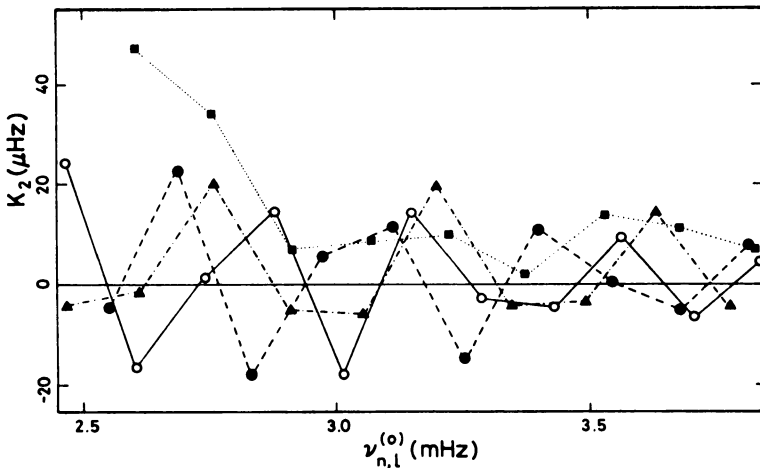


Figure 2. Frequency perturbation coefficients $K_2^{(n,l)}$ for the confined magnetic field given by Equations (1) and (6) with $r_c = 0.7R$, $d = 0.05R$ and $B = 10^7 G$. Open circles represent $l = 2$, filled circles are for $l = 10$, triangles for $l = 20$ and squares for $l = 30$. The symbols are connected by straight lines to aid the eye; had ν been engineered to vary continuously, the results would have been smooth oscillatory functions for modes that penetrate beneath the magnetic layer.

In the case considered the width of $B(r)$ at half maximum is $0.04R$, and a half wavelength of a vertically propagating 3mHz sound wave is about $0.05R$. Thus any low-degree five-minute mode, which penetrates well beneath the base of the convection zone, exhibits the oscillatory behaviour. As frequency increases, the wavelength of the mode decreases, and therefore the amplitude of the oscillatory variation of $K_{2s}^{(n,l)}(\nu)$ diminishes. As l decreases at fixed ν the spatial phase difference between the base of the convection zone and the upper turning point increases (cf Deubner and Gough, 1984). Consequently the phase of the oscillation of $K_{2s}^{(n,l)}$ with respect to ν advances.

Acoustic modes are particularly sensitive to conditions in the vicinity of $r = r_t$, for there the constituent waves travel almost horizontally and therefore remain near that level for a comparatively long time. Consequently when r_t is within the region of strong magnetic field, as is the case when $t_1 = 30$ and $\nu \approx 2.5 \text{ mHz}$, the frequency perturbation is large. As l increases beyond that value the waves no longer reach the field, and the frequency perturbation declines.

4. ANALYSING FREQUENCY DATA

It is evident that the amplitude and phase of $K_{2s}^{(n,l)}(\nu)$ depend on the width d of the field concentration and the radius r_s about which it is located. Therefore in principle these properties of a field, if it exists, could be determined from data such as those that Duvall *et al* (1986) have obtained from the sun (cf. Gough and Thompson, these proceedings). The odd- m component of the frequency splitting can be used to infer the angular velocity, and thence the rotational contribution to the even- m component can be calculated and subtracted from the data. The l and m dependence of the remaining splitting provides the information about the radial and latitudinal dependence of a magnetic field, or any other cause of symmetry-breaking. It is important to appreciate that the oscillation in $K_{2s}^{(n,l)}(\nu)$ is poorly sampled, and superficially resembles random scatter; under no account should data from different modes be indiscriminately averaged merely to improve the apparent quality of the results. Nevertheless, averaged data do contain useful information. It is instructive to compare the even- m component of the averaged splitting data of Duvall *et al* (1986) with comparable averages produced by the field defined by Equations (1) and (6). Duvall *et al* represent their frequencies essentially by the formula

$$\nu_{nlm} - \nu_{nl0} = L \sum_{i=0}^5 a_i P_i(-m/L) \quad (7)$$

where $L^2 = l(l+1)$, and average the coefficients a_i over different ranges of l and all observed n . It can be shown that for the relatively large values of l considered these coefficients can be related approximately to ours according to

$$L a_{2i} = (-1)^i \frac{(2i)!}{2^{2i} (i!)^2} K_{2i}^{(n,1)} \quad (8)$$

if the symmetry axis of the field coincides with the rotation axis. In Table II we compare our computations with the observations. The contribution from rotation is small. The observed coefficients do not decline rapidly with l once l is high enough for r_t to be above the perturbation, and we conclude, in particular, that the even- m component of the observed splitting could not have been produced solely by a perturbation confined near the base of the convection zone.

| l | Confined field | | Duvall <u>et al</u> | |
|-----|--------------------|--------------------|---------------------|--------------------|
| | $L \overline{a_2}$ | $L \overline{a_4}$ | $L \overline{a_2}$ | $L \overline{a_4}$ |
| 20 | - 1.3 | - 0.2 | 0.46 | 0.01 |
| 30 | - 12.9 | 3.8 | 0.16 | 0.31 |
| 40 | - 13.9 | 5.1 | 0.37 | 0.15 |
| 50 | 6.8 | 1.0 | 0.45 | 0.15 |
| 60 | 7.0 | - 1.2 | 0.56 | 0.26 |
| 70 | - 0.07 | 0.04 | 0.33 | 0.26 |
| 80 | - 0.09 | 0.06 | 0.34 | 0.25 |
| 90 | - 0.08 | 0.05 | 0.61 | -0.28 |
| 100 | - 0.04 | 0.05 | | |

Table II Theoretical mean splitting coefficients (in μHz), averaged over frequency between 2 and 4 mHz, computed from the confined field used for Figure 2, and corresponding observed frequencies which in addition are averaged over degree l between the values for which the theoretical coefficients are tabulated.

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