THERMOSOLUTAL CONVECTION

Herbert E. Huppert

Department of Applied Mathematics and Theoretical Physics,
Silver Street,
Cambridge CB3 9EW
England

1. Introduction

The aim of this contribution is to survey a relatively new form of convection, which is very easy to investigate in the laboratory, plays an important role in the oceans and many chemical engineering situations and is likely to prove essential in the understanding of some areas of stellar convection. Thermosolutal convection (or double-diffusive convection as it is often called) owes its existence to the presence of two components of different molecular diffusivities which contribute in an opposing sense to the locally vertical density gradient. The different sets of components studied have covered a wide range including

a. heat and salt - two components relevant to the oceans and a number of laboratory experiments;

b. heat and helium - two components relevant to certain stellar situations;

c. salt and sugar or two different solutes - components useful for laboratory investigations; and

d. heat and angular momentum - components which are likely to be relevant to some stellar situations. In each case, the most rapidly diffusing component has been listed first. Thus, while in the paper the terminology of heat and salt will be used, different components can be envisaged by reference to the above examples.*

Aside from its many applications, thermosolutal convection has received considerable attention because it can induce motions very different from those predicted on the basis of purely thermal convection, that is, convection with only one component. In particular, diffusion, which is known to have a stabilizing influence in thermal convection, acts in a destabilizing manner in thermosolutal convection. By the action of diffusion, instabilities can arise and vigorous motion take place in situations where everywhere throughout the fluid heavy fluid underlies relatively lighter fluid.

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*Ed. Spiegel paraphrases this by the maxim: for salt, think helium. Is this his secret of gourmet cuisine?

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An example of naturally occurring thermosolutal convection which highlights its counter-intuitive nature is afforded by Lake Vanda. Situated in Antarctica, approximately 5 km long, 1½ km wide and 65 m deep, Lake Vanda has a permanent ice cover of 3 - 4 m. Just below the ice the water temperature is 4.7°C and the temperature increases with depth, often in a step-like fashion, until at the bottom the temperature is 24.8°C (Figure 1). There is a corresponding increase in density, from 1.004 gm cm^{-3} just beneath the ice to a maximum of 1.10 gm cm^{-3}

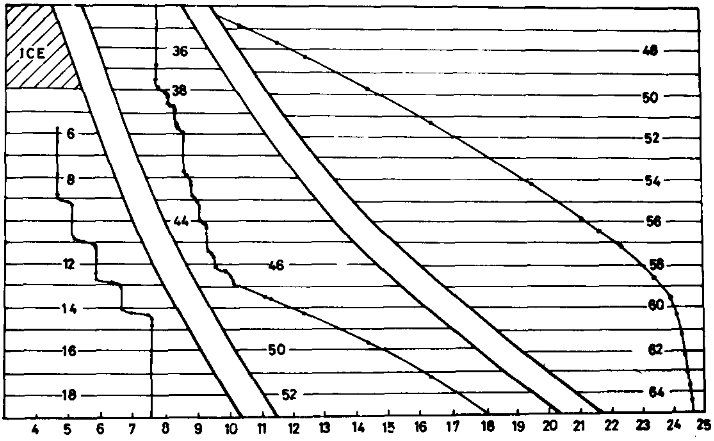

Figure 1. The temperature profile in Lake Vanda as a function of depth indicated in meters (taken from Huppert and Turner, 1972). Note the existence of a layer of uniform temperature (7.6°C) between 14.2 and 37.9 m which has been partially omitted from this figure.

at the bottom, due to the presence of salt. Vigorous convective motions take place in the upper portions of the lake, maintaining the regions of uniform properties, which are the hallmark of thermosolutal convection. Any model of the lake based solely on considerations of temperature, or density, is doomed to failure. Only by incorporating thermosolutal effects can a successful model be derived (Huppert and Turner, 1972).

The plan of this survey is as follows. The two fundamental mechanisms of thermosolutal convection are described physically in §2. These form the foundation of the quantitative analysis of a suitable Rayleigh - Bénard convection problem, whose linear and nonlinear aspects are discussed in §3. The mechanism by which a series of layers and interfaces can be maintained, as in Lake Vanda, is considered in §4. In §5 a few ways in which a series of layers and interfaces can originate are described. The structural stability of such a series is investigated in §6. Conclusions are presented in §7.

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2. The Fundamental Mechanisms

The first of the two fundamental mechanisms of thermosolutal convection occurs in a fluid for which the temperature and salinity both decrease with depth, while the (overall) density increases with depth, as indicated in figure 2a. In this

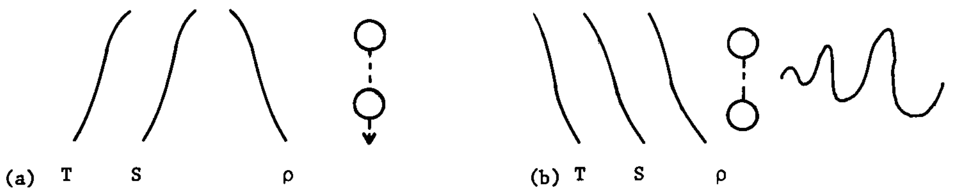

Figure 2. Typical temperature, salinity and density profiles for: (a) the finger situation and (b) the diffusive situation, including a sketch of the motion of a disturbed parcel of fluid.

statically stable situation, the dynamic instability that arises can be examined by considering a parcel of fluid displaced vertically downward. Initially warmer and saltier than its surroundings, the parcel comes to thermal equilibrium before its excess salinity can be diffused. It is thus heavier than its surroundings and continues to descend. The ensuing motion consists of adjacently rising and falling cells, interchanging their heat, and to a much smaller extent their salt, much like a heat exchanger. The kinetic energy of the motion is extracted from the potential energy stored in the salt field. Experiments indicate that in typical conditions, the plan form of the cells, called salt-fingers, is squarish with a horizontal length scale of $\{(\alpha g/\kappa_T \nu)\ (d\overline{T}/dz)\}^{-1/4}$, where α is the coefficient of thermal expansion, g is the acceleration due to gravity, κ_T is the coefficient of thermal diffusivity, ν is the kinematic viscosity and $(d\overline{T}/dz)$ is the mean (positive) vertical temperature gradient. This length scale, discussed further in the next section, represents a balance between dissipative effects acting preferentially on small scale motions and the increasing inefficiency of diffusing heat over ever larger horizontal distances.

The second fundamental mechanism occurs in a fluid whose temperature, salinity and (as before) overall density increases with depth, as indicated in figure 2b. Displacement of the typical fluid particle vertically downwards now places it in a warmer, saltier and more dense environment. As before, the thermal field of the parcel begins to equilibrate with its surroundings more rapidly than does the salt field. The parcel is then lighter than its surroundings and rises. But due to the finite value of the thermal diffusion coefficient, the temperature field of the parcel lags the displacement field and the parcel returns to its

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original position lighter than it was at the outset. It thus rises through a distance greater than the original displacement, whereupon the above process continues and leads to a series of growing oscillations, or overstability, which is resisted only by the effects of viscosity. This oscillatory form of motion has been experimentally documented (Shirtcliffe, 1969) and some of its characteristics explored by Moore and Spiegel (1966) in an imaginative paper which develops an analogy between this form of thermosolutal convection and the motion of a flaccid balloon in a thermally stratified fluid. For sufficiently large temperature gradients, steady motion can occur because a large temperature field can overcome the restoring tendency of the salinity field. The criteria at which this first occurs are discussed in the next section.

3. The Rayleigh-Bénard problem

The fundamental mechanisms of the previous section form the basis of all quantitative calculations. The most straightforward and hence frequently considered calculation relates to the extension of the classical Rayleigh-Bénard problem: what is the motion of a fluid confined between two horizontal planes across which there is a temperature difference ΔT and a salinity difference ΔS? A major motivation behind such studies is the expectation that just as the purely thermal problem has successfully explained a variety of phenomena, as summarised by Spiegel (1971), so also will the thermosolutal extension. And indeed this expectation has already been partially fulfilled.

All calculations so far performed have essentially assumed two-dimensional motion, dependent on one horizontal co-ordinate, x, and the vertical co-ordinate z. Considering this restriction and non-dimensionalising all lengths with respect to D, the separation between the planes, time with D^2/κ_T and expressing the velocity q^* in terms of a streamfunction ψ by

$$q^* = (\kappa_T/D)\ (\partial_z\psi,\ -\ \partial_x\psi), \tag{3.1}$$

the temperature T^* by

$$T^* = T_o + \Delta T\ (1 - z + T) \tag{3.2}$$

and the salinity S^* by

$$S^* = S_o + \Delta S\ (1 - z + T) \tag{3.3}$$

where T_o and S_o are constant reference values, we can write the governing Boussinesq equations of motion as

$$\sigma^{-1}\nabla^2\partial_t\psi - \sigma^{-1}J(\psi,\nabla^2\psi) = -R_T\partial_x T + R_S\ \partial_x S + \nabla^4\psi, \tag{3.4}$$

$$\partial_t T + \partial_x\ \psi - J(\psi,T) = \nabla^2 T \tag{3.5}$$

$$\partial_t S + \partial_x\psi - J(\psi,S) = \tau\nabla^2 S \tag{3.6}$$

where the Jacobian, J, is defined by

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$$J(f,g) = \partial_x f \partial_z g - \partial_z f \partial_x g. \qquad (3.7)$$

We have also assumed the linear equation of state

$$\rho^* = \rho_o (1 - \alpha T^* + \beta S^*), \qquad (3.8)$$

where α and β are taken to be constant, in the expression for the body-force term in (3.4).

Four non-dimensional parameters appear in (3.4)-(3.6): the Prandtl number $\sigma=\nu/\kappa_T$; the ratio of the diffusivities $\tau=\kappa_S/\kappa_T$, where κ_S is the saline diffusivity, which is less than κ_T; the thermal Rayleigh number $R_T = \alpha g \Delta T D^3/(\kappa_T \nu)$; and the saline Rayleigh number $R_S = \beta g \Delta S D^3/(\kappa_T \nu)$.

To these equations must be added a series of boundary conditions. Mainly because of their mathematical simplicity, the most frequently used conditions are those obtained by assuming that both horizontal planes are stress free and perfectly conducting to both heat and salt. That such an assumption is a reasonable one for a model which is to apply in the interior region of a star can be fairly well defended (and often has been). One aspect of the defence incorporates the belief that the use of other, possibly more realistic, conditions is likely to lead to only slight quantitative differences. Indeed, Huppert and Manins (1973), in a series of experiments described below, give an example of this. Mathematically, free-free boundary conditions, as the above are often loosely called, are expressed by

$$\psi=\partial^2_{zz}\psi = T = S = 0 \qquad (z = 0,1). \qquad (3.9)$$

a) Linear Disturbances

The equations governing infinitesimal motions are obtained by deleting the nonlinear Jacobian terms of (3.4)-(3.6). The resulting differential system has constant coefficients and a solution in terms of the lowest normal modes

$$\left.\begin{aligned} \psi(x, z, t) &= \psi_o \sin\pi\alpha x \\ T(x, z, t) &= T_o \cos\pi\alpha x \\ S(x, z, t) &= S_o \cos\pi\alpha x \end{aligned}\right\} e^{pt}\sin\pi z \qquad \begin{aligned}(3.10a)\\(3.10b)\\(3.10c)\end{aligned}$$

leads to the dispersion relationship

$$p^3+(\sigma+\tau+1)k^2p^2+ \{(\sigma+\tau+1)k^4 - \pi^2\sigma\alpha^2k^{-2}(R_T-R_S)\}p + \sigma\tau k^6+\pi^2\sigma\alpha^2(R_S-\tau R_T) = 0, \qquad (3.11)$$

where

$$k^2 = \pi^2(1+\alpha^2). \qquad (3.12)$$

Since (3.11) is a cubic with real coefficients its zeros are either all real or consist of one real root and two complex conjugate roots. Exchange of stabilities, which arises when one of the roots equals zero, i.e. $p= 0$, or equivalently $\partial_t\equiv 0$, occurs first for $\alpha= 2^{-1/2}$ and

$$R_T = R_S/\tau + 27\pi^4/4 \tag{3.13}$$

Overstability, which arises when the pair of complex-conjugate roots crosses the imaginary axis, that is $p_r = 0$, occurs first for the same wavenumber, $\alpha = 2^{-1/2}$, and

$$R_T = (\sigma+\tau)R_S/(\sigma+1) + 27\pi^4(1+\tau)(1+\tau\sigma^{-1})/4. \tag{3.14}$$

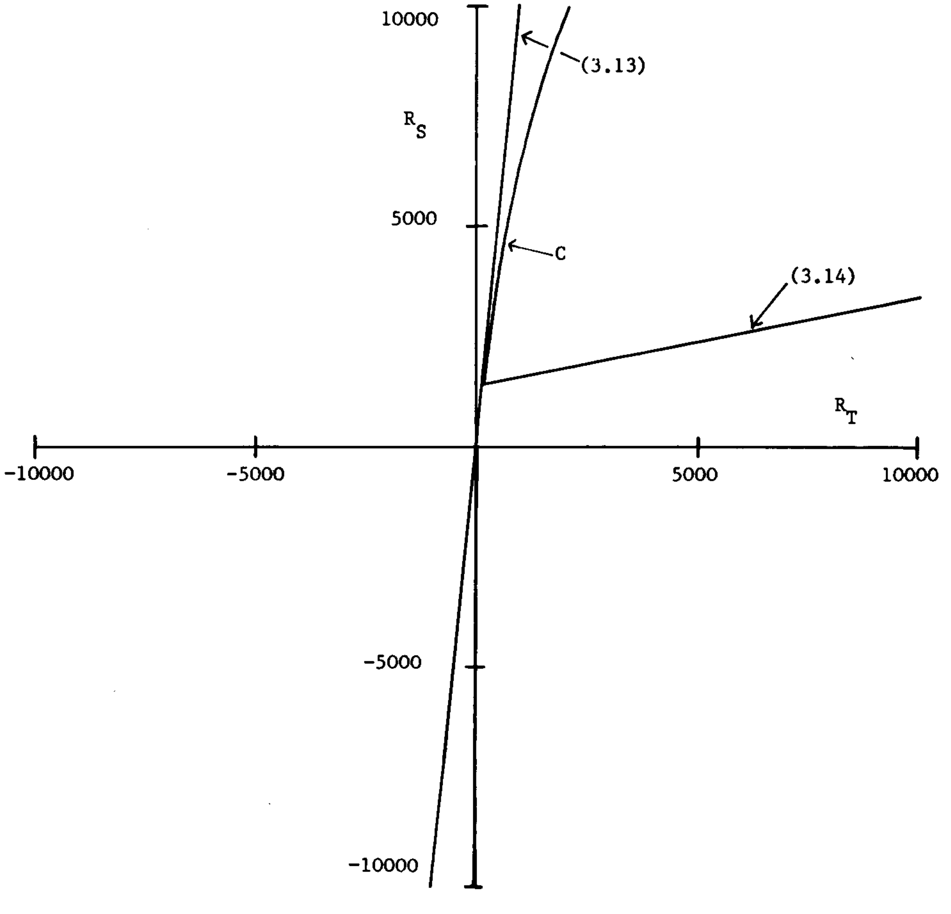

Figure 3. The linear stability results for $\sigma = 10^{-1}$, $\tau = 10^{-1}$. Along (3.13) one of the temporal eigenvalues, p, is identically zero; along (3.14) two of the (complex conjugate) p are pure imaginary; and along C two complex conjugate eigenvalues coalesce on the real axis.

In the R_S, R_T plane the linear stability boundary is a combination of (3.13) and (3.14), as depicted in figure 3, which presents a complete summary of the linear results for $\sigma = 10^{-1}$, $\tau = 10^{-1}$.

An investigation of the fastest growing mode evaluated by linear theory has been presented by Baines and Gill (1969), although owing to the linear constraint, the results are of at most academic interest. In agreement with the result

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originally calculated by Stern (1960), Baines & Gill find that in the salt finger region, the unstable portion of the third quadrant in figure 3, the wavelength of the disturbance of most rapid growth is much smaller than the marginal value, $2^{3/2}\pi$, except very close to the marginal stability line (3.13). This is in accord with the physical description of §2, which indicates that a thinner mode acts as a better heat exchanger.

The following experiment is an example to which results based on linear theory can be profitably applied. A uniform layer of hot, salty water is carefully placed over a uniform layer of relatively colder, fresher water. The temperature and salinity distributions across the initially paper-thin horizontal interface evolve by diffusion, leading to a situation similar to that considered at the beginning of this section. Equating the central gradient of the diffusing distribution to the temperature and salinity gradients which appear in the marginal stability criterion (3.12), Huppert & Manins (1973) calculate that under typical laboratory conditions, specifically R_T, $\tau^{-1}R_S \gg 27\pi^4/4$, salt-fingering should occur if

$$\beta\Delta S/\alpha\Delta T > \tau^{3/2}, \qquad (3.15)$$

where ΔT, ΔS are the initial temperature and salinity differences across the interface. The results of a series of experiments, conducted with a variety of pairs of solutes with different values of τ, are in very good agreement with (3.15).

b) Nonlinear Disturbances

Fully nonlinear, but two-dimensional, investigations have been conducted by Straus (1972) for R_T, $R_S < 0$ and by Huppert & Moore (1976) for R_T, $R_S > 0$.

The former reduces the complexity of the governing equations by assuming that

$$\tau \to 0,\ R_S \to 0 \text{ with } R_S/\tau \text{ fixed.}$$

In this limit, the inertial terms in the momentum equation and the advection of the disturbance temperature, but not the disturbance salinity are negligible. Straus calculates the solutions for a variety of different values of α as R_T increases from the marginal stability value.+ He also tests the linear stability of these solutions. His principal conclusions are that as R_T increases, both extremes of the range of stable wavenumbers increase significantly. Beyond a specific R_T, dependent upon R_S/τ, the range of stable wavenumbers no longer includes the wavenumber at marginal stability. The form of motion with the most stable wavenumber corresponds to the long thin cells of the (somewhat different) experiments and is close to that wavenumber which leads to a maximum salt flux. These results are interesting and suggestive, but the two-dimensional and small τ assumptions might limit the generality of some of the specific conclusions.

The calculations for R_T, $R_S > 0$ of Huppert & Moore aim to follow the form of solutions as R_T increases for fixed R_S, σ and τ. Drawing on the results of a

+The marginal stability point is supercritical, that is, there is only the conductive solution for R_T less than the marginal value.

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number of numerical experiments, they put forward the following general conclusions. There are two rather different branches of solutions. Along one branch, which may be initiated either subcritically or supercritically from the linear oscillatory critical point given by (3.14), the solutions are oscillatory. In

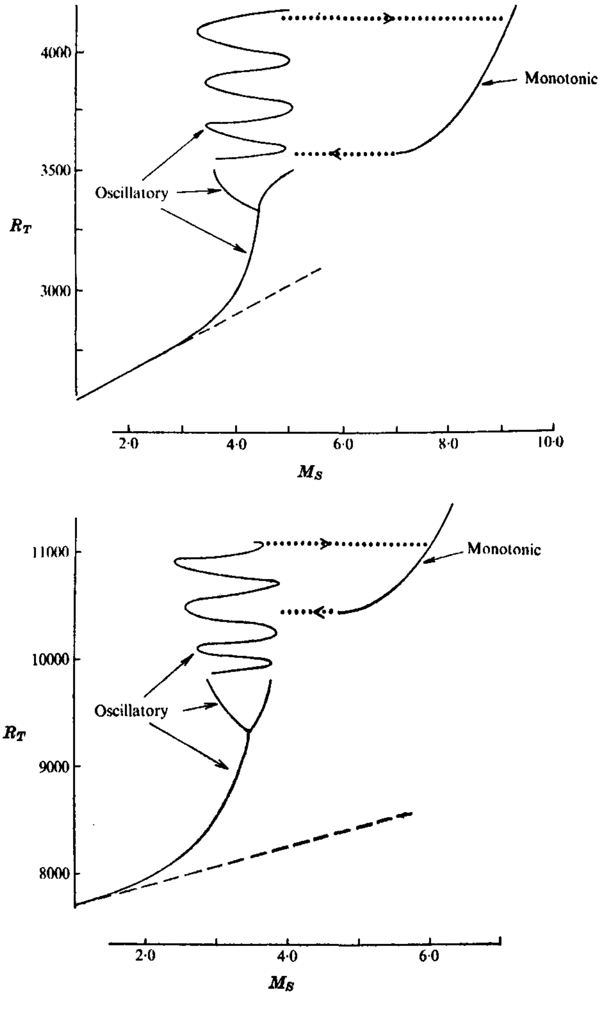
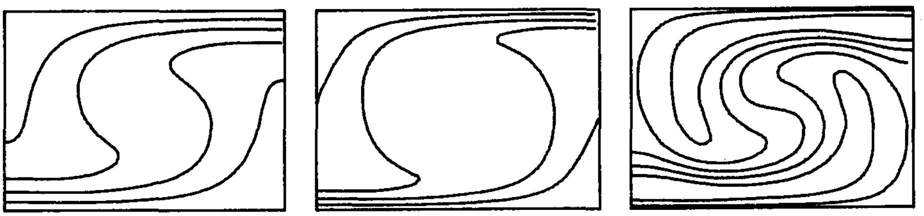


Figure 4. The stable solution branches in a thermal Rayleigh number, maximum Nusselt number at z = 0 plane for (a) $\sigma = 1$, $\tau = 10^{-\frac{1}{2}}$, $R_S = 10^4$ and (b) $\sigma = 1$, $\tau = 10^{-1}$, $R_S = 10^{7/2}$. Where relevant both local maxima are shown and the rapidly oscillating curve indicates that no definite maximum can be assigned to the aperiodic motion in this range. The dots indicate the transitions that can take place between the oscillatory and monotonic branches.

general, as R_T increases, a transition point is attained at which the solution changes from being relatively simple to being fundamentally more complicated, yet still periodic. At a yet larger value of R_T another transition takes place, a transition to aperiodic motion. Finally, beyond a still larger value, stable aperiodic solutions cease to exist and only solutions which are ultimately steady, and make up the second branch of solutions, can be found. The two branches for particular values of σ, τ and R_S are graphed in Figure 4. Steady motion exists in a thermosolutal fluid because of the tendency of the temperature field to cause an almost isosolutal core to be produced, confining all solute gradients to thin boundary layers. The temperature, salinity and density fields for a typical steady solution are presented in figure 5. For given σ, τ and R_S, there is a minimum



T S ρ

Figure 5. The temperature, salinity and density fields for R_T = 10700, $R_S = 10^4$, σ = 1 and $\tau = 10^{-\frac{1}{2}}$.

value of R_T for which steady motion exists and one of the aims of the investigation by Huppert & Moore is to calculate this minimum. For details of this result and others the reader is referred to the original paper. The major finding is that for sufficiently small τ, steady convection can occur for values of R_T less than *that obtained from the linear stability boundary (3.14) (and thus much less than* the value at which linear theory suggests non-oscillatory convection occurs).

The specific results obtained by Huppert & Moore, primarily by numerical computation, were limited to 14 different values of σ, τ and R_S. Guided by these calculations, M. R. E. Proctor and independently Huppert & Gough are currently attempting to obtain analytic expressions for various limiting cases, in particular, the astrophysically relevant situation $\tau \to 0$.

4. Layers and Interfaces

As the experiment of Huppert and Manins, described in the previous section, progresses, the fingers and the interface between the two layers extend in length. Within the interface there is a strong background gradient of density, and the interface is hence an ideal site for internal waves, which are generated by disturbances induced by the salt-finger motion. These internal waves cause the fingers to sway back and forth, like a banner fluttering in the breeze. If the fingers become too long, this motion causes them to loose their vertical coherence, or break, much like a long strut subjected to an oscillatory transverse load. Guided by the Navier-Stokes equations of motion rather than the analogies used above, Stern (1969) argues that for very small τ an established field of salt fingers is limited in length by the requirement that

$$\beta F_S / \left(\nu\alpha \frac{d\overline{T}}{dz}\right) < C, \tag{4.1}$$

where F_S is the salt flux through the interface, to be discussed below, $d\overline{T}/dz$ is the mean temperature gradient and C is a constant of order unity. Thus for a fixed salt flux, the length of the salt fingers and the thickness of the interface increase until equality in the constraint (4.1) is reached.

At the two edges of the interface the salt fingers impart an unstable buoyancy flux on the adjacent layers, which causes the layers to convect. Developing an analogy with purely thermal convection at high Rayleigh number, Turner (1967) argues on the basis of dimensional analysis that the relationship between the saline Nusselt number and the Rayleigh number is of the form

$$Nu_S \equiv F_S\ D/(\kappa_S \Delta S) = f_F(\alpha\Delta T/\beta\Delta S,\ \sigma, \tau) R_S^{1/3}\ , \tag{4.2a,b}$$

where D cancels in (4.2b) and f_F is some function of its three argments. Also, argues Turner, the resultant heat flux, F_T, is related to F_S by

$$\alpha F_T/\beta F_S = g_F(\alpha\Delta T/\beta\Delta S,\ \sigma,\ \tau)\ . \tag{4.3}$$

Turner obtained experimentally the explicit form of f_F and g_F for heat and salt in water and found that for the range of $\alpha\Delta T/\beta\Delta S$ considered, $2 < \alpha\Delta T/\beta\Delta S < 10$, f_F is such that as $\alpha\Delta T/\beta\Delta S \to 1$ the salt flux is approximately 50 times as large as if the same salinity difference were maintained across a region bounded by two solid boundaries, and f_F decreases slowly with increasing $\alpha\Delta T/\beta\Delta S$. The constancy of g_F indicates that, independent of $\alpha\Delta T/\beta\Delta S$, a constant fraction of the potential energy released by the salt field is supplied to the temperature field. Linden (1973) experimentally evaluated f_F and g_F using a different technique and determined the same f_F but a different, yet still constant, g_F. Which result is in error is still not known.

If a uniform layer of hot, salty water is placed below a uniform layer of relatively colder, fresher water, heat and salt are transferred upwards through the thin interface primarily by diffusion, with the resulting unstable buoyancy flux driving convection in the layers as before. For this case, known as the diffusive situation, the relationships equivalent to (4.2) and (4.3) are

$$Nu_T \equiv F_T\ D/(\kappa_T \Delta T) = f_D(\beta\Delta S/\alpha\Delta T,\ \sigma,\ \tau) R_T^{1/3} \tag{4.4a,b}$$

and

$$\beta F_S/\alpha F_T = g_D(\beta\Delta S/\alpha\Delta T,\ \sigma,\ \tau) \tag{4.5}$$

for some functions f_D and g_D. Using the results of another series of experiments by Turner (1965) with heat and salt in water , Huppert (1971) suggests that for this particular case

$$Nu_T = 3.8\ (\beta\Delta S/\alpha\Delta T)^{-2}\ R_T^{1/3} \tag{4.6}$$

$$\beta F_S/\alpha F_T = \begin{cases} 1.85-0.85(\beta\Delta S/\alpha\Delta T) & 1 < \beta\Delta S/\alpha\Delta T < 2 \quad (4.7) \\ 0.15 & 2 < \beta\Delta S/\alpha\Delta T. \quad (4.8) \end{cases}$$

A deductive model of the diffusive interface has not as yet been obtained, though a number of ad hoc arguments, some of them described by Turner (1974), lead to formulae in some agreement with equation (4.8). A few experiments with two solutes, rather than heat and salt, have been performed. For both salt-fingering and diffusive cases, all the experiments indicate a constant value of the flux ratios, (4.3) or (4.5), for a large range of $\beta\Delta S/\alpha\Delta T$. The deduction explanation for this fact is awaited and is one of the major theoretical prizes still to be gained. To be more general, a major advance in the subject would be achieved on building a mathematical model which predicts the heat and salt fluxes for all values of σ and τ.

Notwithstanding our current lack of knowledge, the important conceptual statement that can already be made is that the above mechanisms can be extended to include a series of convecting layers, separated by fingering or diffusive interfaces, as the situation demands. This is the explanation of the profiles of Lake Vanda, those obtained under the drifting Arctic Island T3 displayed in figure 6, and of many other oceanographic examples. The main aim of this review is to support the suggestion that a process which occurs so readily on earth must also play a fundamental role in stellar convection.

5. The Building of Layers

As suggested in the previous section, a series of convecting layers separated by thin interfaces can be easily constructed in the laboratory by carefully placing one layer on top of another. They arise in natural situations by a large number of different mechanisms. A few situations have received a fair amount of quantitative analysis and will be described here.

Consider a fluid with a uniform salinity gradient increasing with depth subjected to a constant heat flux, F_H, at its base. Initially a growing overstable oscillation occurs. Shortly thereafter a convecting layer develops adjacent to the bottom because hot fluid rising from the base can penetrate only a finite height into the stable salinity gradient. As time proceeds the height of this layer, h, grows according to

$$h = (2Bt)^{\frac{1}{2}}/N_s, \tag{5.1}$$

where

$$B = -\alpha g F_H/(\rho c) \quad \text{and} \quad N_S^2 = -g\beta\frac{dS}{dZ}. \tag{5.2}$$

The relationship (5.1) is a consequence of the conservation of heat and salt and the experimentally observed fact that the density (but not the temperature or salinity) is continuous across the top of the layer (Turner, 1968).

This growth does not, however, continue indefinitely. There is a thermal

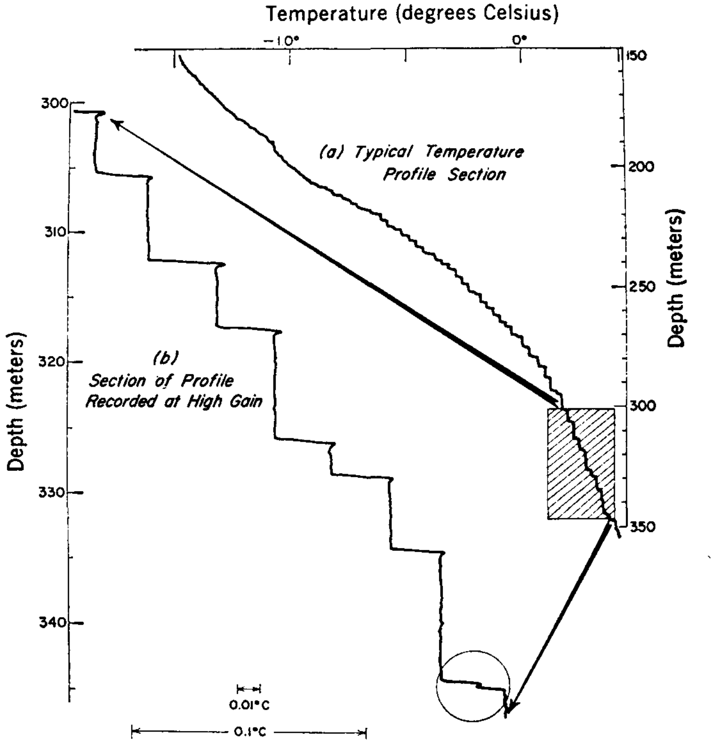


Figure 6. The temperature profile under the Arctic Ice Island T-3 (taken from Neal et al, 1969).

boundary layer ahead of the advancing front and when a critical Rayleigh number, R_c, is reached the region above the first layer ceases to grow. This can be calculated to occur when

$$h = (\tfrac{1}{4}\sigma\ R_c B^3/\kappa)^{1/4}/N_S^2\ . \qquad (5.4)$$

The second layer then grows, the thermal boundary layer ahead of its advancing front becomes unstable, and so in time a series of layers is built up. Heat and salt are transferred across the interfaces, in the manner of the last section, and in the course of time some of the lower interfaces disappear because the density difference across them tends to zero. A combined theoretical and experimental investigation of the depths of all the layers and time scales for their formation is currently being undertaken by Huppert & Linden (197?).

This heating a salinity gradient from below mechanism, but acting in reverse, that is, cooling a salinity gradient from above, produces the layers under T3 shown in figure 6.

The above mechanism involes an entirely one-dimensional model. Many natural phenomena can be expected to be two - or even three-dimensional. As yet such extensions are only in the early stages of investigation.

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In a series of qualitative experiments, Turner & Chen (1974) show that even a relatively small disturbance applied to one side of a thermsolutal fluid, thereby introducing horizontal inhomogeneities, can have significant effects. For example, the raising of a small flap at the wall of a vessel containing a thermosolutal fluid induces, in the salt-finger situation, a rapidly propagating wave motion which is accompanied by the initiation of convection over large horizontal distances. In the diffusive situation, the disturbance propagates horizontally more slowly and can cause local overturning which leads to the initiation of salt fingers.

Another example concerns the introduction of a small source of warm, salty water into a uniform layer of relatively colder, fresher water of exactly the same density. Fluid which commences to fall diffuses its heat to the surroundings, thereby becoming heavier. Neighbouring fluid, having been warmed, is relatively lighter and rises. Large vertical motions, both upwards and downwards, in the form of plumes result. As the motion proceeds, the density difference between each plume and the surroundings increases. Thus, starting only with fluid of uniform density, solely by diffusion both heavier and lighter fluid are formed. If the plumes impinge on horizontal boundaries, they spread out and build a series of layers and interfaces through the entire vertical extent of the fluid.

A final example is afforded by introducing an insulated sloping boundary. In a stably stratified fluid, stratified with respect to only one component, such a sloping boundary induces a thin slow steady upwards motion adjacent to the boundary. The flow provides a convective density flux equal to the diffusive flux in the interior and allows the isopycnals, horizontal in the interior, to bend near the boundary and intersect it at right angles. In a fluid stratified with respect to two components, the curves of constant T and constant S must intersect the boundary at right angles and no steady boundary layer flow can accomplish this. Alternatively, it is not possible for a single boundary-layer to give rise to convective T and S fluxes which balance the unequal diffusive T and S fluxes in the interior. Instead, a series of layers and interfaces from throughout the fluid, as shown in figure 7 to build that characteristic structure of a thermosolutal fluid.

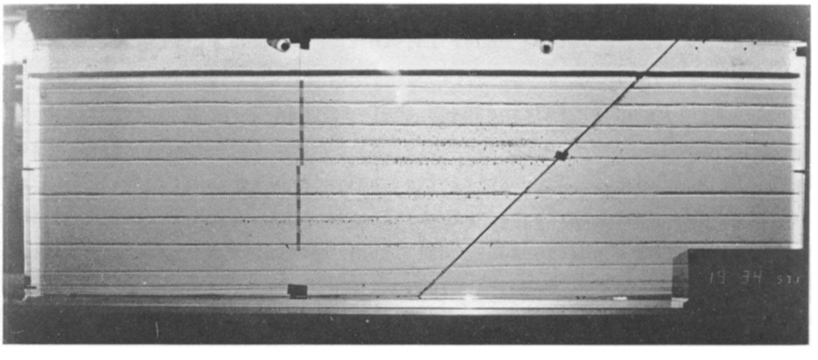

Figure 7. A series of layers and interfaces set up in a laboratory tank by introducing a solid sloping boundary (from Linden and Weber, 1977).

The size of layers and the time-scale of their initiation is dependent upon the angle of the sloping boundary, or more generally their existence is due to its presence. However, this specific example presents another illustration of the basic point: initially small horizontal inhomogeneities in a stably stratified fluid leads to large scale layering with vertical transports considerably in excess of those calculated on a molecular basis.

6. The Destruction of Layers

In the experiment of heating a salinity gradient from below, discussed in section 4, the tendency of the lower layers to merge as the density difference across them tends to zero was mentioned. Such merging is due to the continually imposed flux of heat from the bottom. Layers can merge, or be destroyed by more natural, internally imposed conditions, which will be described in this section.

Consider a three-layer system consisting of two semi-infinite layers of uniform T and S between which there is a finite layer of intermediate properties. All layers are assumed to be convecting, with temperature and salinity fluxes across the interfaces in accord with (4.2) and (4.3) or (4.4) and (4.5). There is then a single-valued relationship between the temperature and salinity in the intermediate layer for which the flux through the lower interface equals that through the upper. The conditions under which such equilibrium situations are stable is partially answered by Huppert (1971). Assuming that merging takes place without any vertical migration of the interfaces, he shows that only if the conditions across each interface are in the 'constant regime', g_F or g_D equals constant, will the layer system persist. Otherwise one or other of the interfaces will disappear and two semi-infinite layers separated by one interface remain. The analysis can be extended to any number of intermediate layers to yield the same result. Thus the prediction is that no stable system of diffusive layers of hot salty water exist if $\beta\Delta S/\alpha\Delta T < 2$. A controlled laboratory experiment to test this prediction has yet to be performed, although Turner and Chen (1974) and Linden (1976) have observed merging which they believe to be due to the above mechanism. Turning to large scale measurements, we can at present report that no series of layers has been observed under T3, in the Red-Sea or elsewhere with $\beta\Delta S/\alpha\Delta T < 2$.

Experiments by Linden (1976) indicate that another form of instability is possible, whereby merging occurs by the vertical movement of one interface to coalesce with its neighbour. A quantitative analysis of this situation has not yet been performed. It would clearly be interesting to know which instability is favoured under specified conditions because layer merging will need to be accounted for in any future quantitative model building.

7. Summary and Conclusions

This review has attempted to bring out the following salient points. Fluids stratified with respect to two (or more) components can exhibit motions very different to singly-stratified fluids. Instabilities can arise even when the overall density is statically (very) stable by drawing on the potential energy stored in one particular component. The typically observed signature of a thermo-solutal fluid is a series of convecting layers separated by thin interfaces through which properties are transported by either diffusion or the action of fingers. This transport is very much larger than one based on consideration of purely molecular diffusion across a quiescent region.

Large stars have a heated helium-rich core surrounded by lighter hydrogen. The composition gradient in the core/envelope regions is thus of the diffusive type and it would be expected that convection of this form predominates. Some attempt has been made to incorporate this process in a semiconvection zone, although quantitative calculations would benefit from a precise description of physics in this zone.

The salt-finger type of instability has been hypothesised to occur in the outer layers of differentially rotating stars (Goldreich and Schubert, 1967), where the two components with different diffusivities are heat and angular momentum. Turner (1974) appears to be of the opinion, however, that such an effect would be obliterated by baroclinic instabilities which occur on a much larger scale. This conclusion should be questioned in view of the large observational evidence for the existence of salt-fingers in the ocean, also subject to baroclinic instability.

We conclude by suggesting that what is achieved so easily in the laboratory and the oceans might also be attained by the stars!

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