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SUBDIRECT PRODUCT OF *PS*-RINGS NEED NOT BE *PS*

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An associative ring R with identity is called a PS-ring if $\mathrm{soc}({}_RR)$ is projective. We construct a non-PS-ring R which is a subdirect product of two PS-rings, thus answering a question of Nicholson and Watters in the negative.

Nicholson and Watters [1] called an associative ring R (with identity) a (left) PS-ring in case $\mathrm{soc}({}_RR)$ is projective, and they showed that a product of rings ΠR_i is a PS-ring if and only if each R_i is a PS-ring. In this note, a non-PS-ring R is presented such that R is a subdirect product of two PS-rings. This answers the question [1, p.447] in the negative.

Let F be a field. Let $S = \begin{bmatrix} F & F \\ 0 & F \end{bmatrix}$ and $T = \begin{bmatrix} F & 0 \\ F & F \end{bmatrix}$. One notes that both S and T are PS-rings. Let R be the subring of $S \times T$ consisting of elements of the form

$$\begin{bmatrix} a & c \\ 0 & b \end{bmatrix} \times \begin{bmatrix} a & 0 \\ d & b \end{bmatrix}.$$

Then $\mathrm{soc}({}_RR) = \left\{ \begin{bmatrix} 0 & c \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 \\ d & 0 \end{bmatrix} \mid c, d \in F \right\}$ is not projective, that is, R is not a PS-ring. Now R has two ideals

$$I = \left\{ \begin{bmatrix} 0 & c \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \mid c \in F \right\}$$

and

$$J = \left\{ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 \\ d & 0 \end{bmatrix} \mid d \in F \right\}$$

such that $I \cap J = 0$, $R/I \cong T$ and $R/J \cong S$. Hence R is a subdirect product of the PS-rings T and S.

REFERENCE

[1] W.K. Nicholson and J.F. Watters, 'Rings with projective socle', *Proc. Amer. Math. Soc.* **102** (1988), 443–450.

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