whole subject); "Completions" (including a new theory of the exact tensor product); "Multiplicities"; "Theory of Syzygies" (without homological methods, hurrah); "Theory of complete local rings"; "Henselian and Weierstrass rings". There are also two appendices, one of which is a very detailed collection of historical comments.

An interesting novel feature is the "principle of idealization": Let R be any commutative ring. With every R-module M there is associated a ring R* as follows: As an additive group, $R^* = R \oplus M$, and multiplication is defined by

$$(r+m)(r'+m') = rr' + (rm'+mr') .$$

R* contains R as well as M, the latter as an ideal, and $M^2 = 0$. The submodules of M are nothing else than the ideal of R* contained in M. This principle enables the author to translate statements about modules into statements about ideals.

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Geometry of Complex Numbers, by Hans Schwerdtfeger. (Mathematical Expositions No. 13) University of Toronto Press, Toronto, 1961. xi + 186 pages. $4.95.

The book under review has three chapters: I. Analytic Geometry of Circles, II. The Möbius Transformation, and III. Two-dimensional Non-Euclidean Geometries.

Chapter I treats standard topics, as indicated by the section headings: Representation of circles by Hermitian matrices, Inversion, Stereographic projection, Pencils and bundles of circles, Cross-ratio. The algebraic machinery used extensively throughout the book, especially $2 \times 2$ matrices, is introduced here. Thus, for example, inversion in a circle has a purely geometric definition which depends on the theorem that for a given circle C and point z (not on C or its center) there is exactly one other point z* lying on all circles through z orthogonal to C. However, proof of this theorem is given in purely algebraic terms, involving the previously established equivalence between Hermitian matrices and circles, the algebraic condition for orthogonality, and - at one stage - a $3 \times 4$ complex matrix which is required to have rank two.

At the end of each section appears an extensive list of examples, some for the student to work out and some worked out in detail by the author. These are clearly intended to be an integral part of the text. For example, two pages are devoted to the problem of finding a circle with respect to which two given circles are mutually inverse, considering the five special cases which arise. Throughout the book the student must be an active participant with pencil and paper, in the body of the text as well as in the examples.

The author's main interest is presumably in Chapter II, where he treats the Möbius transformation $Z = \mathcal{J}(z) = \frac{az+b}{cz+d}$ and the anti-homographies $Z = \mathcal{J}(\bar{z})$ in detail. Some of the terminology and results have direct geometric motivation, and the tools are occasionally geometric - e.g. the characteristic parallelogram of $\mathcal{J}$, defined by the two fixed points of $\mathcal{J}$ together with the poles of $\mathcal{J}$ and $\mathcal{J}^{-1}$. However the author is essentially concerned with an algebraic study of the group of all Mobius transformations (we write M.t.), certain of its subgroups, and their normal forms. The preliminaries, showing the geometrical significance of certain special cases and the proof of the circle preserving properties of $\mathcal{J}$, are followed by a section on the relation between one dimensional perspectivities, projectivities and M.t. The next section discusses similarity of M.t., showing that similar M.t. have similar characteristic parallelograms, and hence that if $\mathcal{J}_0$ is a given M.t. with finite characteristic parallelogram, then every similar M.t. $\mathcal{J}$ with finite characteristic parallelogram can be written $\mathcal{J} = \mathcal{I}\mathcal{J}\mathcal{I}^{-1}$, where $\mathcal{I}$ is a uniquely determined integral M.t. (c = 0). This is followed by a discussion of anti-homographies and a long section on iteration of M.t. (needed for the determination of one-parameter subgroups used in Chapter III). The chapter concludes with characterizations of M.t. and anti-homographies as (i) circle preserving transformations in the completed plane and (ii) collineations in the real projective space.

Chapter III applies the results and methods of the preceding chapters to a discussion of plane hyperbolic, Euclidean and spherical geometries on the basis of their groups $\mathcal{U}_+$, $\mathcal{E}$ and $\mathcal{R}$ respectively. Here $\mathcal{U}_+$ is the subgroup of M.t. mapping the unit circle $Z\bar{Z} = 1$ onto itself (and its interior onto its interior), $\mathcal{E}$ the subgroup of displacements, and $\mathcal{R}$ the subgroup mapping the imaginary unit circle $Z\bar{Z} = -1$ onto itself. Each of those groups has a domain $\mathcal{D}$ (the interior of a circle, the dotted plane, and the completed plane respectively) in which it is simply transitive. One-parameter subgroups of these groups define "straight line", "cycles", and "circles" in $\mathcal{D}$, and hence by means of two-point invariants ("distance") the three geometries are obtained. The case of the hyperbolic plane is worked out in great detail, including trigonometry and area, with slightly less attention paid to the other two geometries. Finally, elliptic geometry is derived from spherical geometry by identification of antipodal points

A brief list of Errata is available from the publisher.

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