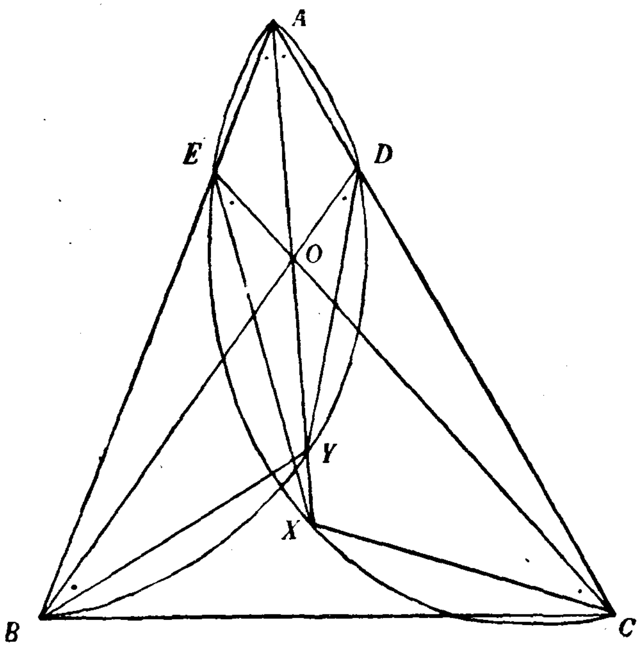
The equal internal bisectors theorem

By C. E. Walsh.

Much interest has always been aroused by this theorem, which asserts that a triangle is isosceles when two of the internal bisectors of its angles are equal. Recently McBride[1] has given a proof, together with a selection from the numerous others which have been published. The following proof, based mainly on Euclid, Book III, differs from any I have come across, and establishes a slightly wider theorem.



Theorem. *Let* BD *and* CE *be two equal lines drawn from the vertices* B *and* C *of a triangle* ABC, *meeting the opposite sides internally at* D *and* E *and intersecting at* O. *Then if* AO *bisects the angle* A, *the triangle* ABC *is isosceles.*

[1] J. A. McBride, *Edin. Math. Notes*, No. 33 (1943), 1-13, where many references will be found.

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PROOF. Let the circles AEC and ABD cut AO in X and Y respectively. Then each of the angles XEC, XCE, YDB, YBD is in the same segment of a circle as OAB or OAC, and is equal to $\frac{1}{2}A$. Consequently the triangles XEC, YDB are congruent, and the four lines XE, XC, YD, YB are equal. It also follows that XE is a tangent to the circle EAO, and YD to DAO, so that, $XO.XA = XE^2 = YD^2 = YO.YA$. Accordingly X and Y must coincide, at the centre of a circle $BCDE$. In this circle the equal chords BD and CE subtend equal angles at the circumference. Therefore angles B and C are equal (or supplementary: this is impossible), and the triangle is isosceles.

74 SERPENTINE AVENUE,
BALLSBRIDGE, DUBLIN.

A note on equilateral polygons

By A. D. RUSSELL.

Theorem. If a circle cut all the sides (produced if necessary) of an equilateral polygon, the algebraic sum of the intercepts between the vertices and the circle is zero; *i.e.*, if any side AB of the polygon be cut by the circle in P and Q, then $\Sigma(AP + BQ) = 0$, the intercepts being signed by fixing a positive direction round the contour of the polygon.

This was proved in a previous note,[1] using Euclid III. 35, for the special case of a *regular* polygon; but the same proof applies to any *equilateral* polygon, which may be re-entrant or self-crossing.

Corollary. If n chords P_iOQ_i $(i = 1, \ldots, n)$ of a circle are drawn through a point O parallel to the sides of an equilateral n-gon, the

[1] "Theorem regarding a regular polygon and a circle cutting its sides," *Mathematical Notes*, No, 22, 1924.

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