GLOBAL CIRCULATION OF THE SUN: WHERE ARE WE AND WHERE ARE WE GOING?

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1. INTRODUCTION

There are several goals I hope to reach in this talk. First, I would like to give this broad audience some idea of our present state of knowledge concerning global circulation of the sun--what the observations tell us (and don't tell us!) as well as current theories. This will comprise a large part of my talk. In doing this, I hope I do not bore the specialists in this area. Second, I would like to put the problem of global circulation of the sun in the broader context of "solar variability", a topic of rapidly growing interest in the solar community, that has implications for stars generally, and which may not have caught the attention of the stellar community. I would like also to put the solar circulation problem in the context of stellar rotation, by making a few admittedly speculative extrapolations to other stars. In addition, I will give a few of my own opinions about what we should be doing in the near future to make more progress in understanding the solar problem. And finally, I hope to provoke a wide ranging discussion of the whole area of circulation and variability in the sun and stars.

2. OBSERVATIONAL KNOWLEDGE OF GLOBAL CIRCULATION OF THE SUN

2.1 Differential rotation

Howard (1978a) has recently given a rather complete review of observations of solar circulation, and I have written a brief summary of some of the most recent developments for the IAU Reports in Astronomy so there is no need to document all the details and references here. The Proceedings of the Catania Workshop on Solar Rotation is now available, in which several new efforts and results are reported.

The best known feature of global circulation on the sun is of course its differential rotation. Its existence was first demonstrated

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systematically by Carrington in the 1860's from sunspots, although it had been noticed by Scheiner more than two centuries before. The equatorial acceleration is substantial: the rotation rate is almost 40% higher near the equator than the poles. The angular velocity also appears to decrease monotonically all the way to the poles. Different rotation rates are obtained by different techniques. Sunspots rotate faster than the photospheric plasma, as measured by the doppler shift, by about 5%. Magnetic fields and filaments show less differential rotation than seen in the doppler shift. (However, Howard (1978b) has recently reported very low rotation rates of magnetic "elements" in polar regions.) Coronal holes seem to show almost no differential rotation, as do the longest lived magnetic patterns. By contrast, the shortest lived magnetic features show more nearly the plasma differential rotation.

It's possible to subdivide even further the various tracer measurements. For example, large sunspots rotate more slowly (by up to 2%) than small spots (Ward, 1966). On the other hand, larger, longer lived x-ray features rotate faster by several percent than the small compact features (Golub and Vaiana, 1978). Regions of strong network field rotate faster than weak field regions (Foukal, 1976). Similar sorts of differences are seen in faculae (Belvedere et al., 1977). Further detailed differences can be seen undoubtedly arising in part from statistical fluctuations in whatever rotation measure is used.

The reality of the difference between sunspot and doppler rotation rate recently came into question, with Stanford Solar Observatory doppler measurements, when corrected for scattered light, appearing to give a plasma rate equal to the sunspot rate. However, Foukal (1979) has convincingly shown that sunspots do rotate faster than their surroundings, by comparing the doppler rate inside and outside individual spots. Beckers (1977) had previously demonstrated that the doppler and proper motion of individual spots agreed to within 1%.

Recently, considerable evidence has accumulated that the solar rotation rate evolves in time. For example, Howard (1976) has reported that the rotation rate at almost all latitudes rose 4-5% between about 1968 and 1975 although the rise was not monotonic. This general increase has been confirmed from Kitt Peak data by Livingston and Duvall (1979) with a smaller sample of data. They also see evidence that the polar rotation rate changes with the solar cycle, by about 8%, with the most rapid polar rotation occuring near or just after cycle maximum. Stenflo (1977) reports changes in the rotation rate of magnetic patterns with time over a solar cycle. These take the form of regions of larger than average shear in rotation that migrate toward the equator on the poleward side of the sunspot zones. Sunspot data are also revealing long term changes in solar rotation. Eddy, Gilman, and Trotter (1976, 1977) found that the equatorial rotation rate was 3-5% faster just before the onset of the "Maunder Minimum" in solar activity in 1644, as compared with the rotation rate seen twenty years before. Eddy et al. (1978) have shown a secular decrease of a few percent in rotation in the first half of twentieth century in sunspots, followed by an apparent leveling off. Perhaps this connects to the rising rate seen slightly later in the doppler measurements by Howard.

Short period changes in the doppler rotation rate have also been reported, particularly in the Mt. Wilson data, on time scales of a week or so, but there are doubts about these measurements. For one thing, rotation can really only be defined from a sample of observations at least one rotation long, so that all longitudes are sampled. Shorter period changes may be other longitude dependent east-west motions moving into view. But more importantly, Mt. Wilson and Kitt Peak do not see the same short term fluctuations, and the fluctuations at Stanford Solar Observatory are much smaller than either. Certain instrumental errors have recently been discovered, involving fringes produced by KDP crystals, which have contributed to these short period changes. Tracer measurements are not of much help in defining such short term changes, because of large scatter and poor coverage on the solar disk.

2.2 Other Global Circulations

A great deal of effort has been expended by solar observers to see other global circulation besides the differential rotation, with disappointing results so far. Three different observatories (Mt. Wilson, Stanford, Sacramento Peak) have recently reported evidence of axisymmetric meridional flow, toward the poles in each hemisphere, of magnitude approximately 10-20m/sec. However, there are serious difficulties in separating such a signal from limb shifts and "ears" (Howard, 1979a) so this result must be regarded with considerable caution. Evidence for giant "cells" or eddies, that is global flow patterns not symmetric about the axis of rotation, has been even harder to obtain, although there are many solar phenomena suggestive of such flows. In the doppler measurements, Howard (1979b) has seen large scale, probably radial flows near the equator which recur for several rotations. But these are seen only occasionally, and hardly can be said to cover the whole sun the way differential rotation or supergranules do. Schröter and several colleagues, e.g., Schröter, et al. (1978) have looked hard at the movement of the calcium network with observations at Locarno, to try to find evidence of other global circulations and rotation rate changes. They find some, but they do not correlate well with either their own doppler velocity measurements or Mt. Wilson measurements. Nor do the Mt. Wilson and Locarno doppler measurements of rotation changes correlate well; the former are highly correlated in latitude, and the latter are not. Part of the difference between doppler and Ca+ velocities may reflect a very complex interaction between the plasma flow and the magnetic fields, but also raises doubts about the accuracy of the measurements, particularly of the doppler velocities.

The very existence of large scale patterns in the solar magnetic field, even though the individual elements of magnetic flux in the pattern are very small in spatial scale, is suggestive of corresponding global velocity patterns which have yet to be measured directly. Similar impressions are gained from coronal holes, particularly ones which last several rotations, and are not being passively sheared apart by the differential rotation. The arrangement of the pattern of large filaments on the sun is sometimes suggestive of underlying global disturbance structure, e.g., Wagner and Gilliam (1976), as are the evolution of Hα neutral lines (McIntosh, 1979). The apparent existence of "active longitudes", areas on the sun where new active regions preferentially arise, suggests there are persistent velocity patterns bringing up the new magnetic flux (see Bogart, 1979, for recent evidence on active longitudes).

There is also the old calculation by Ward (1965) of the correlation of east-west and north-south sunspot motions which has been interpreted as evidence of an equatorward transport of angular momentum by eddies presumably of much larger spacial scale than sunspots. More recently, a similar but usually larger correlation has been seen in movement of Calcium plages by Schröter and Wöhl (1978b) and by Belvedere et al., (1978), but the scatter has been too large to determine the profile of transport with latitude. This is one of the flow properties one would really like to determine from doppler as well as from tracer velocities.

To summarize the status of our observational knowledge of giant cells or global eddies, there are many tantalizing bits and pieces, but they do not corroborate each other well yet, and most are not direct measures of the velocity field itself, but some other pattern or tracer. Various theoretical considerations do favor the existence of giant cells, and their existence is presumed in most global circulation models applied to the sun.

3. WHAT IS NEEDED IN NEW OBSERVATIONS

To me the key observational question yet to be answered is--do global eddies or giant cells exist, and if so, what do they look like; what is their magnitude and their time evolution; how do they relate to the magnetic and other global patterns that we see? In my opinion, we will never be able to demonstrate the existence of such eddies convincingly from tracer measurements alone. Virtually all tracers are magnetic in origin, and so involve the interaction of plasma velocity and magnetic patterns. There is no substitute for the direct measurement of the plasma velocity.

When one looks at how this measurement has been made in the past, it is hard to escape the feeling that the observing programs carried out were not designed to find such motion. Global velocities other than differential rotation are apparently weak (< 50 m/sec) and yet no stable wavelength references have been employed against which to measure. The velocity signal at any instant in time is heavily influenced by large amplitude small scale solar noise: granules, 5 minute oscillations, and supergranules--and yet the typical measurement program has obtained only one or two full disk scans per day. One exception is the short series of observations at Locarno reported in Schröter et al., (1978).

What is needed are observing programs designed from the start to look for weak global eddies, which make use of the best available technology in detectors, reference standards, and devices to analyze the spectrum. The whole solar disk must be observed often enough to average out the granule and oscillations noise, requiring full disk observations at least every minute, and densely enough in time from hour to hour and day to day to reduce the influence of supergranules as far as possible. It appears that supergranules will be harder to get rid of than five minute oscillations and granules, but we should be able to do much better than we have done so far. My own observatory and Sacramento Peak Observatory have been working together on the development of one such new instrument, called a Fourier Tachometer, which we have high hopes will allow more accurate, more stable, faster measurements to be made. The principal scientists involved are Tim Brown at HAO and Jacques Beckers at SPO. I urge other groups in other places to also look at this problem, and design their observing system and programs in such a way as to optimize their chances of measuring the global eddies. Clearly, we need comparable observations from several different observatories in the world. This is not an effort that can be made casually over a short period of time. It requires a long term dedication, and considerable cleverness, because the observation is difficult to make. But the scientific payoffs are potentially large, because knowledge of the global eddy flows on the sun may be the key to understanding the origins of solar activity, and magnetic structures on the sun--the workings of the solar dynamo. More about that later.

4. GLOBAL CIRCULATION IN THE CONTEXT OF SOLAR VARIABILITY

I have given you some details of what we know now about global velocities on the sun, including evidence that the surface rotation rate varies with time on time scales of years to hundreds of years. There are now several other kinds of evidence that the global properties of the sun vary on these time scales. The basic sunspot cycle of about eleven years, and the magnetic cycle of 22 years, are of course well known to everyone. In addition, there is the almost equally well known envelope to the solar cycle, which shows there is a long term modulation to the amplitude of successive cycles. Whether there are true "periods" to this envelope variation is a subject of recurrent argument in the literature. Eddy (1976) has recently demonstrated rather convincingly that this envelope has extremes of high and low solar activity in it, including the "Maunder Minimum", a period from about 1645-1715 when apparently almost no sunspots occurred. He has inferred the existence of other minima further back in time, as well as periods of solar activity much greater even than now, by examining the carbon-14 isotope record in tree rings. Carbon 14 is produced in the high atmosphere by cosmic rays, and more of these get to the earth during periods of low solar activity.

We have strong suspicions that these changing levels of solar activity are connected to changes in the global dynamics of the sun

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responsible for the solar dynamo. Feedback from the magnetic field on the global motions may play a role, but long-term variations in the global velocities simply due to their intrinsic nonlinearity may also be important. A large element of randomness is probably also present. Clearly, understanding the solar dynamo is another reason why we need much better information about global velocities on the sun than we now have.

Other global properties of the sun appear to be changing. Scientists have searched for variations in solar luminosity at least since the beginning of the 20th century. Abbot made a career of it. Reported changes have generally been unconvincing until recently. But rocket measurements in 1976 and 1978 by Willson and by Kendall (private communication) have shown about a .4% rise in total irradiance, and Kosters and Murcray (1979) have seen a similar rise with balloon measurements in 1968 and 1978. Such changes, if real, are most likely also connected to changes in the dynamics of the convection zone, and perhaps to its dynamo behavior.

There also may be secular changes in solar diameter occurring. Eddy and Boornazian (1979) have recently re-examined measurements of solar diameter made at Greenwich from 1836-1953, and find a secular decrease of about 0.1% per century. If this effect is real, it should have important implications for the global dynamics of the convection zone and neighboring layers. For example, it might be an indicator of potential and internal energy of the stratification being released.

Finally, there is of course the specter of the low neutrino flux from the sun hanging over all thinking about the global structure of the sun. Is this low flux also an indicator that the sun is changing and not in complete equilibrium? Are the apparently shrinking solar diameter and the neutrino deficit linked, as Eddy and Boornazian (1979) have suggested? That is, is part of the solar luminosity being supplied by shrinkage of the outer layers, so that the central temperature and therefore the neutrino flux can be less than previously predicted? We do not know, but it further highlights the need to understand the global dynamics of the convection zone.

5. THEORIES OF GLOBAL CIRCULATION OF THE SUN

5.1 General Remarks

Some general remarks are in order first. Virtually all theories of the global circulation of the sun have concentrated on explaining the observed differential rotation. This is for the reason that, as I have discussed earlier, there is so little reliable information on other global motions. Early theories concentrated on the differential rotation alone, and did not include the consequences for other observed properties of the sun, such as the solar cycle, or the uniform heat flux with latitude leaving the solar surface. The ultimate theory must satisfy all these observed constraints and more. No theory does so yet, but we are getting closer.

Implicit in all I say that follows is the assumption that the origin of the sun's differential rotation is in the solar convection zone. The slow decay of solar rotation with solar evolution, due to solar wind torques, is a separate and distinct problem, having little to do with the observed differential rotation profile. It has occasionally been suggested, for example by Schatten (1973), Alfvén (1977), that solar wind torques might contribute significantly to the low rotation rate seen at the poles. However, as Parker (1971) and Gilman (1974) have argued, these torques are far too weak to compete with turbulent mixing within the convection zone.

All models for differential rotation calculate some motions explicitly, and parameterize the effects of others. No model can contain enough spatial resolution to explicitly represent all the scales of motion which might be important for determining differential rotation. After all, individual granules are a factor of 10^3 or so smaller in extent than the differential rotation. Therefore, global motions are explicitly calculated, and smaller scale motions are parameterized. At present, the parameterizations used are very crude. Great improvements are needed, and there are several theorists working on this problem. The most heavily parameterized models are those that are axisymmetric. In these, only a meridian circulation and differential rotation are explicitly determined, and all other motions parameterized by coefficients of turbulent diffusion. Another class of models calculates global scale convection, including departures from axisymmetry, and parameterizes smaller scale motions with similar coefficients. We discuss both below.

Before discussing particular models in detail, it is useful to consider the mechanisms of angular momentum transport available to generate and sustain the solar differential rotation. Since angular momentum can be convected by the fluid itself any flow which has a component in either the radial or latitudinal direction can change the rotation profile in the meridian plane. By convention, these circulations are broken into two types: axisymmetric meridional circulation, and all departures therefrom--global and small scale eddies or convective cells. Their effects are best discussed with the aid of an illustration (Figure 1). At the top, two schematic meridional circulations are shown, of opposite sense. If either pattern started up in a fluid originally in solid rotation, and no other transport processes were acting, high latitudes and deeper layers would tend to spin up, because the circulation would conserve angular momentum. But now if sufficient diffusion is present (most likely associated with small scale motions in the convection zone) to link the different layers of fluid and keep the angular velocity nearly constant, the circulation on the left may produce a net equatorial acceleration. This is because fluid moving toward the equator on the outer branch, for example crossing the dashed line, would contain more angular momentum than fluid moving toward the pole underneath, where the moment arm is shorter.

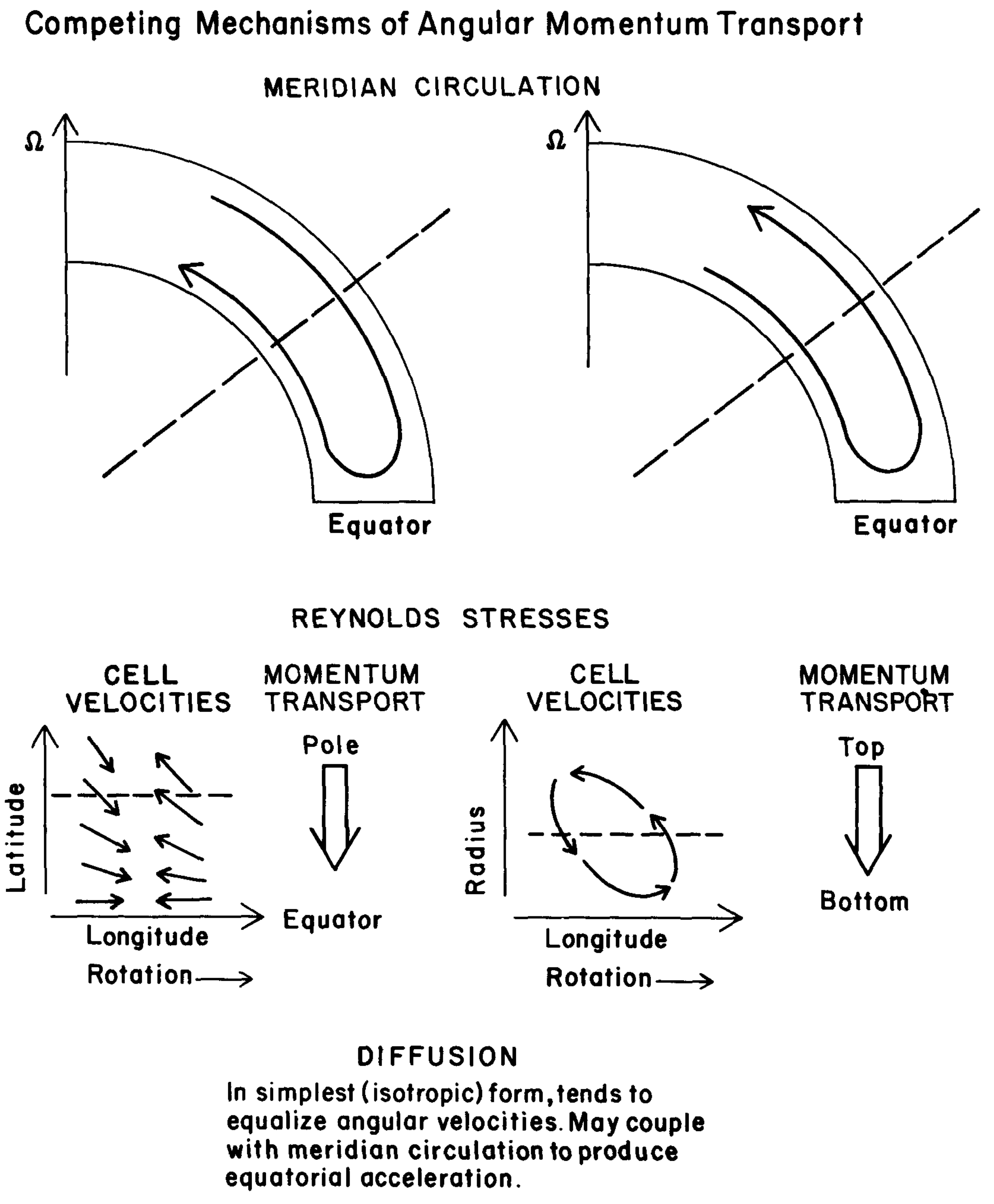

Figure 1: Schematic of various mechanisms of angular momentum transport discussed in text.

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The middle schematics of Figure 1 illustrate angular momentum transport by nonaxisymmetric motions, through correlations between east-west and either north-south or radial motions, called Reynolds stresses. Both left and right middle schematics illustrate common motion patterns, relative to a uniformly rotating reference frame, actually seen in models. On the left is a horizontal flow which leads to angular momentum transport toward the equator. Flow toward the west (in the same direction as the rotation of the whole system) also has a component toward the equator, while flow at adjacent longitudes toward the east has a poleward component. If we average in longitude, say along the dashed line, we get a net correlation which implies momentum flux toward the equator, even if there is no net mass flux. In the right schematic is a typical convective circulation pattern in a local longitude-radius plane which by similar arguments leads to a net flux of angular momentum inwards. In both cases, the necessary tilts in the velocity vectors are induced by coriolis forces acting on the convective motion. These same forces also select which convective modes are preferred.

Other azimuthal torques might be present that could change the angular momentum, such as arising from electromagnetic body forces associated with the presence of the magnetic field. However, in the solar convection zone, these appear to be too small to exert much influence, except perhaps a drag in the neighborhood of magnetic flux tubes. Buoyancy forces, since they act in the radial direction, do not directly contribute azimuthal torques that can change differential rotation, but of course they drive the motions which transport momentum. Similarly, azimuthal pressure torques average to zero when integrated around a complete latitude circle, since the pressure is a single valued function.

Thus, we are left with the motions themselves as the direct determinant of the differential rotation. In the real situation, all the motions compete with each other in determining the resulting differential rotation. Which angular momentum transport by which motion dominates can only be determined by actual model calculation (and ultimately for the sun, by observation). The result is also bound to be somewhat model dependent. Coriolis forces play a crucial role in determining convective mode size and shape, and therefore their momentum transport properties. It follows that a very important parameter determining what kind of Reynolds stresses are produced in the convection is the ratio of turnover time for the convection to rotation time for the whole system. For the sun, this ratio is much less than one for granules, somewhat less than one for supergranules, and probably greater than one for giant cells. Therefore we can expect these different scales of motion to contribute differently to the observed differential rotation. With the above remarks as background let us turn now to some actual model calculations. We consider axisymmetric models first.

5.2 Axisymmetric Models for Differential Rotation

Historically, axisymmetric models have invoked particular parameterizations of either momentum or heat transport to provide the mechanism for driving an equatorial acceleration. In one class of models, the eddy viscosity is assumed to be anisotropic, i.e., there are different transport rates in different directions. The suggestion is that this anisotropy is introduced by the presence of gravity. When the anisotropy is included, solid rotation is no longer a solution of the equations of fluid motion. Meridian circulation and differential rotation result, and with suitable choice of the sign and magnitude of anisotropy, the solar equatorial acceleration can be reproduced. The mechanism is essentially the one I described earlier in connection with Figure 1. The meridian circulation set up has flow toward the equator in the outer branch, flow back toward the pole in the inner branch. With large rotation rate assumed, the rotation is nearly constant on cylinders, so along a radial line, there is more equatorward transport in the outer branch than poleward transport on the inner branch. A typical meridional circulation velocity is only 2m/sec which is smaller than could be observed.

This approach originated with Biermann (1951) and was exploited by Kippenhahn (1963), Cocke (1967) and in greater detail by Köhler (1970). To get equatorial acceleration requires that the eddy viscosity for horizontal momentum transport be larger than that for radial transport. With the reverse, deceleration results. But there is no physical argument which clearly favors this sense of anisotropy. Also, no account has been taken of the influence of rotation upon the eddy viscosity. Finally, no thermodynamics are included in the model.

Another approach which has been carried further has been to assume convective heat flux is weakly influenced by rotation, such that it becomes a function of latitude. This was first tried by Durney and Roxburgh (1971) and later developed further by Belvedere and Paterno (1976, 1977, 1978) and Belvedere, Paterno and Stix (1979). Here again, the sign and magnitude of the variation in heat transport coefficient with latitude is chosen so as to give the observed equatorial acceleration. Again, the dominant meridian circulation has flow in the outer part toward the equator. Belvedere, Paterno and Stix (1979a) have shown that if the eddy diffusion of momentum is assumed to be much less than that for temperature, the required meridian circulation can be very small, and be consistent with extremely small differences with latitude in surface temperature. However, there appears to be little physical justification for this assumption. Later versions of the model also contain a similar density variation with depth as the solar convection zone is thought to have, unlike many previous models. A typical differential rotation with depth produced by the model has angular velocity increasing inwards, being nearly constant on surfaces perpendicular to the rotation axis.

The problem with this particular model is that it relies heavily on the assumption of weak influence of rotation upon convection in the solution procedure (turnover time short compared to rotation time) which assumption is a poor one for the deep parts of the solar convection zone. When the influence of rotation is more accurately taken into account a very different answer may result. There is also some question as to whether the answers obtained are reasonable even in the weakly rotating case. In my own experience with nonaxisymmetric convection in rotating spherical shells, I have found it difficult to construct examples in which convection weakly influenced by rotation could sustain an equatorial acceleration. Virtually always, global convection weakly influenced by rotation gives high latitude acceleration, as well as large oscillations in rotation rate, when enough degrees of freedom are included in the calculation to represent finite amplitude effects reasonably well. This leads me to question the validity of the more heavily parameterized axisymmetric model.

5.3 Nonaxisymmetric Models for Differential Rotation

5.3.1 Historical and philosophical background. As you have probably inferred from my remarks, I am not a proponent of axisymmetric, heavily parameterized models for global circulation. This is primarily because I suspect the parameterizations upon which they depend are too inaccurate. I personally prefer models in which the dynamics responsible for giving the correct differential rotation are explicitly calculated, with parameterizations of unresolved motions relegated to a less critical, more neutral role, so their detailed form is less important. Such models have been developed, so far for physics considerably simpler than the real sun, but nevertheless instructive. In particular, models have been developed for nonaxisymmetric convection of a stratified liquid in a rotating spherical shell. In these models, the shell is heated uniformly at the bottom, cooled at the top, and all small scale diffusion is assumed to be isotropic and independent of position and time. Usually the diffusion rates for temperature and momentum are assumed to be equal (in fluid dynamical parlance, the Prandtl number is unity). Analogies are then drawn to the sun by identifying the model diffusion with small scale eddy diffusion of momentum and heat. These diffusion coefficients are passive, however, in the sense that, in the absence of global motions, solid rotation and uniform heat flux are all that result.

If we think of this model as representing a classical Newtonian fluid in a spherical shell held together by a central gravity, rather than an approximation to a stellar convection zone, then we have a completely well defined physical system with no ad hoc assumptions or parameterizations. We could imagine building such a system as a physical experiment, and have some confidence our model calculation would accurately describe the observed dynamics. In fact, such an experiment is planned, to be flown in orbit on Spacelab I and III by several colleagues and myself. By orbiting the experiment, we escape earth's gravity and produce a central body force by application of an electric

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field across the spherical shell. Because the dielectric constant of the working fluid is a function of temperature, we can simulate a radial buoyancy force.

Early, mostly linear analyses of convection in a rotating spherical shell by Busse (1970, 1973), Durney (1970, 1971), Yoshimura and Kato (1971), and Gilman (1972, 1975), demonstrated the preference for convective modes that transport momentum toward the equator, via the Reynold stress mechanism illustrated in Figure 1. More recent, nonlinear calculations by myself (Gilman 1976, 1977, 1978, 1979) have exploited this fact to determine in detail when equatorial acceleration occurs, and with what amplitude relative to the convection which drives it and relative to the basic rotation rate. As with the thermally driven, axisymmetric models described earlier, some of the early calculations referenced above were done in the limit of weak influence of rotation, mostly as a mathematical convenience. But in that limit, the preference for convective motions which transport momentum toward the equator is a very weak one, easily overpowered when finite amplitude effects are taken into account. Therefore, some of the inferences and extrapolations made from these early papers have not been borne out, even as to sign, by later nonlinear calculations. This is partly because the first attempt in these early papers to represent nonlinear effects involved severe truncations of the system, down to essentially the first unstable mode, which naturally gives too much advantage to this mode in competition with others. Some of these early calculations also ignored the role played by radial transports of angular momentum in determining the final differential rotation profile.

5.3.2 Results of Nonlinear Model Calculations. The basic model I have been using is fully nonlinear. It is formulated for a finite difference grid in the meridian plane, and is Fourier analyzed in longitude. Typically, 16 to 24 longitudinal wave numbers are retained, and all the nonlinear interactions among them included. The calculations are done for a full sphere with no assumptions concerning symmetry about the equator. Some filtering of small scales is performed around the poles to preserve computational stability. Boundary conditions are usually stress free top and bottom, as well as constant heat flux bottom, constant temperature top (other combinations have been tried). I summarize some of the more qualitative results obtained, and then give a few typical solutions for velocity patterns.

Finite amplitude equatorial acceleration is produced in this model only when the influence of rotation upon convection is strong, i.e., the rotation time is less than the turnover time for convection. Under these circumstances, the angular velocity also decreases inward; when the rotational constraint is very strong, the angular velocity predicted is nearly constant on cylinders concentric with the axis of rotation. The angular velocity is fastest at the equator because the equatorward transport of angular momentum from high latitudes by the convection is the dominant mechanism determining differential rotation. If the rotation time is a few orders of magnitude shorter than the turnover time, then the latitudinal profile of differential rotation may be more complicated, but this is an unlikely case for solar or stellar application.

When the convection zone is deep, say 1/3 of the radius or more, the equatorial acceleration profile with latitude is broad, with essentially monotonic decrease in angular velocity to the poles, as seen on the sun. On the other hand, when the convection zone is shallow, say 20 percent or less, and the rotational influence is strong, the angular velocity reaches a minimum in mid latitudes, and then increases again toward the poles. Thus, a polar vortex is present in these thinner layers. The width of the equatorial acceleration is determined by the depth of the layer: the shallower the layer, the narrower the width. The monotonic decrease of angular velocity with latitude for deep layers arises because the Reynolds stresses which transport angular momentum toward the equator reach to higher latitudes, and the moment of inertia of the polar cap is a smaller fraction of the total for the shell, so the poles are easier to spin down.

For both deep and shallow convection zones, with weaker rotational influence (increased convective velocities) the profile switches from equatorial acceleration to deceleration. Angular velocity now increases with depth. There is an intermediate stage in which the angular velocity is highest in mid latitudes and lower near the equator and near the pole, while still decreasing with depth. None of these cases correspond to the sun, but they could to other stars. The changeover from equatorial acceleration to deceleration comes about because inward radial transports of angular momentum in equatorial regions, due to both convective cells and mean meridional circulation, become more powerful than the equatorward transport from high latitudes by the cells in determining the differential rotation profile.

The maximum differential rotation sustainable by the convection is about 40 percent of the average rotation. For larger values, the feedback from the shear on the convection is strong enough to change the dominant patterns and consequently the differential rotation profile, resulting in a new equilibrium with lower amplitude differential rotation. The maximum differential rotation maintainable by the convection has about the same kinetic energy in it as the convection itself. Amplitudes of individual convective modes may change by factors of two or three with time while the differential rotation amplitude changes by only 10% or so.

The model does have certain undesirable thermal characteristics. In particular, even though constant heat flux is assumed at the bottom, the heat flux does vary at the top with time and with latitude by 10-20%. This is, of course, much larger than observed on the sun. The addition of compressibility, and the condition that the stratification not depart very much from the adiabatic gradient, should reduce these effects some, though perhaps not enough to agree with the solar case.

5.3.3 Typical velocity patterns. To give you a somewhat better feeling for the typical solutions for convection in a deep rotating spherical shell, I show several computer drawn velocity patterns, for one case with a depth of 40% of the outer radius, near the maximum possible for the solar convection zone. Figure 2 shows the differential rotation linear velocity, plotted relative to the uniformly rotating reference frame used in the calculation. Solid contours represent rotation faster than this frame, dashed contours, slower velocities. One can easily see the equatorial acceleration, and the tendency for rotation to be constant on cylinders, except near the boundaries where the stress-free boundary conditions prevail. The axisymmetric meridional circulation which accompanies this differential rotation is shown in Figure 3, for the northern hemisphere. Note that there is a single large cell in low and middle latitudes, with poleward flow near the top, equatorward flow near the bottom, with smaller scale cells at high latitudes. This pattern is virtually always the dominant one seen. Note the big cell has the opposite sense to what is invoked in the axisymmetric models to produce an equatorial acceleration. In the present model, its effects are overpowered by the nonaxisymmetric convection in determining the differential rotation.

The structure of the convection patterns driving this differential rotation is seen in the next several figures. In Figure 4 are plotted contours of radial motion about one fourth of the way in from the outer boundary, for a 180° longitude strip. We can see the cells are elongated in latitude at low latitudes, but more cellular or elongated in longitude at high latitudes. Some of the high latitude elongation is due to the distortion of the projection used, of course. The low latitude patterns show strong evidence of symmetry about the equator; the high latitude patterns do not. In the low latitude "cartridge belt" rolls, some are obviously stronger than others. They typically come in packets which tend to move prograde relative to the rotating frame. Here they are strongest near about 180°, 250°, and 340° longitude, and weaker elsewhere. It is these rolls which are primarily maintaining the equatorial acceleration. Shearing of the patterns by the differential rotation is also evident.

The depth dependence of these convective rolls is illustrated in Figure 5, which shows the pattern of east-west and radial convective motion in the equatorial plane. The differential rotation is subtracted out. We can see the rolls extend essentially from the bottom to the top of the layer, but some rolls are much wider than others. The whole pattern of rolls tends to move prograde with time (counter clockwise) as well as evolve. Figure 6 shows the same case, but with the differential rotation added back in. The effect is to see stronger than average rotation in those longitude bands where the horizontal convective velocity is in the same direction as the differential rotation, with gaps in between where the convective and rotational velocities tend to cancel out. Obviously, such a pattern on the sun would produce an apparent time change in observed equatorial rotation, since at any time only 120° or so of longitude would be sampled.

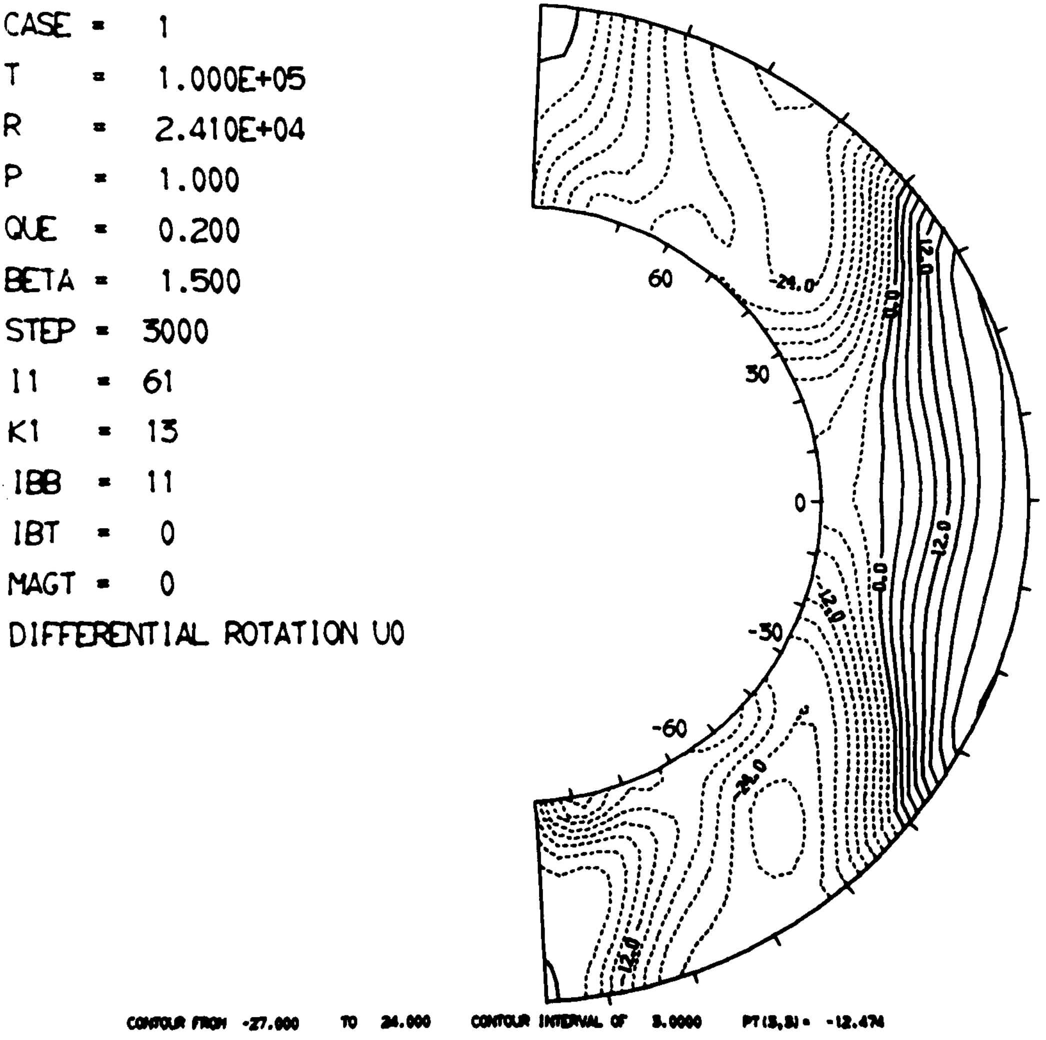

Figure 2: Computer plot of differential rotation linear velocity produced in nonaxisymmetric spherical shell convection model for a typical case. The North Pole is at the top. Solid contours represent rotation faster than the rotating reference frame, dashed contours slower rotation. Magnitudes are in dimensionless units, but for parameters chosen surface values agree with observed solar differential rotation within 10-20% (see Gilman 1979).

Figure 7 shows the horizontal velocity pattern (differential rotation plus convective velocities) near the same level and for the same longitude band as in Figure 4. What we see are a series of vortices of different sizes, arranged more or less symmetrically about the equator.

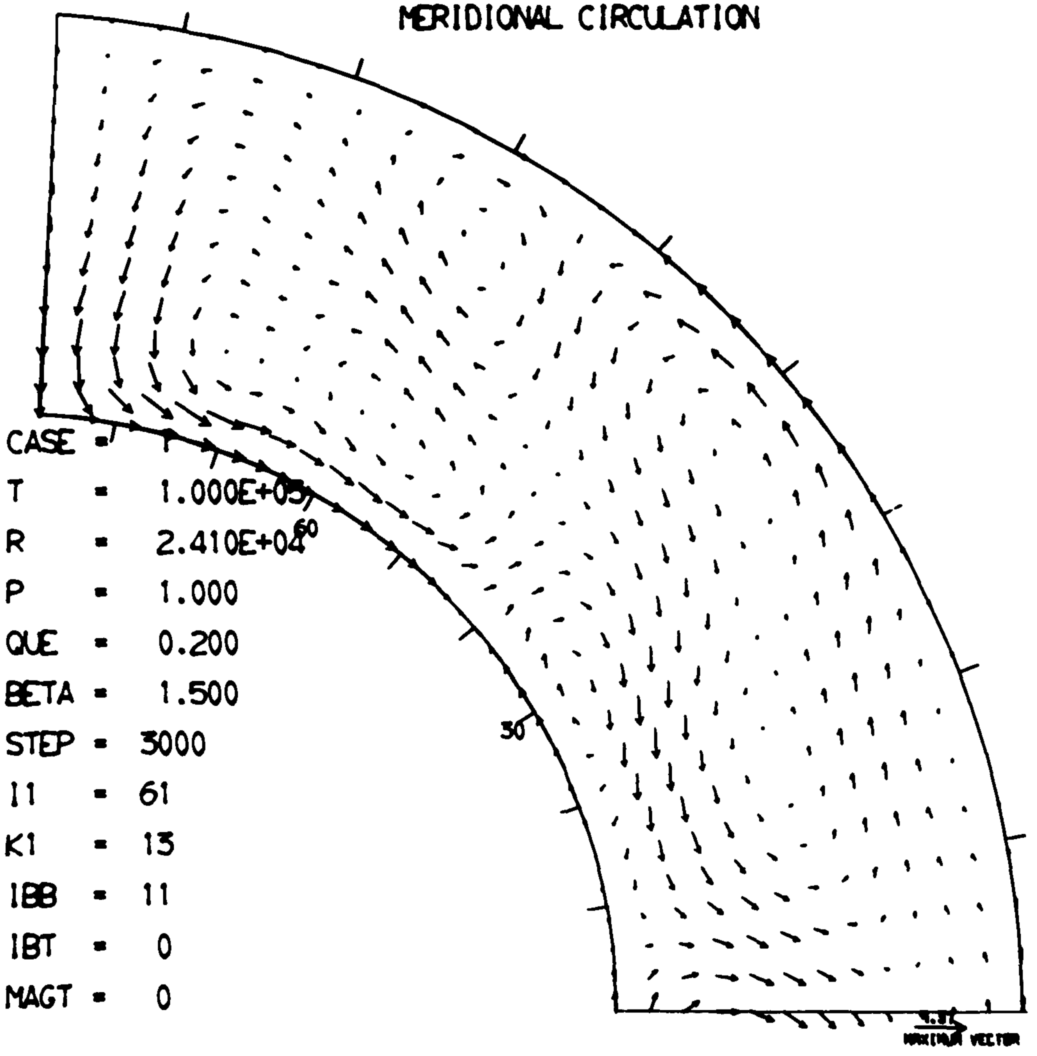

Figure 3: Vectors of meridional circulation produced for same case as Figure 2 for northern hemisphere. North Pole at the top.

The local "differential rotation" or latitude shear in the east-west flow is strongest in the band between 190° and 230°, as well as between 280° and 320°. From Figure 4, this is where the radial convective velocities are weakest and broadest in horizontal scale. The velocity vectors have the characteristic tilt with latitude in each hemisphere needed to give angular momentum flux toward the equator, as first illustrated in Figure 1.

5.3.4 Application to the sun. How are we to extrapolate these results to the convection zone of the sun, which, of course, is a long way from

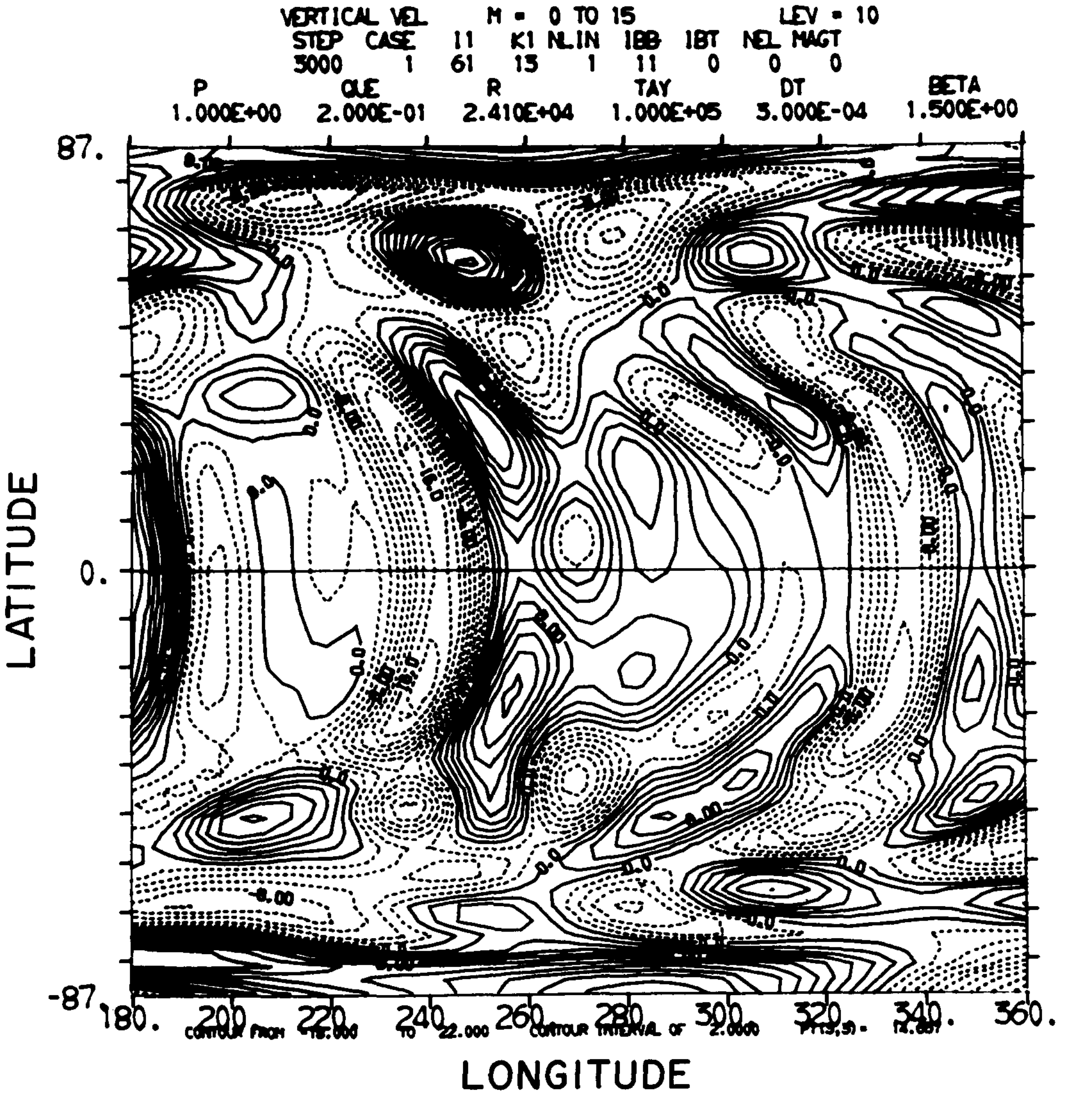

Figure 4: Longitude-latitude plot of contours of radial motion in longitude band 180-360°, about 1/4 of distance in from outer boundary of spherical shell. Solid contours indicate outward motion, dashed contours inward motion.

being a stratified liquid? The theory is generally thought to apply best to the deepest layers of the convection zone, where the density varies more slowly with height, the natural scale for the convection is global, and where the turnover time for convection is likely to be as long as or longer than the rotation time. Thus, our results would say

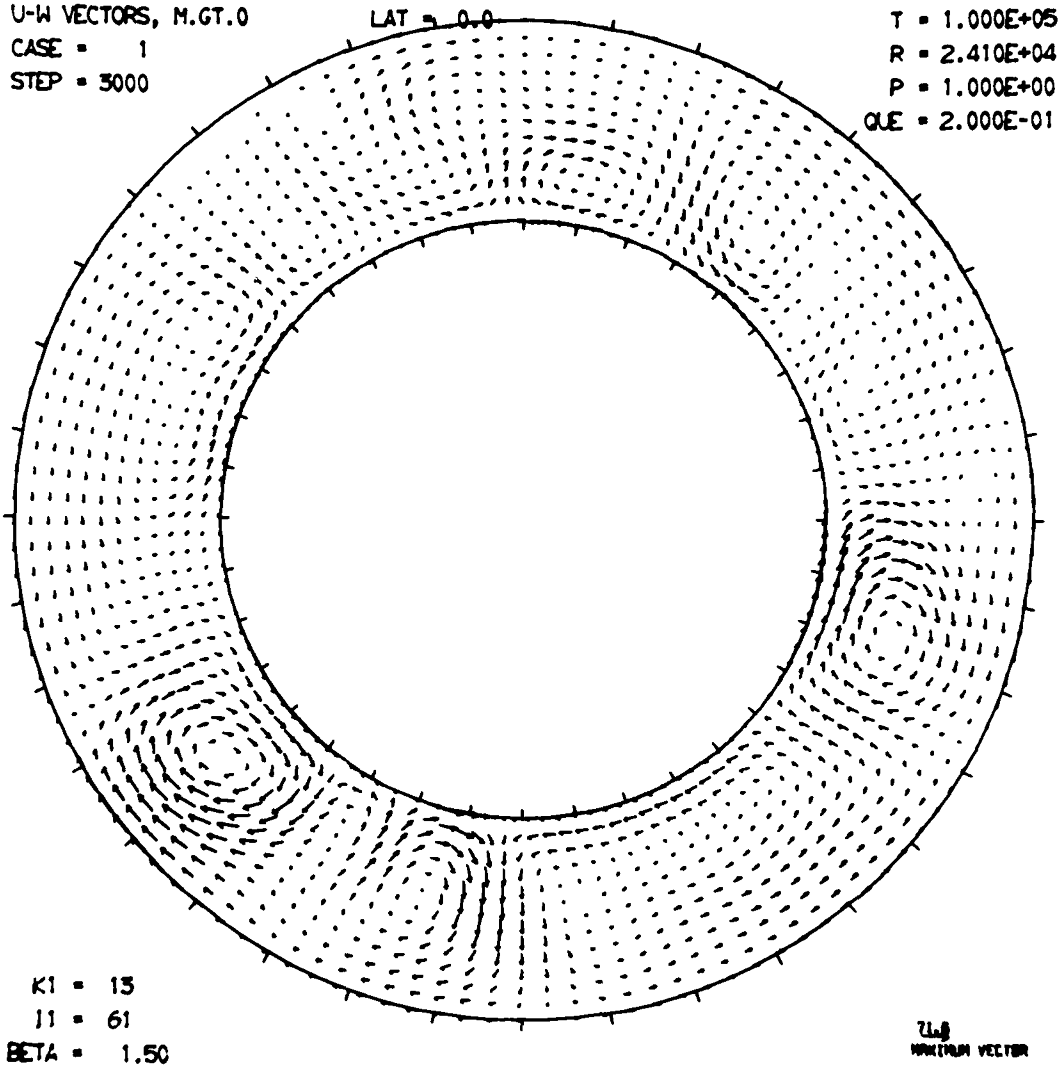

Figure 5: Vectors of nonaxisymmetric convective motion in the equatorial plane of the shell. Sense of basic rotation of system is counterclockwise. 0^{o} longitude is at the top of the figure.

the latitudinal gradient of angular velocity is formed in these deep layers. The results also suggest that since the solar angular velocity does decrease monotonically to the poles, the convection zone must be deep, at least 1/3 of the radius, and perhaps deeper. This has been argued in detail by Gilman (1979). Earlier calculations (Gilman, 1976, 1977) for a depth of 20% had predicted the existence of a polar vortex,

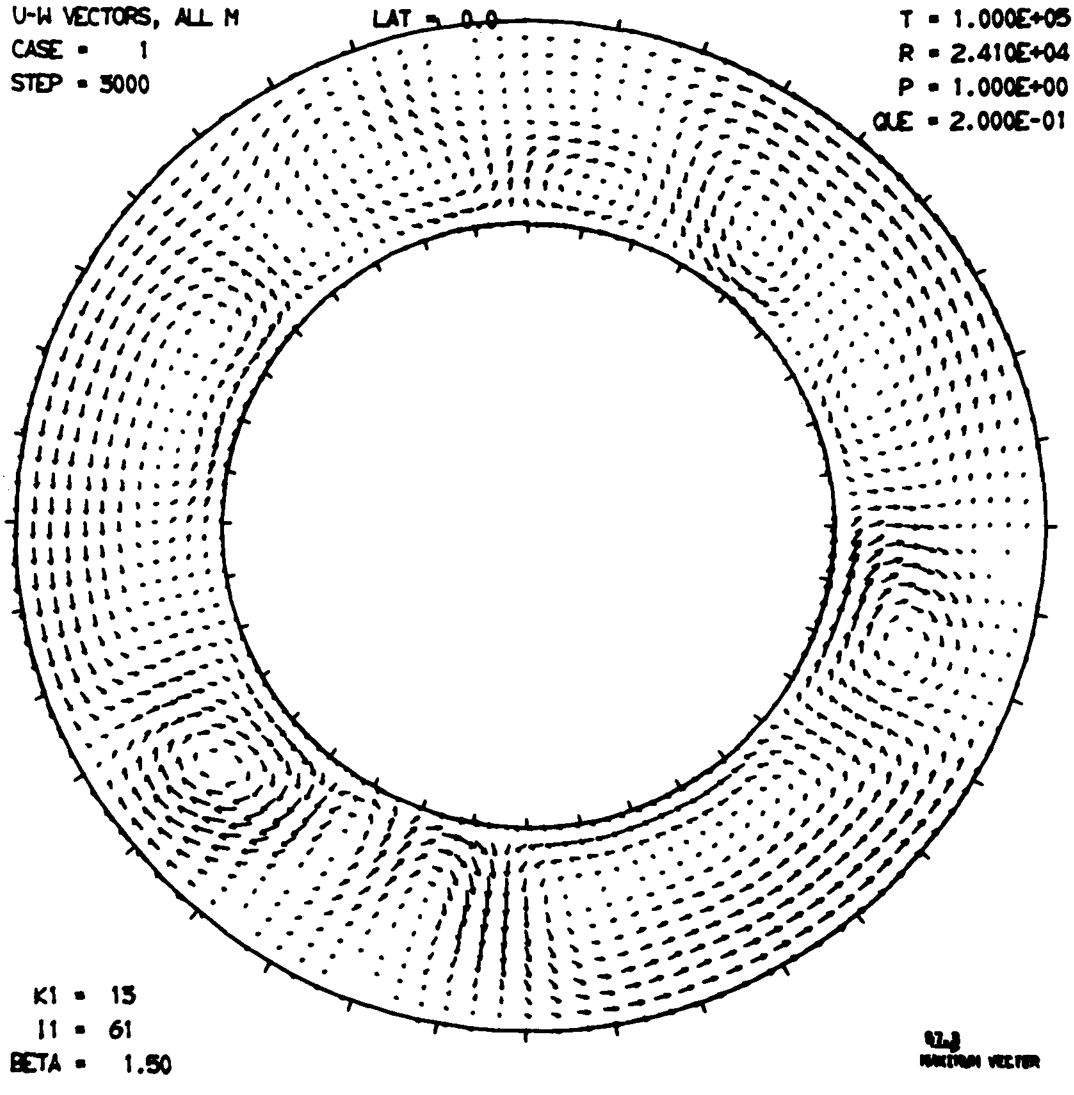

Figure 6: Same as Figure 5, but with differential rotation added back in to velocities.

where the angular velocity increased with latitude. Beckers (1978) searched for it by observing the polar doppler velocities in detail, but could not find it. Howard and the author also looked for it in Mt. Wilson data, also without success.

What about the radial gradient of angular velocity? Foukal (1972) has interpreted the fact that sunspots rotate faster than the photospheric plasma as evidence the angular velocity increases inwards near

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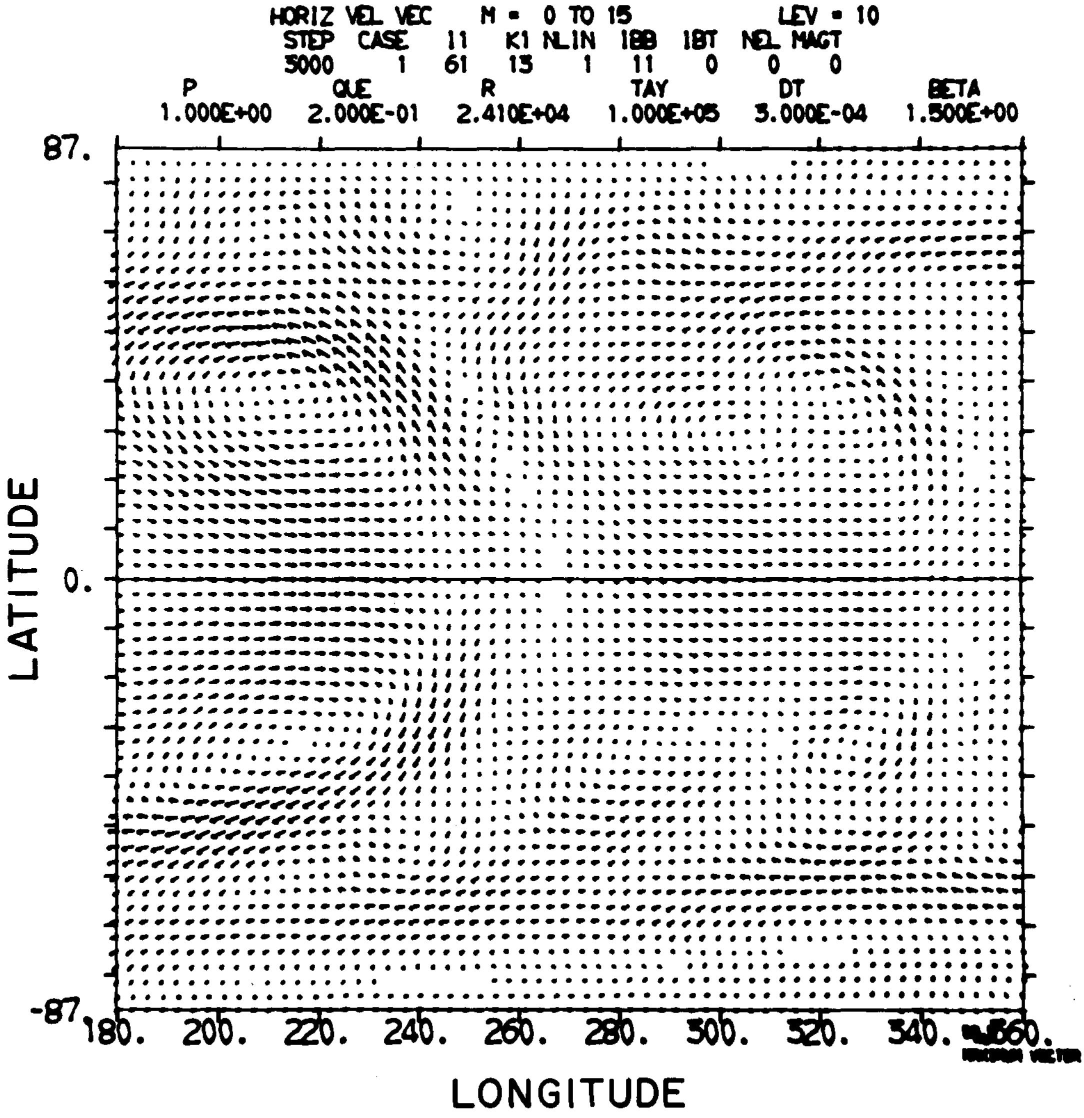

Figure 7: Longitude-latitude plot of total horizontal velocity vectors in longitude range 180-360°.

the solar surface, assuming the spots are "anchored" at a depth several thousand kilometers below the photosphere. Other inferences that the angular velocity increases inwards in these layers have been made by Deubner, Ulrich and Rhodes (1979) from frequency shifts in solar pressure mode oscillations. This was discussed in detail this morning by Deubner in his review. Foukal and Jokipii (1975) suggested such an increase could be produced by the radial flow in supergranules conserving angular momentum producing a thin layer near the top of the solar convection zone in which the angular momentum per unit mass is approximately constant with radius. Gilman and Foukal (1979) tested this notion using the same model I just described above and showed that strongly supercritical convection in a thin rotating spherical shell weakly influenced by rotation will produce a layer which approaches constant angular momentum along radial lines. This would allow for a 5% increase in angular velocity through a depth about 2.5% of the radius, which is a reasonable depth for supergranules to reach.

Putting all of these results together, then, Gilman and Foukal (1979) have argued that the latitude gradient of angular velocity seen at the surface of the sun is produced deep in the convection zone by global convection, and then is transmitted to the surface by supergranules, which locally near the surface cause the angular velocity to decrease outward. According to this view, then, in low and middle latitudes, the angular velocity should increase to a maximum somewhere near the bottom of the supergranule layer, and then decrease inward from there.

How much of the rest of the global circulation patterns are transmitted to the surface is not clear. Granules and supergranules may act to "mask" some of these motions from our view.

5.4 Implications of Global Circulation Models for Solar Dynamo Theory

Once one has constructed a global circulation model for the sun which gives, under certain circumstances, the correct latitudinal profile in differential rotation, one is strongly tempted to find out what kind of hydromagnetic dynamo action it sustains. Can the same dynamics responsible for the differential rotation also simulate solar magnetic cycles? This has been tried, both for axisymmetric and non-axisymmetric models described above.

Time does not permit a review of the history of solar dynamo theory here. For reviews see Stix (1976, 1979). Suffice it to say that dynamo models of the so-called α-ω type have had considerable success in simulating many features of the solar cycle, including the butterfly diagram, change of sign of polar fields, etc. In such dynamos, toroidal magnetic field is produced primarily by stretching out the poloidal field into the azimuthal direction by latitudinal and radial shears in the differential rotation ω. New poloidal field, of opposite sign to the original in a field reversing dynamo, is generated by the "α effect" or upwelling and twisting of toroidal field lines into the meridian plane. When applied to the sun, virtually all such models require angular velocity increasing with depth to simulate the butterfly diagram. To get the correct dynamo period for the sun, they also require α to be of small magnitude.

One paradox noticed previously, e.g., Durney, Gilman and Stix (1976), is that dynamical models for differential rotation which include the strong influence of rotation upon convection give rise to angular velocity decreasing with depth, the opposite of what is needed to simulate the sunspot cycle. However, as Stix (1978) has pointed out, perhaps α in the sun is a sufficiently complex tensor function that the simple relation between it and differential rotation that results in equatorward migration of the zone of sunspot formation breaks down. In any case, dynamo models have previously been able to choose α and the differential rotation independently of each other, without regard to dynamical consistency. In the latest such calculation, Belvedere, Paterno and Stix (1979b) have used their axisymmetric global circulation model to calculate an α-ω dynamo. They find a similar level of success with previous models, but only if α is assumed to be at least a factor of ten smaller than in previous models, for example Yoshimura (1972, 1975).

An apparently new paradox arises when I calculate the dynamo action from my own nonaxisymmetric convection model for differential rotation. In that model all the hydromagnetic induction effects are explicitly calculated. No α is assumed, but the processes represented by an α are present. Furthermore, their magnitude is determined by the dynamics needed to drive the right amplitude and profile of differential rotation. What we find is that the effective α is larger than has been previously assumed by a factor of 10^3 to 10^4. Consequently, the dynamo runs much faster, and gives much shorter reversal periods, when it gives reversals at all. Many aspects of its behavior are much different from those seen previously in α-ω dynamos applied to the sun. The reason for the large effective α is that strong influence of rotation upon convection is needed to give large enough angular momentum transport to drive the equatorial acceleration. This strong influence of rotation produces a lot of swirl or helicity in the motion, which leads directly to large α. Previous claims of success in demonstrating that global convection responsible for driving the differential rotation also gives dynamos which accurately simulate the solar cycle (particularly the long series of models by Yoshimura, e.g., Yoshimura (1975, 1978)) are doubtful. The difficulty is that these models have not consistently calculated the fluid dynamics which lead to both equatorial acceleration and the various hydromagnetic induction effects.

What will be the resolution of this paradox? Clearly, compressible models for nonaxisymmetric global convection and differential rotation have to be built and tested to see whether the paradox continues. I suspect it will. The answer may instead lie in the fact that most of the sun's magnetic field is concentrated into small tubes of flux. Fluid may flow around these tubes, and perhaps the effective α is greatly reduced as a result. Magnetic buoyancy, not presently included in the model, may also be important. It can move flux tubes from one region of fluid to another before the local α acts fully on it.

5.5 Global Circulation Model Improvements Needed

I have already discussed some of the deficiencies of the axisymmetric global circulation models. For the nonaxisymmetric models, the first order of business is to add compressibility. A student of mine, Glatzmaier and I are actively working on this problem, and I am sure there are others also, such as Marcus at Cornell. The so-called "anelastic" equations seem to be the most promising to use. In these, sound waves are filtered out, but the large density variation between the bottom and top of the convection zone is retained in the dynamics and thermodynamics. We have already obtained a few linear results for the compressible case, which suggest that such things as the angular momentum transport profiles are rather similar to the stratified liquid case. We hope within several months to be carrying out numerical simulations of nonlinear, compressible convection in a rotating spherical shell. Clearly, conclusions reached using the stratified liquid model for nonaxisymmetric convection must be tested in the compressible case.

The other major area that needs improvement is the parameterization of the effects of those scales of motion too small to resolve with the model. Durney and Spruit (1979) and also Gough (1978) are actively working on this problem. Marcus is also including simple turbulence closures into his global convection calculations (private communication). Durney, Spruit, and Gough taking the approach of writing the stress tensor for transport of heat and momentum in very general form, and then evaluating the various correlations by estimating the preferred sizes and shapes of the unresolved motions as determined by such effects as rotation and gravity. Ultimately one would like to achieve parameterizations of this type which are functions of the global, explicitly calculated motions. This has been done extensively in dynamical meteorology and oceanography, with some success. More needs to be done to see if any of the formulations used there might carry over to the solar problem. On the other hand, many difficulties have been encountered in demonstrating such parameterizations are valid representations of the real atmosphere or ocean, for which generally better observations are available than for the sun. We should not underestimate the difficulty of this problem. In this regard, Rüdiger (1977, 1978, private communication) has been developing rather general statistical turbulence theory arguments which should be useful in guiding the development of specific parameterizations for the sun.

I would adopt the philosophy that we should rely on parameterizations only when we have to, and explicitly calculate all the processes we can afford to. This is because we have so few ways to test the validity of the parameterization, except by forcing it to give us the right answer for differential rotation! One approach may be to test the parameterizations of convection against the nonaxisymmetric convection model calculations themselves.

There are many other kinds of physics which should be added to such models, but I would rate them as being somewhat lower in priority than the two mentioned above. These include boundary conditions and penetration, partial ionization, and radiative effects on boundaries.

6. DIFFERENTIAL ROTATION OF STARS WITH CONVECTION ZONES

If the differential rotation model calculations reviewed above apply to the sun, they presumably apply in some form to other stars with convection zones. Using a stellar envelope code, we can make an estimate of the depth of a stellar convection zone, as well as its turnover time, based on mixing length velocities near its bottom. Then, if we have some idea of its rotation rate, we can extrapolate from the nonlinear spherical shell convection calculations to guess its likely profile of differential rotation. This argument is carried forward in detail in a paper I am presenting at IAU Colloquium 51 in London, Ontario, so I will only briefly summarize here.

We have carried out the calculation for a series of main sequence stars from late A to early K first used for estimating convection zone depth by Baker (unpublished manuscript). In doing so, we have used a stellar envelope code by Latour. The resulting convection zone depth estimates are plotted in Figure 8, as a function of α, which in this case is the (assumed constant) ratio of mixing length to pressure scale height. Convection zone depths estimated in this way are between about 25 and 35% of the stellar radius around K0, and disappear somewhere between about F4 and A5, depending upon the α chosen. Based on our spherical shell convection calculations, we favor larger α, since for the sun at $\alpha = 1.0$, the convection zone would be less than 20%, for which we could not get an angular velocity decreasing all the way to the poles.

Using Latour's envelope code, and estimates of rotation rate for this range of main sequence stars by Kraft (1967), we have calculated the ratio of turnover time to rotation time, using for our length one pressure scale height at the bottom, and for our velocity the value at a distance one pressure scale height up from the bottom, following Durney and Latour (1978). Figure 9 shows the result. Although the turnover time is shorter for earlier stars, the rotation time is much shorter too, so the relative rates of change are important. From the $\alpha = 2$ curve, and the knowledge the sun has a broad equatorial acceleration, we would estimate the same would be true for later stars in this range because the ratio of turnover time to rotation time is even larger than for the sun, and the convection zone of these stars is deep. For earlier stars, the equatorial acceleration would remain but become more narrowly concentrated about the equator, with a high latitude increase in angular velocity developing by $\log(M/M_{\odot})$ of about .10, or about F4. Beyond that both the turnover time/rotation time ratio and the depth rapidly decrease, and we expect a relatively weak equatorial deceleration to be present. Analogous arguments to the above could be made for red giants.

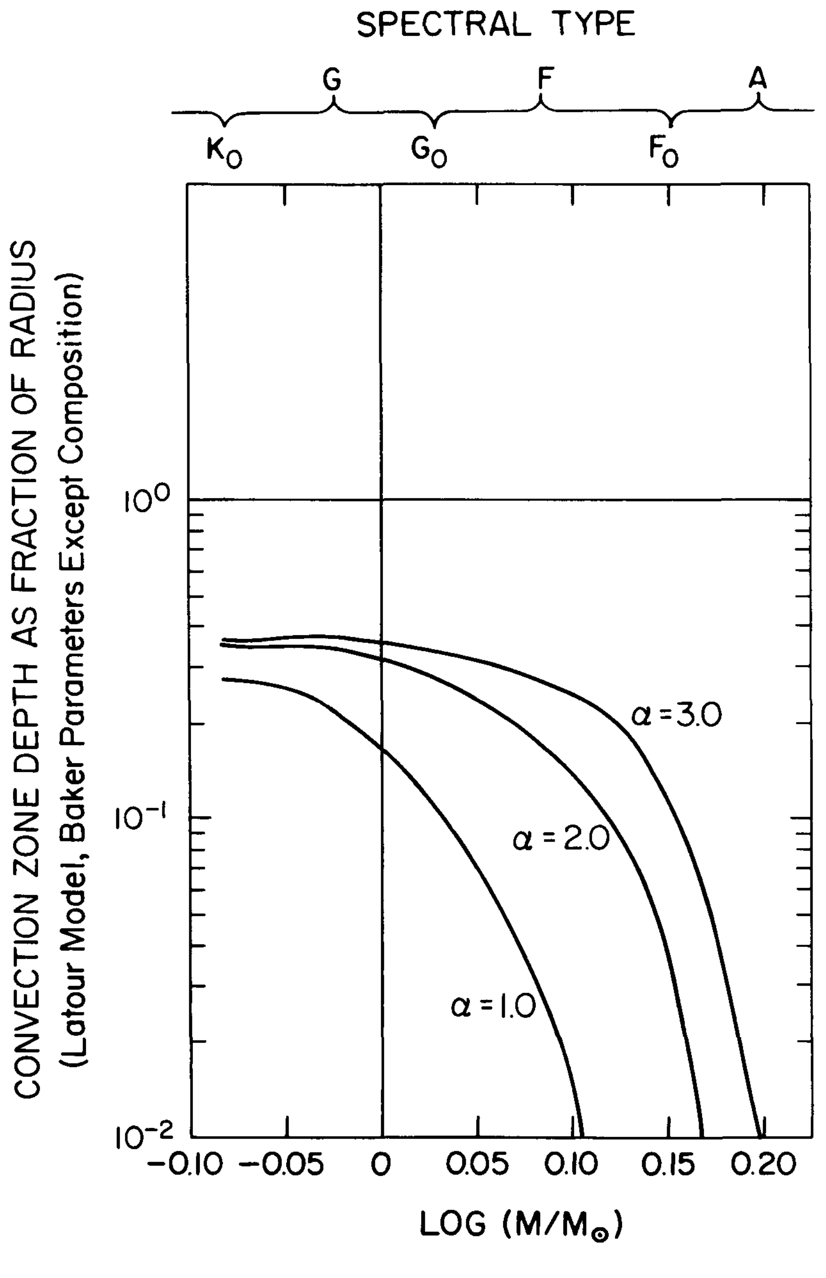

Figure 8: Estimate of convection zone depth as a fraction of stellar radius for series of main sequence stars, made using Latour stellar envelope code. α is ratio of mixing length to pressure scale height assumed in the model.

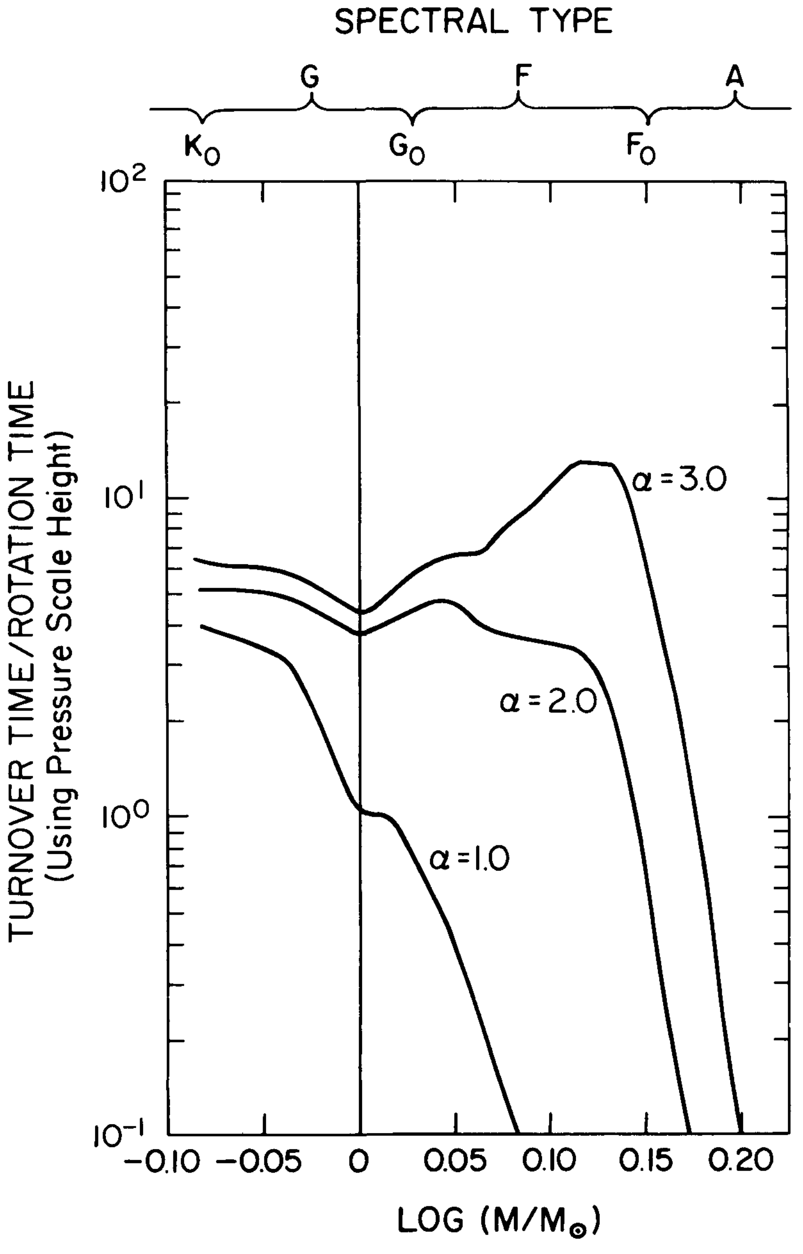

Figure 9: Ratio of convective turnover time to rotation time for same cases as in Figure 8. Rotation information taken from Kraft (1967).

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Clearly, our conclusions are heavily model dependent, both to the α chosen, and to the extrapolation from a stratified liquid spherical shell convection model. These conclusions must be retested when compressibility and other more realistic physics are added. My main point is to illustrate what might be done. I also realize it is extremely difficult to measure differential rotation in any star, and we may have to be content in many cases with predicting the differential rotation of stars for which we have a measure of the average rotation.

7. CLOSING REMARKS

In closing, I have tried to present an up-to-date picture of observation and theory of global circulation of the sun, admittedly with my own biases. I have also tried to connect this subject to neighboring fields of interest, particularly the solar dynamo and solar variability as well as stellar rotation. I have given you some opinions as to where the action should be in the near future. This field is basically healthy, and making good progress, but it is underpopulated with practitioners, particularly in theory, but also in observations, especially in the development of new observational tools which could yield results that are truly comparable from one observatory to another. Both the oscillations measurements and interferometric techniques hold much promise, but are really in early stages of development.

With respect to theory, one of the inhibiting factors is the great cost of developing large computer models, both in computer time and man power. Resources need to be pooled. In this regard, I am considering the possibility for the future of making available the spherical shell convection code we have developed at NCAR, as a facility that can be used by other scientists wishing to test improvements to global circulation model physics they have developed. The idea would be that the model would be modularized and documented well enough that another scientist, with our assistance or collaboration, could interface his part with the rest of the model. Presumably, the model would still be run at NCAR, since it requires a very large fast computer. I would like to get a feeling for how interested other theoreticians would be in this possibility. It would take new resources--money, people, and computing time--to prepare and carry out such an effort, and it is worth doing only if there is a real market for it.

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