

## CORRIGENDUM

# The Toeplitz noncommutative solenoid and its Kubo–Martin–Schwinger states – CORRIGENDUM

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There is an error in [2]. On line 6 of §5 in [2], the set  $\Xi_N$  is defined as

$$\Xi_N := \{(\theta_n)_{n=1}^{\infty} : \theta_n \in \mathbb{S} \text{ and } N^2\theta_{n+1} = \theta_n \text{ for all } n\},$$

where  $\mathbb{S} = \mathbb{R}/\mathbb{Z}$  denotes the circle group. The arguments in the paper are incorrect with this definition, and it must be replaced with

$$\Xi_N := \{(\theta_n)_{n=1}^{\infty} : \theta_n \in \mathbb{R} \text{ and } N^2\theta_{n+1} = \theta_n \text{ for all } n\}.$$

The reasons, as discussed in [1, Remark 2.2], are as follows. In the final displayed equation in the proof of Theorem 6.9, to conclude that  $\theta_j/N^k = N^k\theta_{j+k}$ , we must treat  $\theta_j$  as an element of  $\mathbb{R}$ , not of  $\mathbb{S}$  (there are many solutions to  $N^k\gamma = \theta_j$  in  $\mathbb{S}$ ). Then later, throughout §8, the statements include ‘let  $r_j := \beta/N^j\theta_j$ ’, which makes sense only if  $\theta_j \in \mathbb{R}$ ; and, in particular, in the displayed calculation below equation (8.3) in the proof of Lemma 8.1, which is a calculation about real numbers, it is crucial that  $N^2\theta_{j+1} = \theta_j$  in  $\mathbb{R}$ , not just in  $\mathbb{S}$ .

Note that this reduces the generality of the results substantially: for a given  $\theta_1 \in \mathbb{S}$  there are infinitely many sequences  $(\theta_n)$  in  $\mathbb{S}$  that satisfy  $N^2\theta_{n+1} = \theta_n$  for all  $n$ ; but, given  $\theta_1 \in \mathbb{R}$ , there is just one sequence  $(\theta_n)$  in  $\mathbb{R}$  that satisfies  $N^2\theta_{n+1} = \theta_n$  for all  $n$ .

## REFERENCES

- [1] Z. Afsar, A. an Huef, I. Raeburn and A. Sims. Equilibrium states on higher-rank Toeplitz noncommutative solenoids. *Preprint*, 2018, arXiv:1810.05323 [math.OA].
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