according to the notation introduced in mathematics by Moigno, $R(x+n)\Big/_{x}^{x+n}p(x)$ and $p(x)+R(x+n)\Big/_{x}^{x+n}p(x)$.

This value changes if the next payment is due in the time t; it changes if an annual premium only for m years is charged instead of a premium for the whole term of life. It is evident that the value of a policy depends on so many different circumstances that they cannot be expressed in one single symbol. But perhaps it is convenient to express this value as a function of the net premium (symbol of the function, ϕ), adding the time elapsed since the policy was taken at top left hand, and the time till the next payment of premium at the foot right hand. This last addition ceases when either no further payment of premium is required, or when the next payment is due immediately, viz.:—

$${}^{n}\phi\{p(x)\}=P(x+n)-p(x)R(x+n)$$

$${}^{n}\phi_{1}\{p(x)\}=P(x+n)-p(x)^{1}R(x+n)$$

$${}^{n}\phi_{t}\{p(x)\};\ {}^{n}\phi\{{}_{(m)}p(x)\};\ {}^{n}\phi\{P(x)\}=P(x+n)$$

$${}^{n}\phi\{P_{t}(x)\}=P_{(t-n)}(x+n);\ {}^{n}\phi\{{}^{m}\mathfrak{P}(x)\}={}^{m-n}\mathfrak{P}(x+n);\ {}^{n}\phi\{R(x)\};\ {}^{n}\phi\{{}^{m}\mathfrak{p}(x)\}\ \&c.$$

At § 8. $${}^{n}\mathfrak{P}(x)=W^{n}(x)\rho^{n}={}^{n}R_{1}(x)$$

$${}^{n}\mathfrak{p}(x)=\frac{{}^{n}\mathfrak{P}(x)}{R_{n}(x)}.$$

At § 9. As a symbol for the value of an annuity for two persons payable till both have died, the age being a and $a+h$, $R(a//a+h)=R(a)+R(a+h)-R(a,\ a+h)$ is recommended.

WILHELM LAZARUS.

THE DEMONSTRATION OF CERTAIN FORMULÆ.

To the Editor of the Assurance Magazine.

SIR,—I beg to submit demonstrations of several of the formulæ for whole life, temporary, deferred, and endowment assurances.

Let (a) denote the value of an annuity of £1 on the joint lives of the last v survivors of the lives m, m_1, m_2, &c.; and (A) the value of an assurance of £1 on the same lives.

The present value of a perpetuity of £1 per annum, the first payment due immediately, is made up of the present value of an annuity of £1 during the continuance of a given status, and the present value of a reversionary annuity of £1 to commence at the end of the year in which the status ceases, which latter annuity is equivalent to the present value of an assurance of £$1+\frac{1}{i}$. Now, the reversionary annuity of £1 can be secured by an assurance of £$1+\frac{1}{i}$ during a certain term, together with an endowment of the same sum payable at the end of that term.

Hence, the value of a perpetuity of £1, first payment immediate, equals the present value of a temporary annuity for t years on the joint lives of the last v survivors + present value of an assurance for t years of £$1+\frac{1}{i}$ on the failure of the joint existence of the last v survivors + present value of

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an endowment of £1 $+\frac{1}{i}$ payable at the end of t years, in the event of the last v survivors being alive at that time; or,

$$1+\frac{1}{i}=1+(a)_{\overline{t-1}|}+\left\{(A)_{\overline{t}|}+r^tp_{\overline{(m,m_1,m_2,\&c.)t}}^{\ v}\right\}\overline{1+\frac{1}{i}}$$

$$\therefore\ 1=d(1+(a)_{\overline{t-1}|})+(A)_{\overline{t}|}+r^tp_{\overline{(m,m_1,m_2,\&c.)t}}^{\ v},\ \text{since } d=\frac{i}{1+i}.$$

$$\therefore\ (A)_{\overline{t}|}=1-d(1+(a)_{\overline{t-1}|}-r^tp_{\overline{(m,m_1,m_2,\&c.)t}}^{\ v}\ \ .\ .\ .\ .\ .\ \ (1)$$

which is the ordinary formula for the present value of a temporary assurance for t years.

Increase t without limit, the present value of the endowment vanishes, and we have

$$(A)=1-d(1+(a))\ \ .\ .\ .\ .\ .\ .\ .\ .\ \ (2)$$

the ordinary formula for the present value of a whole life assurance.

The annual premium in the two cases is at once obtained by dividing (1) by $1+(a)_{\overline{t-1}|}$, which gives

$$\frac{1-r^tp_{\overline{(m,m_1,m_2,\&c)t}}^{\ v}}{1+(a)_{\overline{t-1}|}}-a\ \ .\ .\ .\ .\ .\ .\ .\ \ (3)$$

for the annual premium of a temporary assurance for t years, and dividing (2) by $1+(a)$, which gives

$$\frac{1}{1+(a)}-d,\ \ .\ .\ .\ .\ .\ .\ .\ .\ \ (4)$$

for the annual premium of a whole life assurance.

From (1) it is at once apparent, that the present value of an endowment assurance of £1, to be paid on the failure of the joint lives of the last v survivors within t years, or at the expiration of that time, should they be then alive, is

$$(A)_{\overline{t}|}+r^tp_{\overline{(m,m_1,m_2^1,\&c.)t}}^{\ v}=1-d(1+(a)_{\overline{t-1}|})\ \ .\ .\ .\ .\ \ (5)$$

The annual premium is obtained by dividing (5) by $1+(a)_{\overline{t-1}|}$, which gives

$$\frac{1}{1+(a)_{\overline{t-1}|}}-d\ \ .\ .\ .\ .\ .\ .\ .\ .\ \ (6)$$

The simple rule then for the annual premium of an endowment assurance is:—Take the reciprocal of the present value of an annuity of £1 for t years, payable in the beginning of each year, less the discount of £1 for one year.

This rule will be found extremely useful, especially in cases where more than one life is involved.

The corresponding commutation formulæ are—

Single premium for an endowment assurance on a single life,

$$=1-d\frac{N_{m-1}-N_{m+t-1}}{D_m};$$

Single premium for a similar assurance on two lives,

$$=1-d\frac{N_{m-1,\,m_1-1}-N_{m+t-1,\,m_1+t-1}}{D_{m,\,m_1}};$$

Annual premium for an endowment assurance on a single life,

$$=\frac{D_m}{N_{m-1}-N_{m+t-1}}-d;$$

Annual premium for a similar assurance on two lives,

$$=\frac{D_{m,\,m_1}}{N_{m-1,\,m_1-1}-N_{m+t-1,\,m_1+t-1}}-d.$$

Again, the present value of a perpetuity of £1 deferred t years, subject to the last v survivors being then alive, equals the present value of an annuity of £1 deferred t years to continue during the joint lives of the last v survivors + the present value of an assurance of £$1+\frac{1}{i}$ deferred t years and payable on the failure of the joint existence of the last v survivors. Hence,

$$r^t p_{\overline{(m,\,m_1,\,m_2,\,\&c.)t}}^{\,v}\frac{1}{i}=(a)_{\rceil t}+(A)_{\rceil t}\left(1+\frac{1}{i}\right)$$

$$\therefore\ A_{\rceil t}=r^{t+1}p_{\overline{(m,\,m_1,\,m_2,\,\&c.)t}}^{\,v}-d(a)_{\rceil t}\quad .\ .\ .\ .\ .\ .\quad (7)$$

and for the annual premium (7) has to be divided by $1+(a)_{\overline{t-1}|}$.

I remain, Sir,

Your obedient servant,

MARCUS N. ADLER.

London, 15*th March*, 1864.

ON THE PAYMENT OF $\frac{1}{m}$ YEARLY PREMIUMS.

To the Editor of the Assurance Magazine.

Sir,—I find in No. 54 of the *Assurance Magazine* Mr. Laundy's method of obtaining half-yearly and quarterly premiums from the annual premium. He derives it very skilfully from Mr. Orchard's expression given in the introduction of his valuable work, *Single and Annual Assurance Premiums.* I am much surprised to see Mr. Laundy availing himself of this expression, since another is nearer and more directly to be got at. Having, however, not found it till now in any work, it might prove useful to publish it here for general use.

Putting, for the age of x years,

π as the annual premium on a premium paid half-yearly,
p " " " " annually,
$a'^{\frac{2}{2}}$ as the annuity to be paid half-yearly in advance,
a' " " " annually "

we find

$$\pi a'^{\frac{2}{2}}=pa'\quad .\ .\ .\ .\ .\ .\ .\ .\ .\quad (1).$$

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