Ergod. Th. & Dynam. Sys. (1989), **9**, 399–401
Printed in Great Britain

BOOK REVIEW

ERGODIC THEORY AND DIFFERENTIABLE DYNAMICS

By RICARDO MAÑÉ: Translated from the Portuguese by Silvio Levy. Ergebnisse de Mathematik und ihrer Grenzgebiete, 3 Folge-Band 8. Springer-Verlag 1987.

This book is the eagerly awaited translation into English of the I.M.P.A. monograph written in Portuguese and not easily available outside Brazil. The aim of the book is to describe the rudiments of measure-theoretic ergodic theory and apply these to the qualitative study of diffeomorphisms.

The reader is assumed to have some knowledge of differential topology and geometry (for example 'regular value', 'the d operator on forms', '$\partial\psi/\partial X$' where X is a vector field and ψ is a function, are all used without any definitions) but measure theory is described from scratch. Measure-theoretic ergodic theory is discussed in Chapters I and II and some interesting examples are studied. These chapters contain nice descriptive sections on twist maps, K.A.M. theory and billiards. The remaining two chapters of the book contain the smooth ergodic theory.

Chapter III is largely devoted to proving Ruelle's ergodic theorem for Anosov diffeomorphisms. This result concerns a topologically transitive, $C^{1+\alpha}$, Anosov diffeomorphism f of a compact manifold M and asserts the existence of an f-invariant Borel probability measure μ^+ on M such that for almost every $x \in M$ *with respect to smooth measure* we have

$$\frac{1}{n}\sum_{i=0}^{n-1} \varphi(f^i x) \to \int \varphi \, d\mu^+$$

for every continuous $\varphi: M \to R$. Moreover the measure-preserving transformation f on (M, μ^+) is measure-theoretically isomorphic to a Bernoulli shift, and if λ is a smooth probability on M then $\lambda \circ f^{-n} \to \mu^+$ in the weak*-topology on measures. This theorem is proved by obtaining a generalised expanding map on a union of some local unstable manifolds (by using f and projections along local stable manifolds) and proving that these generalised expanding maps preserve a probability equivalent to a smooth measure. This measure is then used to obtain μ^+. The absolute continuity of the stable foliation is proved. The existence of stable manifolds and Markov partitions is assumed. The generalised notion of expanding map includes some important maps of $[0, 1]$, such as the Gauss map $x \to (1/x) =$ fractional part of $1/x$, as well as the usual expanding maps of manifolds.

Lyapunov exponents are studied in Chapter IV and proofs of Oseledec's theorem, Ruelle's inequality, and Pesin's formula are given. The proof of Oseledec's theorem is more geometric than the usual Raghunathan–Ruelle–Ledrappier proof (see

'Random Matrices and their Applications' AMS Contemporary Mathematics, Volume 50, pp 23–30).

The choice of material in the book is excellent and the book would be exemplary were it not for a couple of factors. The first, and comparatively minor, point is that certain mathematical words are used in ways contrasting with the usual accepted uses. For example the word 'automorphism' is used where most people use 'endomorphism'. This is so for endomorphisms of a torus (see pages 48 and 166) and could lead to confusion. It also happens for measure-preserving transformations where any measure-preserving transformation (whether invertible or not) is called an automorphism. This leads, on page 158, to a definition of 'exact automorphism' which is enough to send ergodic theorists in search of a soothing anodyne.

The second issue, which greatly reduces the usefulness of the text, is that a decent proof-reading job has not been done. Inevitably most books have typographical errors (and even untrue statements) but this book goes way beyond this. Inequalities are the wrong way around on several occasions (e.g. the last inequality on page 214; one should have $K>0$ in the definition on page 251; on page 252 the expressions for $W^s_{\varepsilon_0}(\mathcal{O})$ and $W^u_{\varepsilon_0}(\mathcal{O})$ should have $n\geq 0$ and $n\leq 0$ respectively; etc.), the wrong symbols sometimes appear (on page 258 Lemma 9.8 should read $\partial R\subset \partial^s R\cup\partial^u R$; the last term in the definition of Markov partition on page 255 should be $W^u(f^{-1}(x), R_j)$; equation (5) on page 157 should be $T^{-1}\mathcal{A}_0\subset\mathcal{A}_0$; on page 231 '$A^n=0$' should be '$A^n>0$'; etc.), absolute value signs are missing in several places (e.g. in the definition of weak mixing on page 147) and should be missing in others (e.g. in the definitions of $E^s(x)$, $E^u(x)$ on page 266). When commutative diagrams are interpreted in symbols the wrong conjugacy relation is sometimes written (e.g. pages 67, 78), and diagrams are sometimes badly labelled (e.g. on page 183 '$f(R_j)$' should be replaced by '$f(R_i)$' and it would help to have the 's' and 'u' directions marked). The result of Misiurewicz, mentioned on page 238, gives examples where the maximum is *not* reached, and Bowen was not the first to show ergodic toral automorphisms are intrinsically ergodic (see page 266) as Berg, in 1968, proved that ergodic automorphisms of compact abelian groups are intrinsically ergodic. (*Math. Systems Theory 3*, 146–150 (1969).) In several places one is referred to the wrong previous result; on page 74 the reference about expansive homeomorphisms of surfaces should be [03] not [02], on page 137 the proof is of Lemma 7.3 not Corollary 7.3, on page 149 one is referred to [D3] which does not appear in the bibliography.

There are so many of these proof-reading errors in Chapters III and IV that the book is seriously limited as a learning tool. For example the definition of regular point, on page 263, should also include $n\to-\infty$. To see the difficulties facing a student, open the text at pages 268/9. Correct the inequalities at the foot of page 268 and write down the correct definition of $C_\varepsilon(x)$. Adjust the definition of distortion on page 269 to make sense of the subsequent statements and try to find a true inequality involving the symbols in inequality (3). The statement of Proposition 11.2 has '$1/n$' missing in the two limits, and the statement of Lemma 11.3 should have '$C\circ T-C$ is integrable' rather than '$T\circ C-T$'.

It is a great shame that such insight and style is masked by the lack of proof-reading. I think the publisher should produce a new proof-read edition as soon as possible, and give a free copy to everyone who had bought the first edition. The new edition would be a tremendous book; the text has graceful style and is full of interesting insights and observations, and the new edition would deserve a favoured position on the bookshelf of every reader of this journal.

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