Sensitivity of an Atomic Force Microscope Cantilever with a Crack

Haw-Long Lee, Yu-Ching Yang, and Win-Jin Chang

Department of Mechanical Engineering, Kun Shan University, No.195, Kunda Rd., Yongkang Dist., Tainan 71003, Taiwan

The atomic force microscope (AFM) has become an essential tool for the measurement of surface characteristics of diverse materials on a micro- and nanoscale level [1]. The resolution of measurements for the AFM cantilever is related to its vibration sensitivity. Many researchers have much interest in studying the resonant frequency and sensitivity analysis of AFM cantilevers [2 - 4].

Cracks may be induced in an AFM cantilever during the fabrication process [5]. Cracks will affect the sensitivity of AFM cantilevers during scanning. However, there are very few literature reports on the crack effect of AFM cantilevers. In this article, we analyze the sensitivity of AFM cantilever using the modified couple stress theory [6]. Unlike the classical continuum theory, the modified couple stress theory includes one additional material length scale parameter revealing the micro-scale effect on the response of structures to estimate the size-dependent behaviours. Here the effects of the length scale parameter and crack parameter on the sensitivity of a cracked AFM cantilever are investigated.

An AFM probe is considered as a cantilever beam which have the density ρ, Poisson's ratio μ, uniform cross-section A, thickness h, and length L. Assume the probe has a crack at a distance D from the fixed end as shown in Fig. 1. Therefore, the cantilever will be divided into two segments by the crack. A rigidity of the rotational spring simulated the cracked section to connect the two segments of the cantilever. The governing equations of transverse vibration of the cantilever with a crack can be expressed as [3]

$$(EI + GAl^2)\frac{\partial^4 Y_1}{\partial X^4} + \rho A \frac{\partial^2 Y_1}{\partial t^2} = 0 \quad for \quad 0 \le X \le D \quad \text{and} \quad (EI + GAl^2)\frac{\partial^4 Y_2}{\partial X^4} + \rho A \frac{\partial^2 Y_2}{\partial t^2} = 0 \quad for \quad D \le X \le L \qquad (1)$$

where E and G are the Young's modulus and shear modulus, respectively; I is the area moment of inertia; l is the material length scale parameter which indicates the size-dependent behavior of the microcantilever based on the modified coupled stress theory; X is the distance from the fixed end along the center of the probe, t is time, $Y_1(X,t)$ and $Y_2(X,t)$ is the transverse displacement of both segments, respectively.

For compatibility of displacement, moment and shear force for the two adjacent portions of the cantilever can be expressed by the following jump conditions as

$$Y_1(D,t) = Y_2(D,t) \,, \partial^2 Y_1(D,t)/\partial X^2 = \partial^2 Y_2(D,t)/\partial X^2 \,\text{,and} \quad \partial^3 Y_1(D,t)/\partial X^3 = \partial^3 Y_2(D,t)/\partial X^3 \qquad (2)$$

The crack is simulated by a rotational spring and its spring constant W. The angular displacement between the two segments can be related by $W(\frac{\partial Y_2(D,t)}{\partial X} - \frac{\partial Y_1(D,t)}{\partial X}) = (EI + GAl^2)\frac{\partial^2 Y_1(D,t)}{\partial X^2}$. In addition, the boundary conditions of two ends of the AFM cantilever are as follows:

$$Y_1(0,t) = \frac{\partial Y_1(0,t)}{\partial X} = 0 \,, (EI + GAl^2)\frac{\partial^2 Y_2(L,t)}{\partial X^2} = -H^2 K_l \frac{\partial Y_2(L,t)}{\partial X} - MB^2 \frac{\partial^3 Y_2(L,t)}{\partial X \partial t^2} \,, (EI + GAl^2)\frac{\partial^3 Y_2(L,t)}{\partial X^3} = K_n Y_2(L,t) + M \frac{\partial^2 Y_2(L,t)}{\partial t^2} \quad (3)$$

where H and M are the tip height and mass, respectively. B is the distance between the lower edge of the probe and centroid of the cross section. The linear spring constants K_n and K_l denote the contact stiffness between the tip and the sample under the normal and lateral direction, respectively.

In order to know the effect of relative parameters on the sensitivity of a cracked AFM cantilever, we considered the geometric and material parameters of a Si cantilever as follows [3]: E = 170 GPa, μ =0.28, ρ=2330 kg/m^3, L=300//m, B = 2.5 μm, H = 10 μm, M = 2 χ 10^{-13} kg. The lateral contact stiffness was assumed as K_l = 0.9 K$_n$[3]. When the above governing equations with the corresponding

conditions are solved, the sensitivity of the AFM cantilever can be obtained. Fig. 2 depicts the dimensionless sensitivity of mode 1 as functions of the microcantilever thickness to material length scale parameter h/l and dimensionless crack flexibility k_c (= EI/ WL) for H/L =1/30, D/L=0.5, and $-\beta_n = K_nL^3/EI$ = 0.1. The ratio h/l represents the size effect of the cantilever based on modified couple stress theory. The change in sensitivity with the value of h/l indicates the thickness-dependent behavior of an AFM cantilever. The cantilever becomes more sensitive for higher values of h/l. It is noted that k_c = 0 implies no crack in the cantilever. The crack existing in the cantilever makes it locally less stiff because of the added flexibility. Therefore, the sensitivity of the cracked cantilever increases with increasing value of crack parameter k_c.

References:
[1]G. Binnig, C. F. Quate, and C. Gerber, Phy. Rev. Lett. 56 (1986) p.930.
[2]M.H. Kahrobaiyan *et al,* Int. J. Eng. Sci. 48 (2010) p.1985.
[3]H.L. Lee and W.J. Chang, Jap. J. Appl. Phys. 51(2012) p.035202.
[4]A.F. Payam and M. Fathipour, Arab, J. Sci, Eng, 39 (2014) p.1393.
[5]C. Shafai, M.J. Brett, and T.M. Hrudey, Sensors Actu. A 70 (1998) p.283.
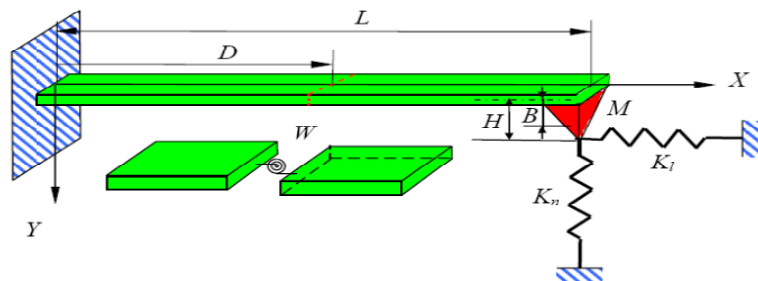[6]F. Yang *et al,* Int. J. Solids Struct. 39 (2002) p.2731.

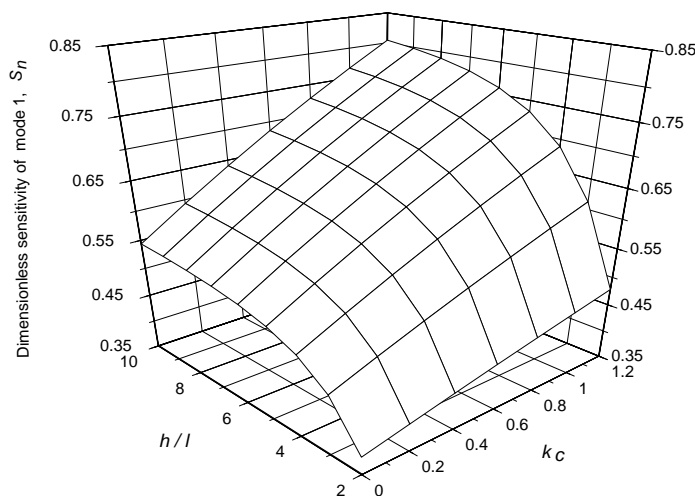Fig. 1. Schematic diagram of a cracked AFM cantilever in contact with a sample.

Fig. 2. Dimensionless sensitivity of mode 1 for H/L =1/30, D/L=0.5, and β_n = 0.1.